A FRACTURE MECHANICS MODEL FOR INTERACTING CRACKS IN THERMAL FATIGUE

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A complex pattern of branched irregular surface cracks develops during thermal cyclic loading of smooth austenitic steel plates. Geometrical simulation models for initiation and propagation of cracks were used to assess and to understand the different stages of damage evolution. A fracture mechanics assessment of a great number of cracks is necessary at each simulation step. The fracture behaviour of branched cracks is described in terms of an effective crack length. The effect of crack branching and of interaction of adjacent cracks is taken into account. The present contribution shows how the effective crack length is determined and shows how different domains of stress amplification and of crack shielding can be defined for adjacent cracks.

Introduction

Under cyclic thermal loading, initiation of cracks on the specimen surface leads to the formation of irregular patterns of kinked and branched cracks of complicated shape. This damage process was analysed for an austenitic stainless steel (German designation 1.4948) in [1]. An example of such a crack pattern is shown in Fig. 1. A stochastic simulation procedure was developed in order to assess the different stages of crack initiation, propagation, and coalescence [2]. Characteristic quantitities were defined which allow to compare different crack patterns in a quantitative way. An essential part within the simulation procedure is the fracture mechanics assessment of branched cracks. This is required because crack propagation in the simulation model is identified with the failure of segments of a potential crack path which starts at a given crack tip and extends along the edges of an underlying mosaic structure. Propagation of a crack is triggered by the stress field ahead of the crack tip. A simplified fracture mechanics analysis of branched cracks is used in the course of the simulation because computational effort becomes prohibitively large if numerical methods such as the Finite Element Method (FEM)

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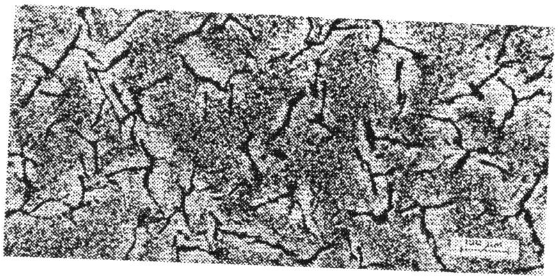


Figure 1: Surface crack pattern after 4500 load cycles

or Boundary Element Method (BEM) is used. The basic ideas of the analysis are explained in the next section. In a later stage of damage, when the crack density becomes large, interaction effects may play an important role and may change the propagation behaviour of the individual cracks.

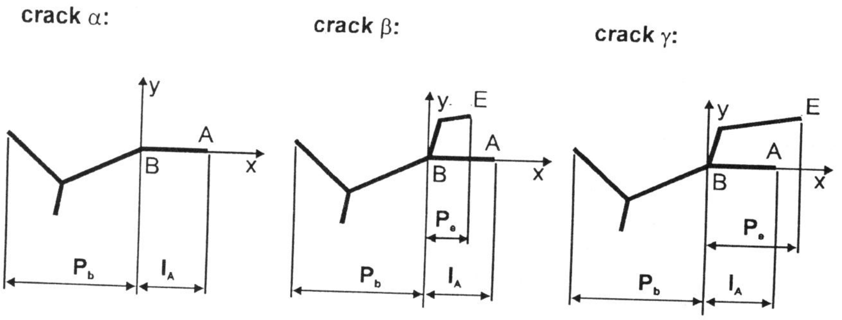
Interaction effects are modelled in terms of domains of shielding and amplification. The basic idea of the interaction model is presented together with selected results for specific configurations of neighbouring cracks. Some remarks on the accuracy conclude the paper.

Stress parameters for branched surface cracks

Surface cracks in thermal fatigue have a complicated three-dimensional shape. The stress intensity factors at the crack tips of the surface cracks are assumed to take the following form:

$$K_{I,II} = \sigma\sqrt{\pi a_{effI,II}}\, F_{TI,II} F_{NI,II} \qquad (1)$$

Mode I and II are present where the mode II loading is related to the asymmetric shape of a branched crack. The most important influence factors are taken into considerations by the multiplicative terms in Eq.(1). The effective crack length a_{eff} takes the branching of the surface cracks into account, F_T describes the influence of the crack extension in depth direction perpendicular to the surface, and F_N is used to describe the influence of interaction from neighbouring cracks. Crack branching is



crack α: crack β: crack γ:

