A NUMERICAL MODEL FOR CRACK GROWTH IN BRITTLE MATERIALS BASED ON THE ELEMENT FREE GALERKIN METHOD

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A numerical model for element free simulation of crack propagation in two dimensions is presented. The model is based on the Element Free Galerkin method which has as key features a topology free way of definition of shape functions and an integration pattern which can be chosen almost independent of the set of shape functions. Therefore, when a crack advances, minor changes have to be made to the discretization for shape functions and to the integration pattern. A description of the main features of the EFG-method is given.

INTRODUCTION

A widely used numerical method for the simulation of crack propagation is the well known Finite Element (FE) method, see for instance Hughes [1]. Since for this method the material under consideration is subdivided into so called elements, the method is based on a mesh, i.e. a topology, of nodal points. Hence, when a crack advances, remeshing is necessary to reflect the changing geometry of the material. However, remeshing is a costly process and a lot of mesh generators can not handle each crack configuration.

The Element Free Galerkin (EFG) method, see Belytschko et al. [2], [3] and Lu et al. [4], is a numerical method which can be seen as a method that is topology free. This makes the method convenient for the simulation of crack propagation, since adaptation of a mesh is not necessary when a crack advances.

The EFG-method is based on Moving Least Squares Approximation (MLSA), see Lancaster and Salkauskas. [5]. For this topology free way of approximation only a set of nodal points in the material, a locally supported weight function for each nodal point and a set of basis functions are necessary, see figure 1a. By means of a least squares

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procedure one obtains a set of shape functions $\{\phi_i\}_{i=1,\ldots,n}$. These locally supported shape functions depend on the nodes, the weight functions and the basis functions and fail in general to have the property that $\phi_i(\mathbf{x}_j) = \delta_{ij}$ (i.e. 1 when $i = j$ and 0 when $i \neq j$), where $\mathbf{x}_j$ is a nodal point. The least squares procedure, however, makes MLSA computationally expensive, since for each point under consideration a linear sytem has to be solved to obtain the values of the shape functions.

As material behaviour linear elasticity is assumed which means that we focus on brittle materials. To apply a certain fracture criterion to decide whether a crack tends to propagate, accurate values for displacements, strains and stresses in the cracked material should be available. Approximate values for these quantities are obtained with the help of the MLSA-shape functions $\{\phi_i\}_{i=1,\ldots,n}$. A weak formulation of the problem description for the deformation of linear elastic material is used where essential boundary conditions are accounted for by a set of (discrete) Lagrange multipliers. With setting

$$\mathbf{u} = \sum_{i=1}^{n} \phi_i \mathbf{d}_i \tag{1}$$

for the displacements and the test functions in this weak formulation, and taking a similar form for the Lagrange multipliers, a linear system of equations is obtained for the unknowns $\mathbf{d}_i$. Solution of this system results in the approximation (1) for the displacements and after derivation, in approximations for the strains and stresses in the material.

To obtain the linear system for the unknowns $\mathbf{d}_i$ integrations over the material and its boundary have to be performed. Therefore, as proposed in [2], a split up of the material and its boundary is made into integration cells, see figure 1b. For each cell the integrals are numerically evaluated by means of Gauss integration. Summing up over all cells results in the linear system.

The shape functions obtained by the least squares process are quite general, and are not, as for the FE-method, piecewise linear or piecewise quadratic. This means that there is no optimal integration pattern. One can say that the integration pattern can be chosen more or less independent of the MLSA-discretization. There should only be accounted for a sufficient accurate integration pattern. Therefore, cell sizes are taken of the order of the mesh size of the nodal distribution and in each cell the number of integration points n_{int} is taken to be

$$n_{int} \approx \max(3n_c, 1), \tag{2}$$

where n_c is the number of nodes in the cell.

