A SIMPLE METHOD FOR THE DETERMINATION OF GEOMETRIC CORRECTION FACTORS IN LINEAR ELASTIC FRACTURE MECHANICS

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A simple method, termed force balance method, is proposed to calculate geometric correction factors in linear elastic fracture mechanics. The method is based upon an equilibrium between the externally applied load/stress and the internal stress existing in the remaining elastic material ahead of the crack tip. The deduced geometric correction factors are found to be in good agreement with the results presented in the literature. In particular, for the standard CCT specimen, a very simple geometric correction factor expression has been derived to be $Y = 1/\sqrt{1-(2a/W)^2}$, where 2a is the crack length, W is the specimen width. This equation lies just in-between Irwin´s equation and the equations reported by Koiter, Tada, Feddersen and Isida.

1. INTRODUCTION

Specimens used in fatigue and fracture tests are of finite dimensions. In order to calculate stress intensity factors at the crack tip in the finite-width specimens, it is essential to know geometric correction factors for the corresponding specimen geometry and loading configurations. Several methods have been proposed to determine the geometric correction factors, such as finite element method, weight function methods [1], experimental calibration method [2] etc. The obtained geometric correction factors have been compiled in several handbooks [e.g., 3-5]. However, in practical problems of the fracture mechanics analysis of engineering structures, structural geometries and loading configurations are often different and complicated so that the available stress intensity factor solutions are inadequate. Therefore, it is necessary to develop some approximate methods which have sufficient accuracy and are easy to apply.

Recently we have proposed a simple approach for the calculation of geometric correction factors, named "force balance method (FBM)" [6-8], since this method is simply based on an equilibrium between the externally applied load/stress and the internal stress existing in the remaining elastic material ahead of the crack tip along the crack-line. In this paper an improved presentation of the proposed FBM is introduced by a few examples for center cracked specimens loaded by different configurations.

2. CONCEPT OF THE FBM

In order to establish the equilibrium equations of forces and/or torques between the

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externally applied load/stress and the internal stress existing in the remaining elastic material ahead of the crack tip along the crack-line direction, it is necessary to know the stress distribution in front of the crack tip. Unfortunately, at present it is not possible to express analytically such a stress distribution for finite-width specimen. Therefore, the following assumption is made: the stress distribution ahead of the crack tip for a finite-width plate has a similar form to that for the corresponding infinite plate, with singular terms being modified by multiplying a magnifying factor. By use of the equation for calculating the stress intensity factor from the stress distribution function, one can immediately find that the added magnifying factor is just the geometric correction factor. This factor can then be determined by means of the equilibrium conditions of force and torques along the crack-line direction. In the following two examples shown in Fig.1 are taken to indicate the applicability of the proposed FBM.

3. EXAMPLES OF GEOMETRIC FACTORS CALCULATED BY THE FBM

3.1 Standard Center Cracked Tension (CCT) Specimen

It is well known from Westergaard's theory [9] that the stress distribution ($\sigma_{yy,inf}$) ahead of the crack tip along the crack-line for an infinite plate, corresponding to Fig.1(a), with a crack length of $2a$ loaded by a remote tensile stress σ can exactly be expressed as,

$$\sigma_{yy,\inf} = \frac{\sigma}{\sqrt{1-(a/x)^2}} \qquad |x|>a, \tag{1}$$

where a is the half-crack length, x is a distance from the center of the crack.

As described above, the stress distribution (σ_{yy}) for the finite-width plate could be expressed as,

$$\sigma_{yy} = \frac{Y_a \sigma}{\sqrt{1-(a/x)^2}} \qquad |x|>a. \tag{2}$$

Using the following formula for the determination of the stress intensity factor from the stress distribution:

$$K = \lim_{x \to a} \sqrt{2\pi(x-a)}\,\sigma_{yy}, \tag{3}$$

one can easily obtain the stress intensity factor expression:

$$K = Y_a \sigma \sqrt{\pi a}. \tag{4}$$

Obviously, the added factor Y_a is just the geometric correction factor. From equation (2) and referring to Fig.2 (the lower half of the specimen is schematically plotted only), one can determine a mean internal stress in the remaining elastic material ahead of the crack tip in the crack-line direction,

$$\bar{\sigma}_{yy} = \frac{2}{W-2a}\int_a^{W/2} \sigma_{yy}\,dx = Y_a \sigma \sqrt{\frac{W+2a}{W-2a}}, \tag{5}$$

where W is the specimen width. It can be seen from Fig.2 that the externally applied stress is solely carried by the internal stress in the elastic material in front of the crack tip, i.e., the following force balance equation holds true,

$$\bar{\sigma}_{yy}(W-2a) = \sigma W. \tag{6}$$

Inserting equation (5) into equation (6) one obtains a simple geometric correction factor formula for the standard CCT specimen:

$$Y_a = \frac{1}{\sqrt{1-(2a/W)^2}}. \tag{7}$$

3.2 Center Cracked Specimen Loaded by Uniformly Distributed Stresses Acting on the Central Portion of Crack

For the case of an infinite plate corresponding to Fig.1(b), loaded by uniformly distributed stresses acting on the central portion of crack, the stress distribution ahead of the crack tip in the crack-line direction is given below [4],

$$\sigma_{yy,\inf} = \frac{2\sigma}{\pi}\left\{\frac{\sin^{-1}(b/a)}{\sqrt{1-(a/x)^2}} - \tan^{-1}\left[\sqrt{\frac{1-(a/x)^2}{(a/b)^2-1}}\right]\right\} \qquad |x|>a. \tag{8}$$

