AN INTERFACE CRACK WITH BONDING IN ITS END REGIONS AND THE ADHESION FRACTURE ENERGY

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A mathematical model of the end region with the linear elastic bonds is considered for interface crack. Normal and shear bond stresses occurring under the action of the external loads are searching for by solving the system of two singular integral-differential equations. Stress intensity factors at the crack tip are calculated taking into account both the external loads and compensating bond tractions. Energetic characteristics such as the energy release rate and rate of the elastic energy dissipation by the bonds are considered. The sensitivity analysis of the force and energetic characteristics to the end region size bond compliance and limit stretching is performed.

INTRODUCTION

The aim of this paper is consideration the mathematical model of the end region of the interface crack. We use bridged crack model and suppose that the crack surfaces interact in some region starting from the crack tip.

A mechanism of the interaction is adjusted by the crack scale and distance from the crack tip as well as by the joint materials microstructure. Intermolecular forces are essential at small distances from the crack tip while "mechanical" bonds are more important at relatively large distances from the crack tip.

STATEMENT OF THE PROBLEM. BASIC EQUATIONS

Let us consider the plane elasticity problem on a crack ($|x| \leq \ell$) at the interface ($y=0$) of two dissimilar joint half-planes. Assume that the uniform load, σ_o normal to the interface is acted at infinity. The crack surface interaction is supposed to be existing in the end regions, $\ell - d \leq |x| \leq \ell$.

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As a simple mathematical model of the crack surfaces interaction we will assume that the linearly elastic bonds act through the crack end regions.

Denote by $\sigma(x)$ the stresses arising in the bonds such that

$$\sigma(x) = \sigma_{yy}(x) - i\sigma_{xy}(x), \quad i^2 = -1, \tag{1}$$

where $\sigma_{yy}(x)$ and $\sigma_{xy}(x)$ are the normal and shear components of the bond stresses. The crack opening, $\Delta u(x)$, at $\ell - d \le |x| \le \ell$ is determined as follows

$$\Delta u(x) = \Delta u_y(x) - i\,\Delta u_x(x) = \frac{H}{E_b}(\varphi_1(x)\sigma_{yy}(x) - i\varphi_2(x)\sigma_{xy}(x)), \tag{2}$$

where Δu_x, Δu_y are the projections of the crack opening on the coordinate axes, H is a linear scale related to the thickness of the intermediate layer adjacent to the interface, E_b is the effective elasticity modulus of the bond and $\varphi_{1,2}$ are the dimensionless functions of the coordinate x.

By incorporating the problem linearity one can represent the crack opening as follows

$$\Delta u(x) = \Delta u_o(x) - \Delta u_\sigma(x), \tag{3}$$

where $\Delta u_\sigma(x)$, Δu_o are the crack opening caused by the bond stresses $\sigma(x)$ and external loads σ_o. According to (England (1)) the value Δu_o is equal to

$$\Delta u_o(x) = 2A\sigma_o\sqrt{\alpha}\sqrt{\ell^2 - x^2}\left(\frac{\ell - x}{\ell - x}\right)^{-i\beta}, \alpha = \frac{\mu_1 + \mu_2\kappa_1}{\mu_2 + \mu_1\kappa_2}, \tag{4}$$

where $\kappa_{1,2} = 3-4\nu_{1,2}$ and $(3-4\nu_{1,2})/(1+\nu_{1,2})$ for the plane strain and plane stress, respectively, $\nu_{1,2}$ and $\mu_{1,2}$ are the Poisson ratios and shear modulae of the joint materials 1 and 2,

$$\beta = \ln\alpha / 2\pi, \quad A = \frac{0.25}{(1+\alpha)}\left(\frac{k_1 + 1}{\mu_1} + \frac{k_2 + 1}{\mu_2}\right). \tag{5}$$

By incorporating formulae (2)-(3) one can obtain a system of the integral-differential equations relative to the bond stresses $\sigma(x)$.

Introduce in (2)-(4) the dimensionless variable, $s = x/\ell$, and differente the relations then we obtain that

$$\frac{H}{\ell}\frac{\partial}{\partial s}\left[\varphi_1(s)\sigma_{yy}(s) - \varphi_2(s)\sigma_{xy}(s)\right] + \Delta u'_\sigma(s)E_b = \Delta u'_o(s)E_b, \tag{6}$$

where the right side is the given function of the coordinate.