EFG IN FRACTURE MECHANICS

The present work differs from that presented in [2], [3] and [4]. A different approach is used to model a crack in the material, a special function is added to the basis to capture the singular behaviour in strains and stresses near the crack tip and an integration pattern is proposed which requires only minor changes to reflect crack propagation.

For a correct performance of the EFG-method it is necessary that the supports of the weight functions are of order two or three times the mesh size of the nodal distribution. Hence, in the neighbourhood of a crack the shape functions will be continuous over the crack, which is in contrast with the fact that quantities in general are discontinuous over the crack. Therefore, for the nodal points in the neighbourhood of a crack its weight function is modified such that it becomes discontinuous over the crack, but continuous in the material, see figure 2a. This yields shape functions which are continuous in the material, but which are discontinuous over the crack. Such a modification is obtained by means of premultiplication of the initial continuous weight function with a function which is discontinuous over the crack. In [2], [3] and [4] a modification of weight functions is used which leads to shape functions which are also discontinuous over a line in the material.

In the neighbourhood of a crack tip stress concentrations occur. In case of linear elastic material behaviour, the stresses near the crack tip behave like $1/\sqrt{r}$, where r is the distance to the crack tip. In the EFG-method this singularity in the stresses can be easily captured by means of adding a (local) function to the set of basis functions which behaves like $\sqrt{r}$. Addition of such a function has the consequence that the MLSA-shape functions behave like $\sqrt{r}$ in the neigbourhood of the crack tip. Hence, the derivatives show a singular behaviour. Special integrations cells are then necessary near the crack tip for sufficient accurate integration.

In each computation step the pattern of integration cells has to match with the crack configuration, i.e. it is not allowed that a (part of a) crack lies in the interior of a cell. To avoid the problem of a complicated redefinition of the cell pattern after a propagation step, integration cells are used which themselves can account for a crack. When a (part of a) crack is present in the cell, it is subdivided into a small set of triangular cells which match the crack, see figure 2b. The cell contribution to the linear system is then obtained with the help of these triangular cells. Moreover, these cells are also used in the next computation steps. Hence, almost a fixed cell pattern is used; only cracked cells in this pattern will be replaced by a subdivision.

COMBINATIONS OF EFG and FE

The least squares procedure to come to shape functions makes the EFG-method computationally expensive. Therefore, it is more convenient to make use of the EFG-method in combination with the more cheaper FE-method. The part of the material which is cracked is handled by means of the EFG-method and for the remaining part of the material the FE-method is used. When the crack propagates into the FE-area, elements which are cracked are replaced by an EFG-discretization. Such a combination still requires minor changes to reflect crack propagation.

When a material is subdivided into a FE-area and an EFG-area, one should account for a coupling between both parts. Several couplings have already been studied, see for instance Hegen [6] and Belytschko et al. [7].

NUMERICAL RESULTS

A simple two dimensional example is presented. A flat plate with initial crack as shown in figure 3a is considered. The initial cell pattern is shown as well. The plate is loaded by means of a vertical traction on the horizontal boundaries of the plate. A fixed set of nodes is used together with an extra radial pattern which is situated around the crack tip, see figure 3b. The crack is represented as piecewise linear. In each step, the value of the J-integral is computed by means of contour integration around the crack tip. The new crack tip is then chosen by means of a fixed step in the direction given by the J-integral.

In figure 4a the computed crack path is shown together with the initial cell pattern. In figure 4b the cell pattern of the last step is shown.

CONCLUDING DISCUSSION

From the previous considerations one can conclude that the EFG-method is a flexible numerical method for the simulation of crack growth. Propagation of a crack can be reflected by only some small changes to the discretization for approximation and to the integration pattern. Moreover, the singular stresses around the crack tip can be easily captured by means of addition of a special function to the basis.

Drawback of the EFG-method is that the method is computationally expensive. Therefore, a coupling with the FE-method is recommended. Furthermore, due to the freedom for choices for sizes of supports of weight functions, for nodal distributions and for integration patterns, it is not possible to find an optimal set up for the method.

