A WEIGHT FUNCTION FOR STRESS INTENSITY FACTOR EVALUATION IN STEEL BRIDGES ORTHOTROPIC DECKS

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The paper is focused on the evaluation of the Weight Function (WF) for an orthotropic deck, employed as structural element for building welded steel bridges. To this end, a special technique is employed, which allows to obtain the WF with a reduced computational effort. The comparison of Stress Intensity Factor values obtained by the Finite Element method and calculated by the WF showed a fairly good agreement, both under axial and pure bending loading.

INTRODUCTION

Welded steel bridges, whose main load carrying elements are the "orthotropic decks" (Fig. 1), are widely employed for roads and railway applications, due to their attractive economical and durability characteristics.

The rather large dimensions of the decks' elements, coupled with relatively low stress levels, suggests that a significant part of the structure fatigue life is likely to be expended in the stable propagation of cracks. Therefore, it appears quite attractive to set-up analysis tools capable of predicting the growth of typical deck's cracks under cyclic loading, also taking account the effects of residual stress fields, which are likely to be produced by the manufacturing cycle.

Recent analyses (Beghini and Bertini (1), Beghini et al. (2)) demonstrated that the Weight Function (WF) approach is a quite effective method for predicting crack growth under general fatigue loading, also including the effect of residual stresses.

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Therefore, a technique developed by the authors (Beghini et al. (3-4)), was employed to derive the WF for the orthotropic deck, assuming a crack emanating from the lower fibre and growing symmetrically (Fig. 1).

THEORY

The Stress Intensity Factor (SIF), K_I, for a body carrying an embedded crack under symmetric loading (Fig. 1) can be obtained as:

$$K_I = \int_0^a \sigma_n(x) \cdot h(x,a)\, dx \tag{1}$$

where $\sigma_n(x)$ is the nominal stress and $h(x,a)$ is the WF. The WF for a generic geometry is usually very complex to be determined, requiring a lot of accurate SIF evaluations by numerical (e.g. Finite Element, FE) methods.

The method proposed in (3-4) is based on the observation that, given a crack of length $b<a$, the following (fundamental) relationship holds:

$$\int_b^a \sigma(x,b) \cdot h(x,a) \cdot dx = \int_0^a \sigma_n(x) \cdot h(x,a) \cdot dx \tag{2}$$

where $\sigma(x,b)$ is the stress acting in the body carrying the crack of length b. The WF for the embedded crack is then assumed (4) to be given by the following double power expansion :

$$h(x,a) = \sqrt{\frac{2}{\pi \cdot W}} \cdot \sum_{i=1}^{n} \sum_{j=1}^{m} \alpha_{ij} \cdot \left(\frac{a}{W}\right)^{i-3/2} \cdot \left[\left(1 - \frac{x}{a}\right)^{\frac{(2j-p-2)}{2p}} + \left(1 + \frac{x}{a}\right)^{\frac{(2j-p-2)}{2p}} \right] \tag{3}$$

where α_{ij} are non-dimensional coefficients, some of which can be fixed in order to satisfy the known asymptotic properties of the WF at the crack tip (4), n and m define the number of terms and p, equal to 1 or 2, selects the exponents employed.

Assuming an approximate expression for $\sigma(x,b)$, given by the asymptotic (singular) term nearby the crack tip and by a FE analysis at some distance from crack tip (3), and substituting $h(x,a)$ from Eqn. 3 into Eqn. 2 an equation containing the α_{ij} as unknowns is obtained for each crack length $b<a$. Coupling these equations with Eqn. 1, written for a number of cracks for which the SIF is known, an overdetermined linear system having the α_{ij} as unknowns is obtained, which can be solved by a best fit technique (e.g. the Normal Equation Method).

The main advantage of the present method is that it only require a few SIF value and a very rough approximation of the stress distribution in the cracked body, as can be obtained by coarse FE models.

