A NEW SIMPLIFIED METHOD FOR CALCULATING J INTEGRAL

THE ELASTIC PLASTIC STRESS STATE METHOD

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Two simplified methods are currently used for calculating J integral in cracked structures, the R6 rule and the EPRI method ; the aim of the paper is to present the basis of a third simplified method referred as the Elastic Plastic Stress State Method (E.P.S.S.M.).

INTRODUCTION

The stability of cracks in elastic structures is assessed by comparing the stress intensity factor K_I to a critical value K_{IC}. The Rice's J Integral is related to K_I :

$$J = \frac{1 - \nu^2}{E} K_I^2 \text{ and } J_{IC} = \frac{1 - \nu^2}{E} K_{IC}^2$$

In elastic plastic structures, K_I cannot be defined, but the stability of a crack is assessed by comparing the J integral to a critical value J_{IC}.

Some numerical methods, such as finite element method, make it possible to calculate J. They are mainly used for defining and assessing simplified methods which allow to calculate easily J for current industrial applications.

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In elastic structures, the simplified Fracture Mechanics computations are based on the superposition principle. In elastic plastic structures, simplified methods of two categories are currently used :

- the methods based on the CEGB R6 rule [1, 2],
- the methods based on the EPRI formulation [5].

The aim of this paper is to present a new approach referred as the "Elastic Plastic Stress State Method" (E.P.S.S.M.). This new method is based on several principles, which are more general, but similar to the principles used in current simplified methods. The main principles of the current simplified methods and of the E.P.S.S.M. will be presented. This will allow to point out both the similarities and the differences of the methods.

USE OF THE SUPERPOSITION PRINCIPLE TO CALCULATE THE STRESS INTENSITY FACTORS IN ELASTIC STRUCTURES

The Fracture Mechanics problem to be analysed

An elastic cracked structure is subjected to several loads such as fixed displacements, thermal expansion and external forces. The stress state near to the crack tip is characterised by stress intensity factors K_I, K_{II}, K_{III}, variable along the crack front.

Method based on the superposition principle

The first step of this method consists in calculating the elastic stress state in the uncracked structure. Special attention is paid to the stress σ existing in the plane where a crack will be considered in the structure.

The method consists in considering the cracked structure which is subjected only to the stresses σ and $-\sigma$ applied on the crack faces. The resulting stress intensity factors K_I, K_{II}, K_{III} are the same as in the initial problem.

In many cases, the stress components can be approximated by polynomials depending on a few parameters. Some formulations are available for calculating K_I, K_{II} and K_{III} from the parameters. So K_I, K_{II} and K_{III} are easily calculated from the stress σ. The value of J can be derived from the stress intensity factors :

$$J = \frac{1 - \nu^2}{E} \left(K_I^2 + K_{II}^2\right) + \frac{1 + \nu}{E} K_{III}^2$$

J CALCULATION IN ELASTIC PLASTIC STRUCTURE - SIMPLIFIED METHODS OF THE FIRST CATEGORY

The simplified method of the first category consists in dividing the stresses into several different components : at least, the primary stresses balancing applied forces and the secondary stresses resulting from imposed displacements or strains.

The first development of this method was performed by C.E.G.B. (Nuclear Electric). The present status of this method is given in [1, 2]. Several authors have improved and adapted the method to more sophisticated loading conditions [3, 4].

All these methods consist in calculating the elastic J values from all the stress categories and in taking into account the amplification effect due to plasticity for each category, by using appropriate models.

They can be applied for any stress-strain law.

SIMPLIFIED METHOD OF THE SECOND CATEGORY

The methods of the second category have been mainly developed by E.P.R.I. [5, 6]. They assume that the stress-strain curve has a Ramberg-Osgood representation :

$$\frac{\sigma}{\sigma_0} = \frac{\varepsilon}{\varepsilon_0} + \alpha \left(\frac{\sigma}{\sigma_0}\right)^n$$

J may often be approximated by $J = J_{el} + J_{pl}$

In many cases :

$$J_{el} = \frac{1 - \nu^2}{E} F^2 \sigma_{el}^2 \pi a$$

$$J_{pl} = \alpha\, a\, \sigma_0\, \varepsilon_0\, h_1 \left(\frac{\sigma_{el}}{\sigma_0}\right)^{n+1}$$

a = crack depth

σ_{el} = applied elastic stress

F = elastic shape factor

h_1 = elastic plastic shape factor

σ_0 = yield strength

E = Young modulus

ν = Poisson's ratio

The parameter h_1 has been tabulated in several publications [5, 6, 7] for typical crack configurations and simple loading conditions (force or moment or pressure, and some combinations of force and moment).

THE ELASTIC PLASTIC STRESS STATE METHOD (E.P.S.S.M.)

The E.P.S.S.M. was originally developed by FRAMATOME for materials exhibiting a Ramberg-Osgood stress-strain curve.

$$\frac{\sigma}{\sigma_0} = \frac{\varepsilon}{\varepsilon_0} + \alpha\left(\frac{\sigma}{\sigma_0}\right)^n$$

In many cases, it can be adapted for materials whose stress-strain curve has a plateau.

The first step of the method consists in calculating the elastic plastic stresses in the uncracked structure. This step is similar to the first step of the method based on the superposition principle for elastic structures, but the material is supposed to be elastic plastic.

The elastic plastic stresses are used for calculating the J integral ; J is divided into two terms J_{el} and J_{pl} similar to the two terms of the simplified method of the second category.

$$J = J_{el} + J_{pl}$$

But the meaning of J_{el} and J_{pl} are different : they are calculated from the true elastic plastic stress existing in the plane where a crack will be assumed, and not from an elastic applied stress.

In the case where the stresses are uniformly distributed, the same formulation can be used in the simplified method of the second category and in the E.P.S.S.M. The shape factors F and h_1 have the same values in the two methods.

PRESENT STATE OF DEVELOPMENT OF THE E.P.S.S.M.

The method can be used in any case where the factors F and h are available or can be easily calculated.

This includes any tridimensionally cracked structure when the stress in the cracked zone is quasi uniformly distributed. The stress state can be uniaxial or multiaxial. The normality of the principal stress to the plane of the crack is not necessary. The stress-strain curve may have a plateau.

In particular, the method can be used to any small crack in any structure. For small surface cracks, h_1 and F are already available in open literature [5, 6, 7]. For embedded cracks, the functions F and h_1 must be evaluated.

FURTHER DEVELOPMENT OF THE METHOD

Developments are mainly necessary for structures subjected to non uniform stress stresses.

CONCLUSIONS

Three categories of simplified methods are available for calculating J in elastic plastic structures.

The methods referred to here as the methods of the first category are based on the C.E.G.B. R6 rule. The stresses must be divided in several categories (primary and secondary, membrane and bending). The method for calculating J from each stress category must be validated for each typical configuration. This development work has not been achieved for complex loading conditions. In the cases where it has been achieved, the methods permit to calculate J easily.

The methods referred to here as the methods of the second category are based on the EPRI formulation. They provide accurate calculations of J. They can be used only for some simple loading conditions.

The main topic of this paper is the Elastic Plastic Stress State Method (E.P.S.S.M.) ; it is a new simplified method for calculating J. This method is accurate and reliable ; it necessitates to know the elastic plastic stresses in the plane of the crack ; it can be used for any complex loading condition. It is a systematical method and its results are indisputable. It will be probably more and more used in the future years.

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