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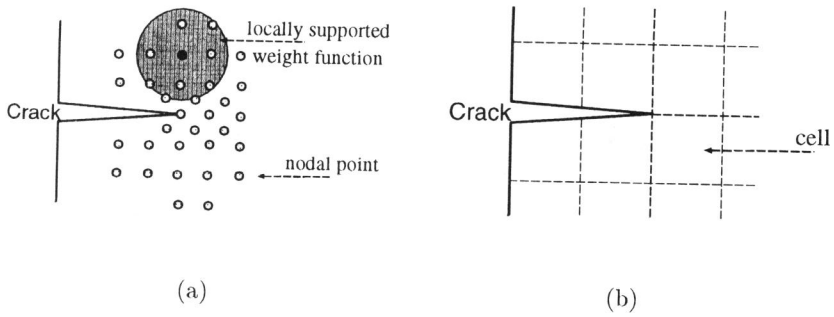
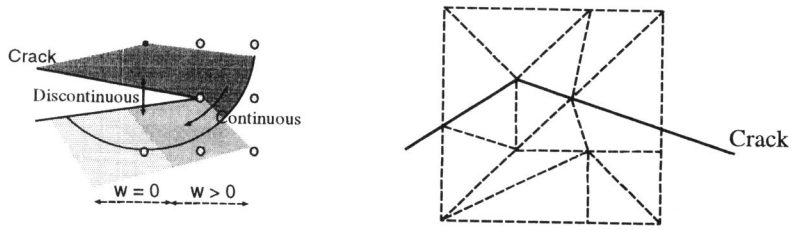


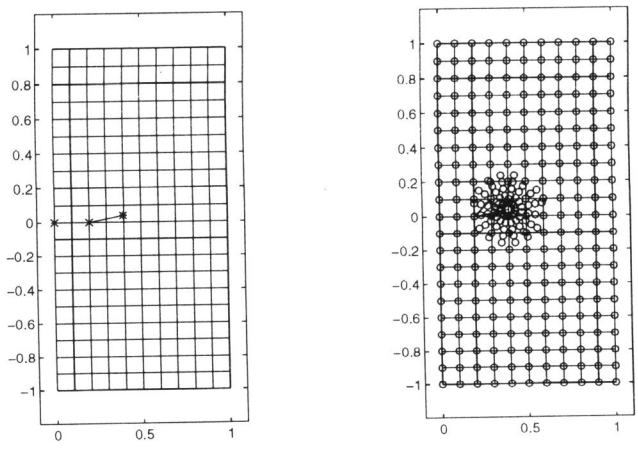
(a) (b)

Figure 1: (a) MLSA-discretization, (b) pattern of integration cells



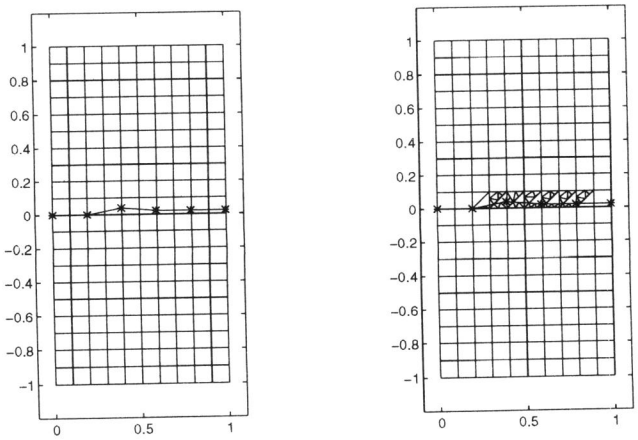
(a) (b)

Figure 2: (a) Weight function modification, (b) internal subdivision of integration cell



(a) (b)

Figure 3: (a) Initial cell pattern and crack (b) Nodal distribution



(a) (b)

Figure 4: (a) Computed crack path (b) final cell pattern