ANALYSIS OF J-INTEGRAL DEPENDENCE ON STRESS HARDENING EXPONENT FOR FINITE NON-LINEAR ELASTIC CRACKED BODY

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The Calladine and Drucker theorem of nesting surfaces is applied to power-law hardening cracked body. For simple geometry of a body with a crack the energy of the body and *J*-integral are expressed via a parameter of remained load. When the stress hardening exponent tends to infinity the remained load tends to the limit load. This determines the character of dependence of *J*-integral on the size of the crack and stress hardening exponent. The expression of *J*-integral is presented in the terms of load ratio and the strain energy. A simple conservative estimation of *J*-integral via the parameters of reference stress for any value of hardening exponent is presented.

INTRODUCTION

The estimation of *J*-integral for plastic bodies is connected with essential difficulties, that are common in the theory of plasticity. In reference (1) it was proposed the approximation for J-integral in the form:

$$J = J_e(a_{eff}) + J_p, \tag{1}$$

where $J_e(a_{eff})$ is the value of *J* for linear part stress-strain relation, a_{eff} is the size of the crack with Irvin type correction to account the size of contained plastic zone, and J_p is the value for identical body from material correspond non-linear relationship between equivalent stress and strain of the form:

$$\varepsilon_{eq}/\varepsilon_0 = \alpha(\sigma_{eq}/\sigma_0)^n . \tag{2}$$

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In the case of general plasticity *J*-integral and load-point displacement due to the crack Δ_c may be expressed in terms of dimensionless functions h_1, h_3 as (1):

$$J = \alpha\varepsilon_0\sigma_0 b h_1(a/w,n)(P/P_0)^{n+1}, \tag{3}$$

$$\Delta_c = \alpha\varepsilon_0\sigma_0 b h_3(a/w,n)(P/P_0)^{n+1}. \tag{4}$$

Where a is the crack length, w is the section thickness, b=w-a, P is applied load and P_0 is the normalizing load. The General Electric Fracture Handbook (1) contains tabulated functions which are normalized finite-element solutions for J for power-law plastic materials.

Using this numerical data the reference stress approach has been developed by Ainsworth (2). It is based on empirical approximation:

$$h_1(a/b,n)/P_0^{\,n+1} = h_1(a/b,1)/P_y^{\,n+1}. \tag{5}$$

The comparison of these approaches in Ref. (3) shows good agreement of estimated load from the viewpoint of practical applications. However, the non-conservatism of the referens stress approach in compare with the finite-element results is observed for small cracks.

J-ESTIMATION AT FIXED BOUNDARY DISPLACEMENT

Calladin and Drucker (3)-(5) showed that in the case of power law material the complementary energy in finite body may be presented in the form:

$$\Omega(P) = \frac{\alpha\varepsilon_0\sigma_0}{(n+1)}\left(\frac{P}{P_y}\right)^{n+1} g_n^{\,n+1}\, V,$$

$$g_n \le g_m \le 1, \quad n < m, \tag{6}$$

where P_y is the limit load, V — is the volume of the body. Respectively, the *J*-integral can be written as:

$$J = \frac{d}{dl}\Omega(P)\Big|_{P = const}. \tag{7}$$

In order to analyse the dependence of *J*-integral on the stress hardening exponents it is necessary to have a relation between g_n and the size of the crack.

To obtain a simple expression for g_n let us consider the case of finite plate with central crack (Fig.1). In order to evaluate the energy of the cracked body let us close the crack by applying the additional stress acting on its surface at fix boundary displacement (Fig.2). On the base of energy conservation law we obtain:

$$W(a,\Delta) = W(0,\Delta) - \int_L \frac{n}{n+1} \sigma_{ij} n_j \Delta u_j \, dl, \tag{8}$$

where σ_{ij} is the stress component in the body without crack with displacement Δ on the bound. Here $W(a,\Delta)$ is elastic (nonlinear) energy in the body. For the finite plate with crack this equation reduces to the following one:

$$W(a,\Delta) = W(0,\Delta)\left(1 - \frac{V_c}{4w\Delta}\right), \tag{9}$$

where V_c is the volume of the crack. The parameter $V_c/(4w\Delta)$ depends only on the form and size of the body and stress hadening exponent. From the equation (9) one can obtain:

$$\left(\frac{\Delta_{nc}}{\Delta}\right)^{1/n} = 1 - \frac{V_c}{4w\Delta}. \tag{10}$$

Let P_r be a remained load that is the load that remained from $P_b = 4w\sigma_0/\sqrt{3}$ after appearance of the crack with the length $2a$. Taking into account the non-compressibility of the body Δ_c can be decomposed by two components:

$$\Delta_c = \frac{V_c}{4w} + \frac{V_b}{4w}. \tag{11}$$

Here V_b is the volume caused by additional displacement on edges due to the crack. Moreover, the second component in equation (11) according to (10) equal zero at n=1. Using equations (7) and (9) J-integral can be presented in the form:

$$J = \alpha\varepsilon_0\sigma_0 \frac{n}{n+1}\left(\frac{P}{P_r}\right)^{n+1}\left(-\frac{\partial(P_r/P_b)}{\partial\xi}\right)H, \quad \xi = a/w. \tag{12}$$

Here P_r can be expressed as:

$$P_r / P_b = 1 - \frac{V_c}{4w\Delta} = \left(1 - \frac{V_b}{4w\Delta_{nc}}\right)^{2/(n-1)} . \tag{13}$$

According to (6) and (13) P_r/P_y is monotonously decreasing function of n and tends to unity if $n \to \infty$.

