

ANALYSIS OF PROPAGATION OF FATIGUE CRACK ON GEARS ON THE  
BASE OF EXPERIMENT

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Gearing of different sizes and with different manners of loading are indispensable component of machines and devices. When optimizing such gearing, one of the most important parameters is their service life. In order to calculate the service life as precisely and reliably as possible we need good mathematical models for describing loadings, geometry, properties of materials and fracture mechanics parameters. Because of particularity of the elements dealt with the accuracy and reliability of these models can be compared only with appropriate experimental results. To this end we have used and developed a series of methods and test pieces. Comparisons of results show that the used models and approaches are good since they have fulfilled our expectations.

INTRODUCTION

For designing machines and devices the dimensioning with respect to service life is increasingly taken into account. This applies also for gearing which are still today one of very important components of almost all machines.

Various standards such as American AGMA standards or German DIN standards already indicate the possibility of dimensioning the so-called time gearing. However, all these standard models are rough and give only approximate results since they do not take into account the actual conditions. Therefore we decided to develop models and procedures of calculations, that will give more reliable and, in particular, more accurate results. We limited ourselves to the tooth root, i.e., the defects occurring there. In practice the following two basic problems arise when calculating the service life:

- a) During designing, the individual elements and the entire products are optimized particularly with respect to the service life.

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The basic requirement is that the service lives of the individual elements are approximately equalised.

b) A defect, i.e., a crack is detected during periodic inspection by non-destructive method.

For this two basic problems we are develop suitable mathematical models.

### MATHEMATICAL MODELS

In our model we propose a new parameter by which we describe the fracture mechanics conditions in the tooth root where the defects, occur statistically most frequently as shown Figure 1. We must emphasise that this factor does not apply in general, but only for gears and for calculation of conditions in the tooth root, i.e., for calculation of their service life. We named that factor the tooth stress intensity factor Z. There are three forms of the factors Z:

- If  $a = 0$ , i.e., if there is no crack, the factor Z is proportional to the stress concentration factor by Peterson (1) occurring in the tooth root,
- If  $0 < a < \gg 10$  grains (according to Taylor (2)), the factor Z depends on local plasticizing of grain, grain size, shear module, grain orientation, shape factor,
- If  $a > 10$  grains, such defect can be considered to be a long crack or a linear elastic fracture mechanics type crack is obtained.

When we want to write the tooth stress intensity factor in a more compact form for all three areas, the equation obtains a more complex form Aberšek and Flašker (3):

$$Z = \left( 0.2 \sigma + e^2 \right) \ln \sqrt{\pi a} \left| \frac{\sigma Y(a/S) \sqrt{\pi a} (\sigma Y(a/S)) \sqrt{\pi a} - K_{th}}{K_{th}} \right| \quad (1)$$

#### Calculation of service life of gears

Lifetime calculation of crack are carried out with the following expression :

$$\Delta N_i = \int_{a_{j-1}}^{a_j} \frac{1}{C (\Delta Z)^m} da \quad (2)$$

The factor  $C$  and exponent  $m$  are the random function of material constants and are approximately identical to the two material functions of the Paris equation for long crack.

#### DETERMINATION OF CRACK GROWTH RATIO - TEST RESULTS

It can be established whether a mathematical model is precise and reliable only by a comparison of results of the methods such as analytical methods in case of simple problems and experimental method when real complex structures are dealt with. Since gears and gearings belong to the second group by correctly selected and developed test pieces and carefully planned experiments we obtained results with which we confirmed and justified the use of the presented mathematical models (3) for calculating the tooth stress intensity factor  $Z$  as well as the service life of gears.

To confirm mathematical models we used two types of test pieces:

- a.) Standard test piece for determination of fracture mechanics parameters of material AISI 4130 in the area of initiation and growth of short cracks according to ASTM-E606, and the previous tests on three point test piece.
- b.) Non-standard test pieces for determination of parameters of gears. This type of test piece were gear test pieces developed during previous researches by Aberšek et al (4).

As we wanted to observe particularly initiation and propagation of short cracks and to determine the conditions occurring in this case (3), we used mixed experimental methods, and this methods were combined as follows:

- photoelastic examination with strain and crack gauges
- strain and crack gauges and
- gauges with replica method.