By incorporating formulae (2) and (4) we will search for the stresses $\sigma(x)$ in the following form

$$\sigma_{yy}(s) - i\sigma_{xy}(s) = (q_y(s) - iq_x(s))\left(\frac{1-s}{1+s}\right)^{-i\beta}, \tag{7}$$

where $\sigma_{yy}(s)$, $q_y(s)$ are the even functions and $\sigma_{xy}(s)$, $q_x(s)$-the odd ones.

Taking into account the results (Slepjan (2)) one can obtain the following formula for the derivative of the function Δu_σ

$$\Delta u'_\sigma(s) = iA(1-\alpha)\sigma(s) - \frac{2A(1+\alpha)}{\pi\sqrt{1-s^2}}\left(\frac{1-s}{1+s}\right)^{-i\beta} \times$$
$$\int_{1-d/\ell}^{1} \frac{\sqrt{1-t^2}}{t^2-s^2}\left[sq_y(t) - itq_x(t)\right]dt \quad (8)$$

Substituting expressions (7), (8) in relation (6) and separating the real and imaginary parts one obtains the system of the singular integral-differential equations relative to the bond stresses $\sigma_{yy}(s)$ and $\sigma_{xy}(s)$.

STRESS INTENSITY FACTORS AND ENERGY RELEASE RATE

Having the solution of system (6) one can calculate the stress intensity factors (SIF) K_I, K_{II} (Hutchinson et al (3))

$$K_I + iK_{II} = \lim_{\delta\to 0}\sqrt{2\pi\delta}\left(\sigma_{yy}(\delta) + i\sigma_{xy}(\delta)\right)\delta^{-i\beta}, \quad (9)$$

where $\delta_{yy}(\delta)$, $\sigma_{xy}(\delta)$ are the stresses ahead the crack tip caused by the external loads and bond stresses, δ is the small distance to the crack tip.

On the other hand, the SIF can be written as follows

$$K_I + iK_{II} = (K_I^{ext} + K_I^{int}) + i(K_{II}^{ext} + K_{II}^{int}), \quad (10)$$

where $K_{I,II}^{ext}$ and $K_{I,II}^{int}$ are the SIF caused by the external loads and bond stresses.

By incorporating the formula for the stress distribution ahead the crack tip at the arbitrary loads (Slepjan (2)) and representation (7) we obtain

$$K_I + iK_{II} = \frac{\sqrt{\pi\ell}}{(2\ell)^{i\beta}}\left[\sigma_o(1+2i\beta) - \frac{2\cosh(\pi\beta)}{\pi}\int_{1-d/\ell}^{1}\frac{(q_y(t) + itq_x(t))}{\sqrt{1-t^2}}dt\right] \quad (11)$$

Bonding in the crack end region reduces the SIF. As a measure of that reduction we will use the relative value of the SIF modulus

$$K_r(d/\ell) = K_o / \sqrt{(K_I^{ext})^2 + (K_{II}^{ext})^2}, \quad K_o = \sqrt{K_I^2 + K_{II}^2} \quad (12)$$

The deformation energy release rate can be written for the interface crack as follows (Salganik (4))

$$G_{tip}(d,\ell) = \left(\frac{\kappa_1 + 1}{\mu_1} + \frac{\kappa_2 + 1}{\mu_2}\right)\frac{K_o^2}{16\cosh^2(\pi\beta)} \quad (13)$$

In case of the bond deformation in the crack end region one can also evaluate the energy dissipation in the end region

$$G_{bond}(d,\ell) = \frac{\partial}{\partial\ell}\left[\int_{\ell-d}^{\ell}\int_0^{\Delta u(x)}\sigma(x)\,du\,dx\right] \quad (14)$$

When the state of the limit equilibrium occurs at the crack tip the following condition is satisfied

$$G_{tip}(d^*,\ell) = G_{bond}(d^*,\ell) \ , \qquad (15)$$

where d^* is the critical size of the end region. The values d^* and σ_o corresponding to the beginning of the bond rupture in the crack limit equilibrium can be determined using condition (15), the criteria of the bond rupture at its limit stretching, $\Delta u(d^*) = \delta_{cr}$, and crack length increment, $K_o = K_{cr}$. The energy release rate $G_{tip}(d^*,\ell)$ corresponding to $K_o = K_{cr}$ determines the adhesion fracture energy of the dissimilar materials with the interface crack.

