ANALYSIS OF ORTHOGONALLY CRACKED LAMINATES UNDER SHEAR LOADING

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In long fibre composite cross-ply laminates submitted to shear loading, matrix cracking appears in both 0° and 90° layers. The number of such cracks increases throughout both quasi-static and fatigue tests. A 2-D shear lag analysis for progressive damage in composite laminates is then developed to model this crack geometry. A general governing equation is derived for $[0_m/90_n]_s$ laminates that involves only in-plane stresses in both layers. Stress distributions, elastic constants and strain energy release rate are obtained as functions of the crack densities in 0° and 90° layers of the laminate. Experiments give the critical value of the strain energy release rate associated with damage initiation.

INTRODUCTION

In the aeronautical industry, designers need good estimates of the performance of structural parts made from polymer-matrix composite materials. Under both quasi-static and fatigue loading, the first damage mechanism to occur is generally matrix cracking. It can be tolerated, but its development must be carefully controlled, as it is usually followed by other harmful degradation mechanisms which can entail the ultimate failure of the part.

So, many authors put their interest in developing models allowing the evolution of damage in long fibre composite laminates to be predicted.

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Numerous models are focusing on the case of cross-ply laminates submitted to different loadings, but with cracks only in the 90° layers. Very few models are studying cross-ply laminates with cracks in both 0° and 90° directions (Hashin (1), Tsai and Daniel (2))

The shear lag analysis presented here is part of a model whose aim is to predict matrix cracking evolution in cross-ply laminates submitted to general in-plane and thermal loadings, leading to cracks in every layer (Henaff-Gardin et al (3)). This paper will adress the case of pure in-plane shear loading.

ASSUMPTIONS ON CRACK GEOMETRY AND DISPLACEMENTS

Schematic Representation Of The Cracked Laminates

In the present analysis, the studied cross-ply laminates are assumed to present a symmetric and periodic stacking sequence such as $[0_m/90_n]_s$. The laminate is subjected to a global shear stress $\overline{\tau_{xy}}$ in the x-y mean plane, z being the ply thickness direction (see Figure 1). The total 0° and 90° ply thicknesses are denoted $2t_0$ and $2t_{90}$ respectively.

According to experimental observations (Tahiri et al (4), Henaff-Gardin et al (5)), the damage pattern consists of a regular array of cracks along both 0° and 90° plies. In order to represent in a simple analytical way the crack distribution in the specimen, we have chosen two damage parameters : the crack density d_{90} in the inner 90° layer, and d_0 in each outer 0° layer. The cracks are spanning either the entire specimen width or length.

Then, we can consider that the laminate, which presents a double periodicity along both x and y axes, is constituted by a double stacking of "repeating unit cells" limited by two pairs of consecutive cracks in 0° and 90° plies (Figure 1).

Displacement Fields

Let us consider the displacements u and v, respectively in the x and y directions in the inner 90° layer (see Figure 2). We assume that, along the 90° layer thickness, u has a parabolic evolution while v is a constant.

The displacement expressions are analogous in the 0° plies. These assumptions are in accordance with the results of a FEM analysis (El Mahi et al (6)).

EXPRESSION OF THE SHEAR STRESSES

We want to obtain the expression of the mean shear stress averaged over the ply thickness in the 0° plies τ_{xy}^{0} as function of the x and y variables. Stress in the 90° layer τ_{xy}^{90} is then obtained by writing the laminate equilibrium, as follows :

$$(t_0 + t_{90})\overline{\tau_{xy}} = t_0 \tau_{xy}^{0} + t_{90} \tau_{xy}^{90} \qquad (1)$$

Deriving the equilibrium conditions in the layers, and expressing the constitutive equations, allow us to obtain the following differential equation for τ_{xy}^{0}

$$t_0^2 t_{90} \frac{\partial^2 \tau_{xy}^{0}}{\partial x^2} + t_0 t_{90}^2 \frac{\partial^2 \tau_{xy}^{0}}{\partial y^2} = 3 \frac{G_{23}}{G_{12}} (t_0 + t_{90})\left(\tau_{xy}^{0} - \overline{\tau_{xy}}\right) \qquad (2)$$

where G_{23} and G_{12} are the out-of-plane and in-plane shear moduli of a ply.

The knowledge of boundary conditions gives us the expression of the shear stress τ_{xy}^{0} :

$$\tau_{xy}^{0} = P \cosh(\psi_x x) + Q \cosh(\psi_y y) + \overline{\tau_{xy}} \qquad (3)$$

where ψ_x, ψ_y, P and Q depend on the elastic constants of an uncracked ply, and on geometric and damage parameters. An analogous expression for τ_{xy}^{90} is then derived from equation (1).

Examples of shear stress fields are given in Figure 3. The maximum values are observed, in 0° layer, at the mid-distance between the 0° cracks, in the vicinity of the 90° cracks. An analogous observation can be made in the 90° layers.

STRAIN ENERGY RELEASE RATE - MODE II OPENING

A fracture mechanics analysis is then applied to develop a tool capable of predicting the onset of matrix cracking. The cracked surface variation is related to the strain energy release rate, G, which is defined as :

$$G = \left(\frac{\partial U_{el}}{\partial S_c} \right)_{\sigma} \qquad (4)$$

where U_{el} and S_c are respectively the stored elastic energy and the cracked surface area in an unit volume of the laminate.