The aim of these combinations was to obtain as complete information about the individual influences as possible and to determine the interaction between these magnitudes.

#### CONFIRMATION OF MODELS AND COMPARISON OF RESULTS

Accuracy of the mathematical model was compared with experiment results. We have used the following data for gear: material AISI 4130, module  $m = 10$  mm, number of teeth  $N = 18$  teeth, dimensions in critical cross section  $B \times S = 20 \times 15,7$  mm according to Figure 1. On the basis of experiment and by applying standard methods we determined the diagram  $\Delta K - a$  and  $\Delta K - da/dN$  and we compared it with a similar calculated diagram i.e.  $\Delta Z - a$  and  $\Delta Z - da/dN$ . The comparison between the

methods described above is shown in Figure 2. Figure 3 shows a comparison of the results of crack propagation.

### CONCLUSION

It can be seen above that for accurate calculations of service life of gearing the use of deterministic methods according to different standards is not sufficient. Therefore, for calculations we have developed the gear stress intensity factor  $Z$  which takes into account not only the stress of homogeneous material in critical cross section but also the stress concentration due to defects occurring there. On the basis of the factor  $Z$  we have developed two models, i.e., the model of initiation and growth of short cracks and the model of growth of long cracks by means of which it is possible to calculate accurately the actual service life.

Comparisons of results show that the used models and approaches are good since they have fulfilled our expectations.

### SYMBOLS USED

$a$  - real crack length  
 $C, m$  - material dependent random variables  
 $Y(a/S)$  - shape factor  
 $Z$  - Tooth stress intensity factors  
 $\sigma$  - General sign for stresses  
 $K_{th}$  - Threshold stress intensity factor  
 $B$  - Gear width  
 $S$  - gear tick

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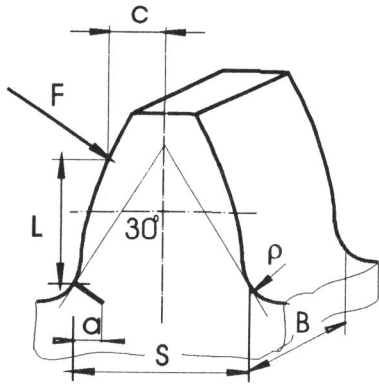


Figure 1: Critical cross section

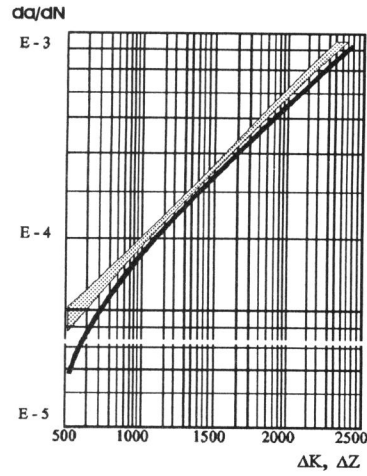


Figure 2: Comparison of results

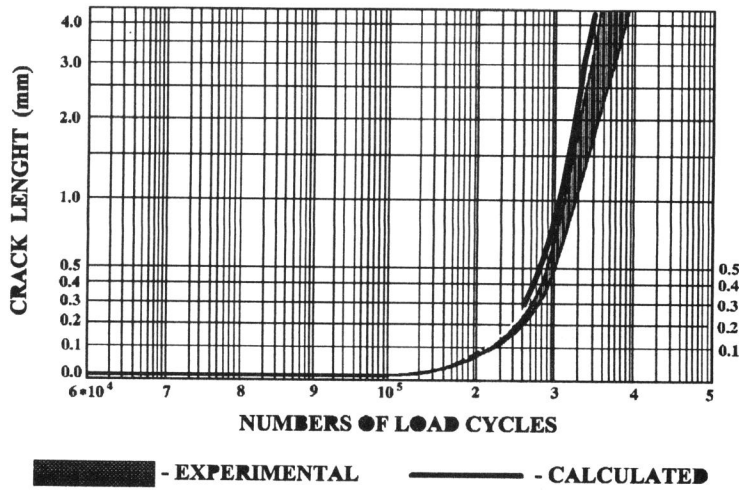


Figure 3: Lifetime curves for gear tooth