ANALYSIS OF NUMERICAL RESULTS

Solving eq. (6) by a collocation scheme we calculated the SIF and energetic characteristics of the end region at the following values of the parameters: μ_1 = 50GPa, μ_2 = 9.26GPa, $\nu_1=\nu_2=0.35$, $E_b=2\mu_2(1+\nu_2)$. The relative size of the end region was varied in the range $0 < d/\ell \le 1$. The relative bond compliance H/ℓ (see, eq.(6)) was assumed to be in the interval $0{,}025 \le H/\ell \le 0.8$.

The characteristic distributions of the normal and shear bond stresses are given in Figs.1,2 for various values of d/ℓ. Maximal stresses occur at the inner edge of the end region. The maximal shear stresses are 3-5 times less if compared to the maximal normal stresses. Note, that the bond stresses have an absolute maximum at a certain value of $(d/\ell)_m$. The hardening factor, K_r, dependencies on the value d/ℓ at various relative bond compliances H/ℓ are given in Fig.3. In a certain range of the end region size the SIF modulus is 1.5-5 times less if compared to the case of the bond absence.
The dimensionless dependencies $G_{tip}(d,\ell)/G_{tip}(0,\ell)$ and $G_{bond}(d,\ell)/G_{tip}(0,\ell)$ are given in Fig.4. The value $d^* \approx 0.04\ell$ respects to fulfillment of condition (15). The value of external load at the crack growth beginning σ_o equals to 85MPa and the adhesion fracture energy $G_{tip}(d^*,\ell)$ is equal to ≈100Joule/m^2 if K_{cr}=1MPa.m$^{1/2}$, 2ℓ=1mm and H/ℓ=0.1.

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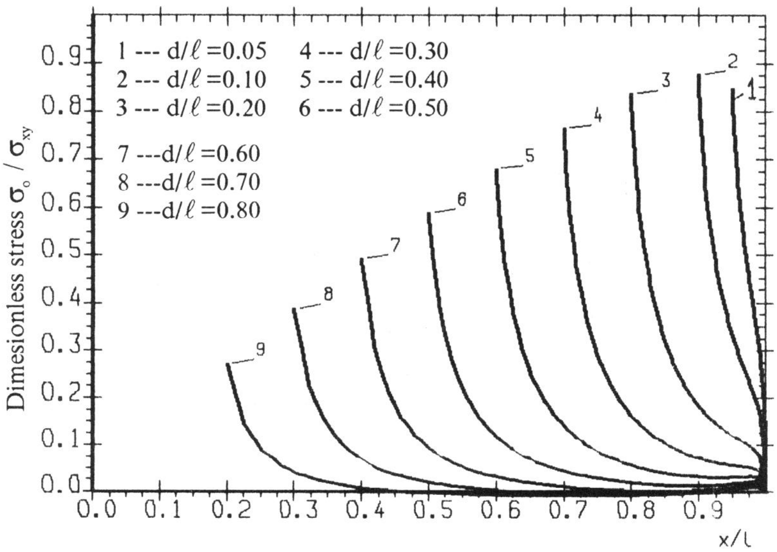