Figure 2: Characteristic length for calculation of effective crack length

certainly the most important factor, at least in the early stage of damage. Later, interaction effects may become important. The extension of cracks in depth direction is very difficult to analyse due to the stress gradients in thermal loading. We will therefore restrict our attention on the behaviour of branched cracks in an equibiaxial stress field and on the assessment of crack interaction. For an isolated planar branched crack, Eq. (1) yields

$$K_{I,II} = \sigma\sqrt{\pi a_{effI,II}} \qquad (2)$$

A specific effective crack length is attributed to each crack tip separately. The effective crack length depends on characteristic dimensions of the branched crack and on suitably defined influence parameters describing the deviation from a straight line. Three typical crack geometries are shown in Fig. 2. The crack tip for which the stress intensity factor is calculated is called A, whereas B denotes the second end point of the crack edge which serves also as the origin of a local coordinate system. **In case of Mode I loading**, the cracks α and β can be analysed as follows: The crack branch on the left side of the y-axis leads to an increase of loading at point A, whereas crack branches to the right of the y-axis have a shielding effect on crack tip A. Therefore, the following relation for the effective crack length is assumed:

$$2a_{effI} = l_A + k_b p_b - k_e p_e \qquad (3)$$

where l_A denotes the length of the crack from branching point B to the crack tip A. The characteristic lengths p_b and p_e are projected lengths of the loading and shielding branches (see Fig. 2). k_b and k_e are weighting factors depending on the crack geometry. If l_A is small compared to the other dimensions of the crack (case γ) the crack branch can be modelled as a tilted edge crack with the dominant crack line defining the free surface which leads to:

$$a_{effI} = k_c l_A \qquad (4)$$

with the weighting factor k_c depending on crack geometry. **For Mode II loading**, a similar procedure is used.

The weighting factors in the above equations are determined using handbook solutions (e.g.[3]) and BEM results of suitably selected cracks [4]. A selection of cracks used for BEM analysis is shown in Fig. 3. The crack geometry is described using quantities which are related to the extension of the crack and to the relative contribution of loading and shielding branches of the crack. Details of the analysis can be found in Refs. [4,5]. The fracture behaviour of branched cracks of rather general shape can be evaluated with this procedure. The accuracy of the K_I-Factors is surprisingly good, even in the more difficult cases β and γ. For Mode I and cases α and β, about 80% of the investigated crack geometries show a deviation of less than 10% from BEM- or handbook solutions [4]. Of course, real crack geometries may be even more complicated than those used for the BEM

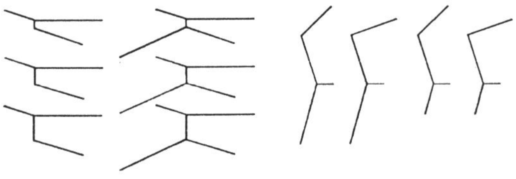


Figure 3: Selection of cracks used for BEM analysis

calculations, but most of the crack geometries occuring in the crack pattern formed during thermal cycling loading are very similar to the investigated geometries. Hence the error is anticipated to be of the order of 10%.

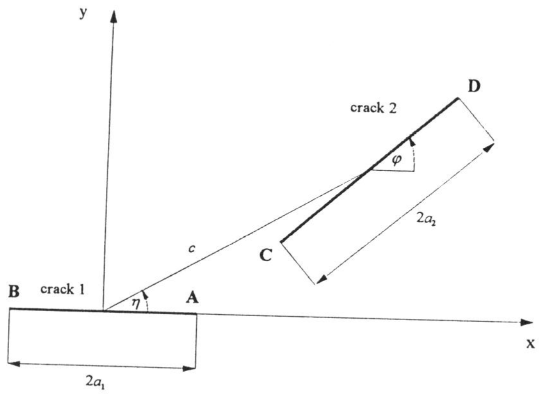


Figure 4: Geometric quantities in case of two interacting straight cracks.

Interaction of cracks

As crack branching is taken into account by the effective crack length, it is sufficient to consider two straight cracks with effective lengths a_1 and a_2, respectively, and with different relative orientations (Fig. 5 shows the notation). In the sequel, the first crack with length $2a_1$ is fixed with the crack tip A at the origin of a local coordinate system. The second crack with length $2a_2$ serves as a „test crack" and is used to identify domains of amplification and shielding of the crack tip loading at A. The crack tip C of the second crack is kept fixed and the crack is moved around in the x-y plane (i.e. vary the angle η in Fig. 5 with fixed orientation φ) the boundaries of amplification and shielding domains are obtined. Amplification occurs if the stress intensity factor exceeds the single crack value by more than 10%, whereas shielding occurs if a decrease of more than 10% is observed. Symmetry relations can be used to obtain different configurations of interacting cracks from one single BEM calculation [5]. It turned out that for selected ratios a_1/a_2 the characteristic domains of amplification and shielding may be characterized in the same