Similarly, for the finite-width plate (Fig.3, also only the lower half of the specimen is schematically plotted), the stress distribution becomes:

$$\sigma_{yy} = \frac{2\sigma}{\pi}\left\{\frac{Y_b \sin^{-1}(b/a)}{\sqrt{1-(a/x)^2}} - \tan^{-1}\left[\sqrt{\frac{1-(a/x)^2}{(a/b)^2-1}}\right]\right\} \qquad |x|>a, \tag{9}$$

where a geometric correction factor Y_b is added to the singular term of the stress distribution. Substituting equation (9) into equation (3) yields

$$K = Y_b \sigma \sqrt{\pi a}\,\frac{2\sin^{-1}(b/a)}{\pi}. \tag{10}$$

It should be pointed out that the magnifying factor could be added to either the singular term only or to the whole expression of the stress distribution, which does not have much effect on the real situation close to the crack tip and on the stress intensity factor expression. However, it has been indicated by means of finite element calculations that the addition of the geometric correction factor to the singular term appears more reasonable than that to the whole expression [10]. With reference to to Fig.3 and from equation (9), the average internal stress ($\overline{\sigma}_{yy}$) can be derived to be,

$$\overline{\sigma}_{yy} = \frac{4\sigma}{\pi(W-2a)}\left\{Y_b\sqrt{(W/2)^2-a^2}\,\sin^{-1}\left(\frac{b}{a}\right) + b\tan^{-1}\left[\sqrt{\frac{(W/2)^2-a^2}{a^2-b^2}}\right] - \frac{W}{2}\tan^{-1}\left[\sqrt{\frac{1-(2a/W)^2}{(a/b)^2-1}}\right]\right\}. \tag{11}$$

Substituting equation (11) into the following equilibrium equation:

$$(W-2a)\overline{\sigma}_{yy} = 2b\sigma, \tag{12}$$

one can deduce the following geometric correction factor formula:

$$Y_b = \frac{\frac{\pi b}{2} + \frac{W}{2}\tan^{-1}\left[\sqrt{\frac{1-(2a/W)^2}{(a/b)^2-1}}\right] - b\tan^{-1}\left[\sqrt{\frac{(W/2)^2-a^2}{a^2-b^2}}\right]}{\sqrt{(W/2)^2-a^2}\,\sin^{-1}(b/a)}. \tag{13}$$

It should be noted that equation (13) corresponds to the stress intensity factor expressed by equation (10). If the stresses are uniformly distributed over the whole crack, i.e., $a=b$ in Fig.1(b), then equations (10) and (13) can be simplified as,

$$K = Y_b\left(a=b\right)\cdot\sigma\sqrt{\pi a}\,, \tag{14}$$

$$Y_b\left(a=b\right) = Y_a = \frac{1}{\sqrt{1-\left(2a/W\right)^2}}. \tag{15}$$

As a result, this special case is identical to the case of the standard CCT specimen (i.e., Fig.1(a)). This is true from the viewpoint of the superposition principle in fracture mechanics [4, 11].

4. COMPARISON WITH THE LITERATURE AND DISCUSSION

In Fig.4 a comparison is presented of the geometric correction factor derived by the FBM with the literature results [4, 12] for the commonly-used CCT specimen (Y_a) and for the center cracked specimen loaded by uniformly distributed stresses acting on the entire crack ($Y_b(a=b)$). It can be seen that our equation lies just in-between the well-known Irwin´s formula and the equations of others. The good coincidence indicates the validity of the proposed FBM.

The FBM is applicable not only to the cases of centric crack and symmetric loading, e.g., currently considered cases, but also to the cases of eccentric crack and unsymmetric loading when the equilibrium of torques is considered [7,8]. However, it should be pointed out that although no approximation is made during the calculations, the assumption concerning the stress distribution in front of the crack tip for the finite-width plate has been applied. Recent finite element calculations [10] have indicated that such assumed stress distributions could basically stand for the situation in the vicinity of crack tip, but they do exhibit some discrepancy close to the free-boundary of specimens especially for larger crack length. Thus, even though the geometric correction factors derived for various cases are found to be in good agreement with those reported in the literature, the proposed FBM should be considered as a simple approximate approach with sufficient accuracy. This can be seen from Fig.5, where the relative error of our equation, denoted by "CWS", to Isida´s equation expressed as a polynomial of 36 terms up to the 70th power of $2a/W$ [12] and to Tada´s equation [4] is plotted. The maximum error of our equation is 14% at $2a/W \rightarrow 1$ relative to Tada´s value, and is less than 11% for $2a/W \leq 0.98$ relative to Isida´s value. The corresponding error for Irwin´s equation relative to Tada´s and Isida´s equations is found to be 23% and 19%, respectively. In fact, for very long crack, i.e., $2a/W \rightarrow 1$, the difference between Tada´s and Isida´s equations themselves becomes also very large. If one considers only the practically-used crack length range, say, $2a/W \leq 0.7$, the error of our equation relative to both Tada´s and Isida´s equations is less than 6%. Therefore, the simple geometric correction factor deduced by the FBM should be applicable.