Figure 1 Tangential stress distribution along the crack end region, $H/\ell=0.1$

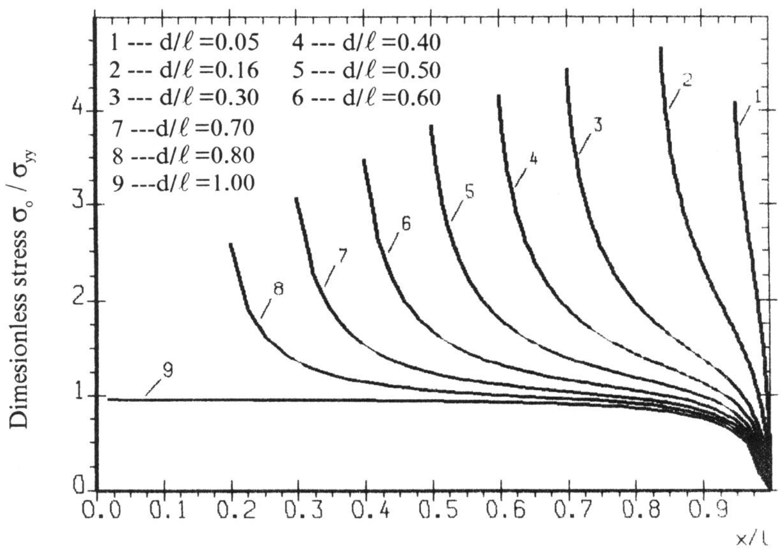

Figure 2 Normal stress distribution along the crack end region, $H/\ell=0.1$

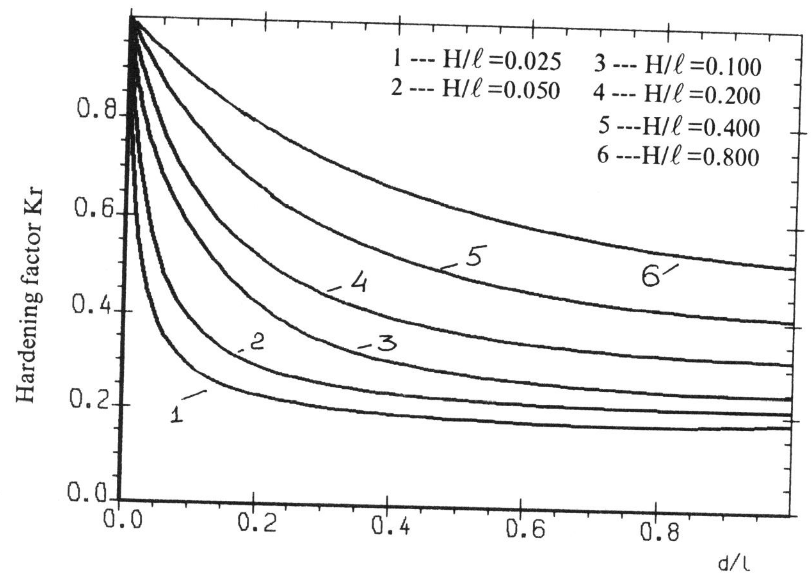

Figure 3 Hardening factor due to bonds vs the crack end region size

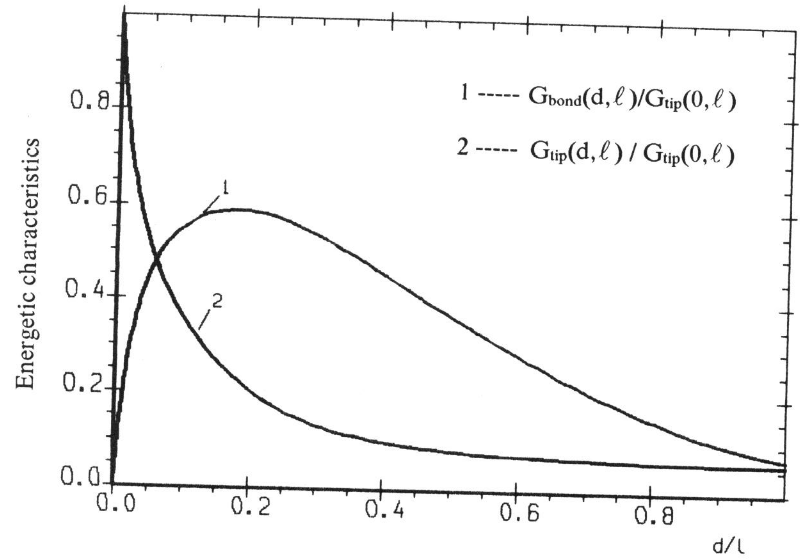

Figure 4 Energetic characteristics of the crack end region $H/\ell = 0.1$