In the case of shear loading, where the crack initiation is mainly due to mode II opening, the strain energy release rate becomes :

$$G_{II} = \frac{t_0 + t_{90}}{2} \frac{\partial \left(\frac{\overline{\tau_{xy}}^2}{G_{xy}^c} \right)}{\partial (t_0 d_0 + t_{90} d_{90})} \qquad (5)$$

where G_{xy}^c, the shear coefficient of the cracked laminate, is obtained after applying the classical laminate theory. G_{xy}^c depends on d_0 and d_{90}, and, as expexted, decreases with increasing crack densities.

EXPERIMENTS : UNIAXIAL TENSION ON $[+45_2/-45_2]_{2s}$ LAMINATE

Quasi-static and fatigue tensile tests have been performed on T300/914 $[+45_2/-45_2]_{2s}$ laminates (Tahiri et al (4)). In a first approach, the laminate $[+45_2/-45_2]_{2s}$ under tension is considered similar to $[0_2/90_2]_s$ under shear loading. During quasi-static tests, the first ply failure stress has been determined. The model then allows to obtain an estimation of the critical strain energy release rate value under mode II opening, G_{IIc} ~ 280 J/m^2 (the corresponding value with this model under mode I opening being G_{Ic} ~ 84 J/m^2) : the damage initiation is, as expected, far much difficult under mode II than under mode I.

Work is in progress in order to obtain a propagation law under fatigue loading. Such a law has already been obtained under mode I loading, based on experiments on different cross-ply laminates (to be published). Note that the thermal residual stresses can easily be taken into account (Henaff-Gardin et al (3)).

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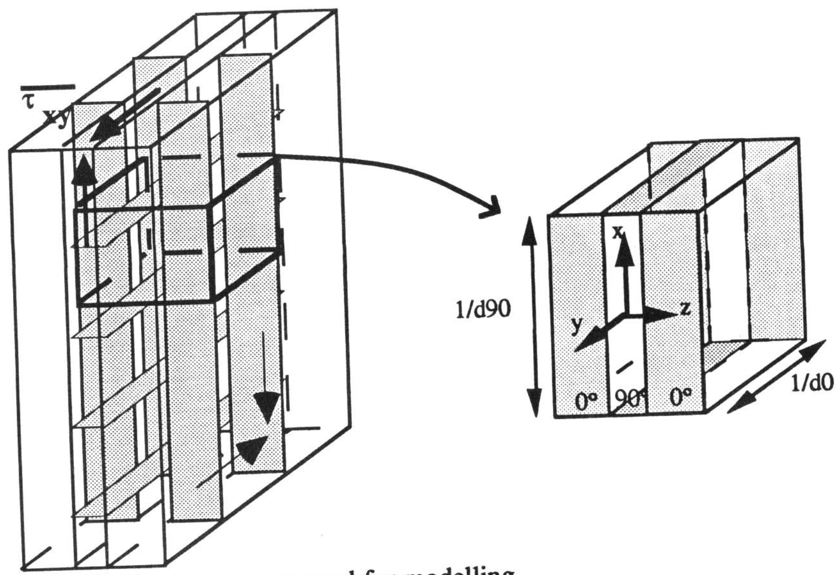

Figure 1 Crack geometry used for modelling the matrix cracking under shear loading

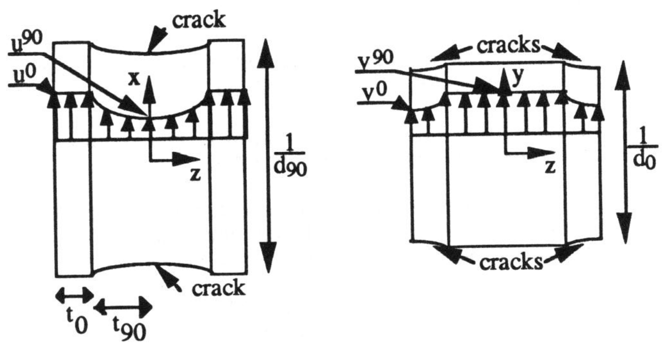

Figure 2 Aspect of the displacement fields in the representative unit cell

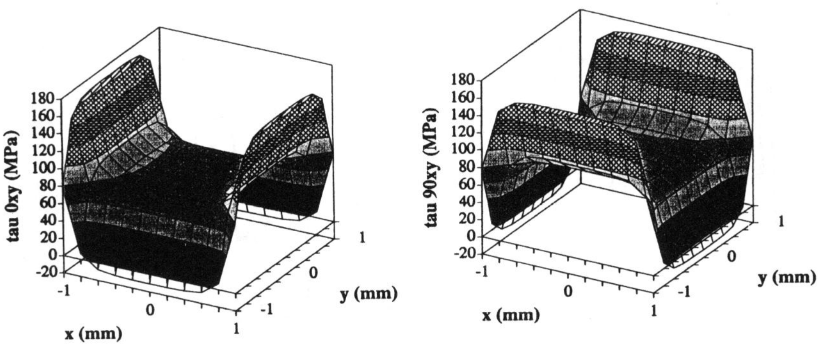

Figure 3 Example of shear stress fields in cracked 90° and 0° layers of $[0_2/90_2]_s$ laminate, $\overline{\tau_{xy}} = 78$ MPa