ACKNOWLEDGMENTS

This investigation is part of a cooperative project between the National Science Foundation of Austria (P8032-TEC and P10160-PHY) and the National Natural Science Foundation of China (A13 and A18). The financial support by the Fonds zur Förderung der Wissenschaftlichen Forschung, Vienna, is gratefully acknowledged. One of the authors (DLC) would like to thank Dr.C.H.Liu, Department of Mechanical and Aerospace Engineering, Carleton University, Ottawa, Canada, for his interests and useful discussion.

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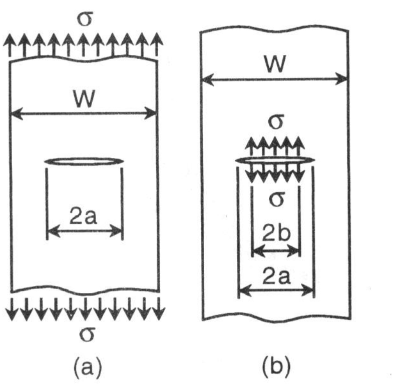

Figure 1 Examples considered:
(a) standard CCT specimen, (b) center cracked specimen loaded by uniformly distributed stresses acting on the central portion of crack.

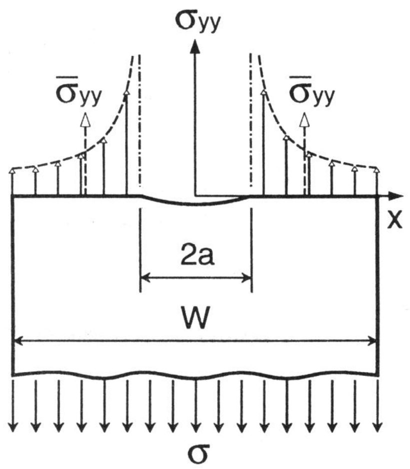

Figure 2 Equilibrium between forces for the finite-width CCT specimen, schematic.

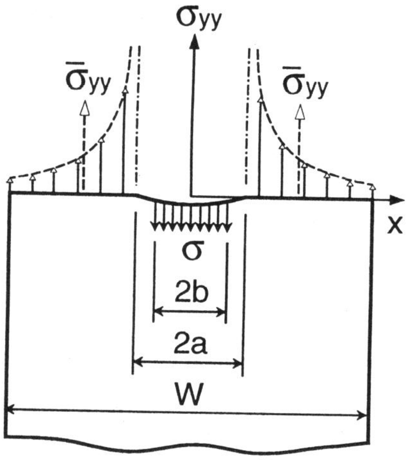

Figure 3 Equilibrium between forces for a finite-width plate containing a center crack of length $2a$ loaded by uniformly distributed stresses acting on the central portion of crack, schematic.

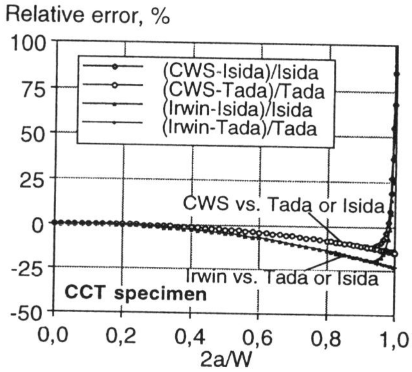

Figure 5 Relative error of the deduced geometric correction factor for the CCT specimen.

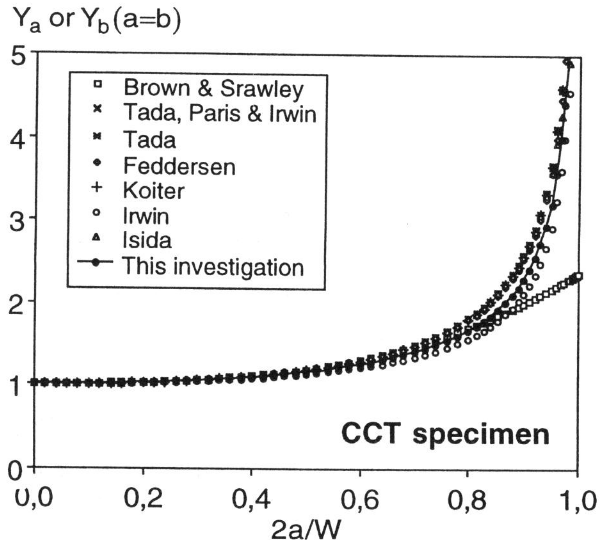

Figure 4 Comparison of the deduced geometric correction factor with those presented in the literature [4, 12].