The expression (12) one can present in the terms of load ratio P/P_y or strain energy W:

$$J(P) = \alpha\varepsilon_0\sigma_0 h_1(a/w,n) b \left(\frac{P}{P_y}\right)^{n+1} , \tag{14}$$

$$J(W) = \eta(a/w,n) W/(2b) , \tag{15}$$

where

$$h_1(\xi,n) = -\frac{n}{n+1}\left(\frac{P_y}{P_r}\right)^{n+1} \frac{\partial(P_r/P_b)}{\partial\xi}\frac{H}{b} , \tag{16}$$

$$\eta(\xi,n) = -\frac{\partial(P_r/P_b)}{\partial\xi}\frac{(1-\xi)}{2}\frac{P_b}{P_r} . \tag{17}$$

SOME APPROXIMATIONS FOR J

When n tends to infinity then according to (13) and (17) 2η tends to unity excepting the cases l=0 and l=w. Additional consideration shows that in the case l=w 2η tends to unity too. From equation (16) in the case n tends to infinity we obtain:

$$h(a/w,n) = \xrightarrow[n\to\infty]{} \exp\left(\frac{-\left(\partial P_r / \partial\mu\right)\Big|_{\mu=0}}{P_0}\right)\frac{H}{b} , \quad \mu = \frac{1}{n} \tag{18}$$

For correct estimation of $h(a/w,n)$ according to equation (18) it is necessary to know the dependence of P_r on n. Let us assume that in the case n tend to infinity $\Delta(P_y)/\Delta(P_r) \to b/H$. Tnen

$$h = \frac{\Delta(P_y)}{\Delta(P_r)}\frac{H}{b} \to 1 \tag{19}$$

If $l/w \to 0$ we can use the known expression for J for crack in infinite plate (6):

$$J = \alpha\varepsilon_0\sigma_0\sqrt{n}\left(\frac{P}{P_b}\right)^{n+1}\pi a \qquad (20)$$

On the base of equations (19) and (20) we proposed the following approximation for J :

$$J = \alpha\varepsilon_0\sigma_0\left(\frac{P}{P_y}\right)^{n+1} h_1\left(\frac{\sqrt{na}}{\sqrt{na}+b},1\right) b \qquad (21)$$

COMPARISONS AND CONCLUSION

The comparison of relation (21) with Ainsworth approach and numerical calculations (1) is shown in Fig. 3 in terms of parameter:

$$F_p = (h_1(a/w,n)/h_1(a/w,1))^{1/(n+1)} \qquad (22)$$

This ratio in Ainsworth approach is equal to 1. For curves that lie over F_p=1 (Ainsworth approach) the value of J is greater than the value of J proposed by Ainsworth and vice versa. As it can be seen from Fig.3 for small l/w we obtains good agreement with numerical calculations (1). For any value of l/w the presented estimation is conservative. The deviations between results obtained from (21) and numerical finite element calculations for all n and l is not greater than 11%. Ainsworth approach yields slightly lower value for J for small cracks.

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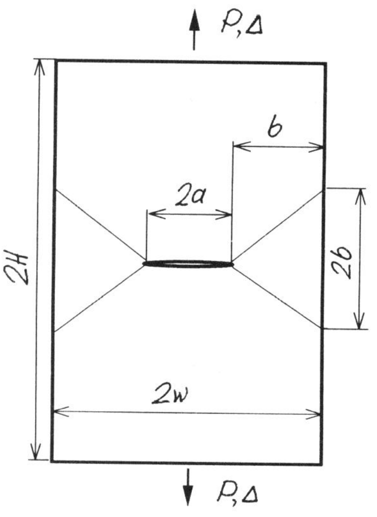

Figure 1 Geometry of central cracked plate and slip lines

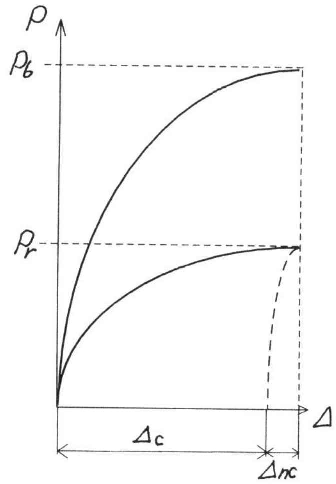

Figure 2 Load—displacement relationship

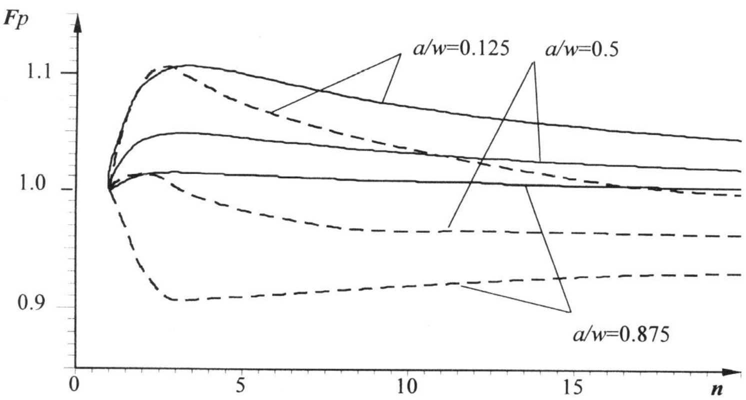

Figure 3. Dependence of F_p on n calculated by finite elements method (dashed lines) and by equation (21) (solid ones)