FINITE ELEMENT MODEL AND ANALYSIS

A FE model (Fig. 2) was developed, making use of the ANSYS 5.0 Code, in order to evaluate the values of K_I for several crack lengths a and for two loading conditions (i.e. axial loading and pure bending). Quarter point elements were employed at the crack tip and the SIF was evaluated by averaging the J-integral calculated over a series of concentric paths to improve accuracy. The comparison of K_I calculated from the different paths and by near tip displacement allowed to estimate that the error on the SIF value was less than 1%.

The WF was determined by using the results of the uniform axial stress analysis only. Ten $K_I(a)$ values, with a ranging between 0 and $0.6W$ were employed and a convergence study showed that assuming n=4, m=5 and p=1 a sufficiently accurate representation of the WF is achieved.

The calculated α_{ij} coefficients are reported in Table 1, while a comparison between the values of $K_I(a)$ calculated by the WF and by the FE analysis for the case of uniform axial loading is reported in Fig. 3. It can be observed that, for a/W<0.6, there is a fairly good agreement between the two types of data, the maximum difference being about 3% (usually not exceeding 1%). In the same figure, the SIF for a Griffith crack of equal length and stress level is also reported. It is worth noting how the use of this very simple formulation, except for very short cracks, would produce significant underestimates of K_I.

The SIF calculated for the pure bending loading were employed for qualification purposes, in order to assess the WF accuracy under stress distributions different from the one employed for its evaluation. The comparison of WF and FE $K_I(a)$ curves is performed in Fig. 4. It can be observed that, also in this case, the agreement of the two types of data appears fairly good, errors being contained within 3%. As before, the SIF for the equivalent Griffith crack underestimates the correct value.

CONCLUSIONS

A numerical technique, based on rather simple Finite Element (FE) analysis was applied to derive the Weight Functions (WF) for an orthotropic deck, employed as main structural element in welded steel bridges.

The WF proved to be capable of evaluating the Stress Intensity Factor (SIF) for a symmetric crack with fairly good accuracy (errors of few percent) both under axial and pure bending loading.

These WF could therefore be efficiently employed for the prediction of fatigue crack growth life of bridges, also taking account of complex stress distributions, such as those produced by Residual Stress fields.

SYMBOLS USED

a, b = crack length
W = section height
K_I = stress intensity factor (Mode I)
σ = stress
σ_n = nominal stress
h = Weight Function
x = curvilinear abscissa

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TABLE 1 - Weight Function α_{ij} coefficients (Eqn. 3).

i \ j	1	2	3	4	5
1	1.000	350.372	-221.029	-271.063	142.226
2	0.000	-1643.460	910.196	1473.240	738.968
3	0.000	3973.870	2079.043	3739.593	1848.398
4	0.000	3328.206	1786.137	3039.762	1508.787

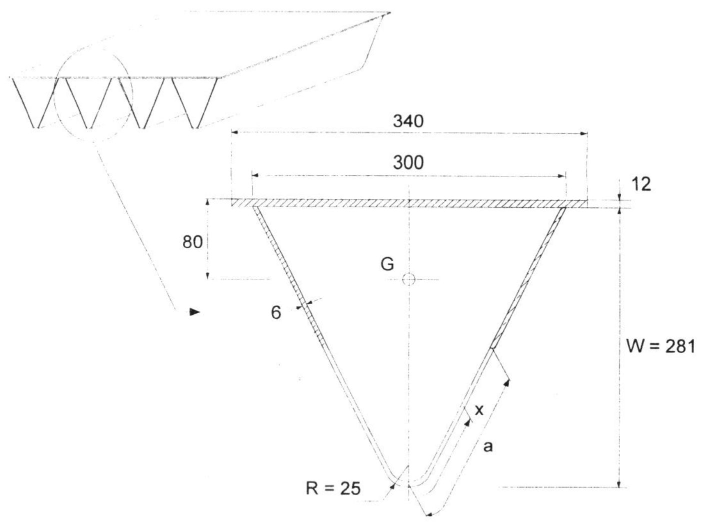

Figure 1 Geometry of the structure

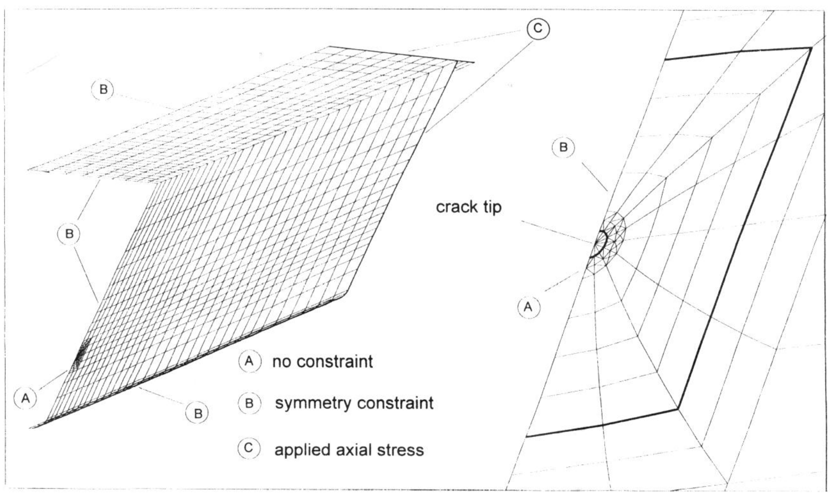

Figure 2 FE model employed for analysis

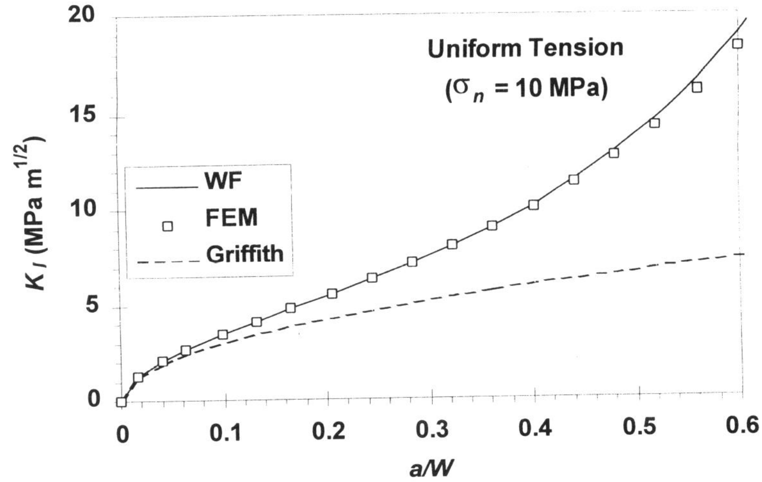

Fig. 3 SIF vs. crack length for the pure tension case

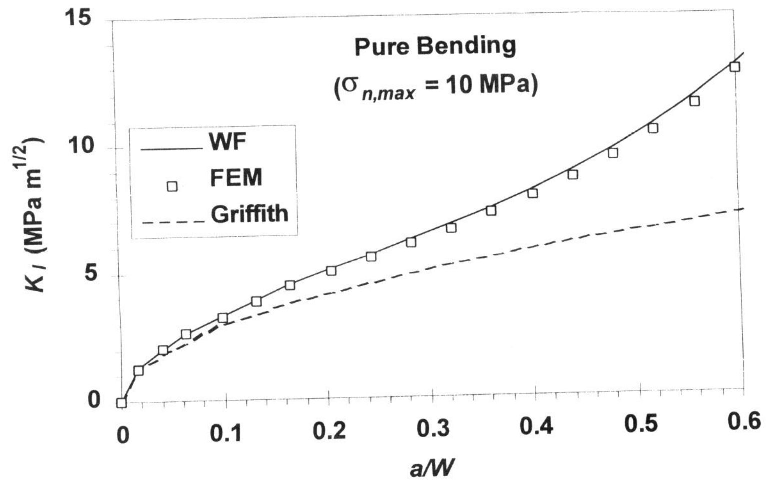

Fig. 4 SIF vs. crack length for the pure bending case.