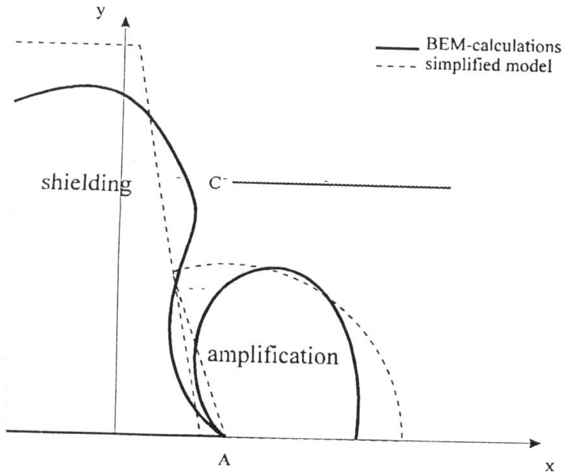


Figure 5: General shape of amplification and shielding domains

way as shown in Fig. 4 for the special case of parallel cracks of the same length. For symmetry reasons, it is only necessary to show the upper half of the domains. In the more general case of an arbitrary oriented second crack which may have different size, the characterization of the domain was as shown in Fig. 6 for two selected cases:

- We have two domains of amplification (denoted by **I**) for crack tip loci C ahead of the crack tips A and B which can be approximated by circular sectors with origin in A or B' (B' is related to B and depends on a_2 and φ).
- We have a domain of shielding (denoted by **II**) for crack tip loci C above crack 1 which can be approximated by a parallelogram.
- Outside the two domains, the interaction effects are negligible (denoted by **III**).
- For nonparallel cracks, we have also a prohibited area to avoid intersection of cracks.

The parameters of the domains depend on the length ratio and on the orientation of the two interacting cracks. For a variety of different configurations, the domains were determined. The parameters of the domains, i.e. the radius of the amplification domain and the lenght values and the angle of the shielding domain parallelogram depend on the length ratio a_1/a_2 and on the angle φ of the relative orientation of the two cracks. From Fig. 6, it can be seen that the amount of shielding and amplification reduces if crack 2 is small compared to crack 1, as expected.

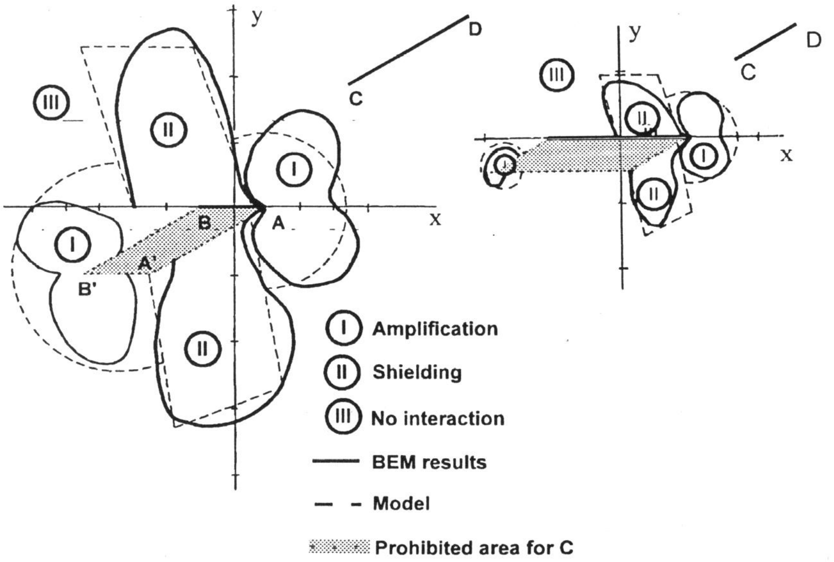


Figure 6: Domains of shielding and amplification for two selected crack configurations.

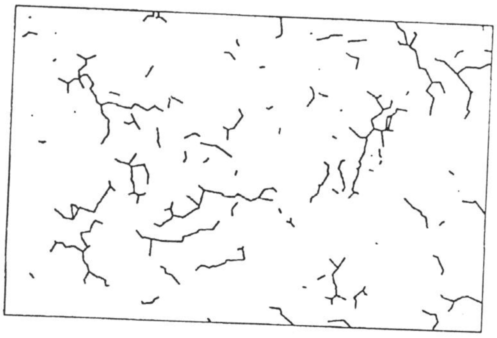


Figure 7: Simulated crack pattern

Summary

A fracture mechanics model for the analysis of branched cracks under equibiaxial loading was presented including the analysis of crack interaction. The model is used for the simulation of crack initiation and propagation under thermal loading. Therefore, it is essential that a large number of cracks with very different shapes can be analysed in a very efficient way. This is achieved by a characterization of crack geometries via characteristic crack lengths depending on the branches of the crack which lead to loading or shielding of the crack tip, respectively. Within the framework of this model crack interaction of branched cracks can be analysed in a straightforward way by consideration of 2 straight cracks with different orientation and length ratio. BEM analyses were used to obtain shielding and amplification domains which could be characterized by simple geometric shapes. The accuracy of the approximation results is quite satisfactory for cracks with few branches and is sufficient even for complicated cracks. An example of the simulated crack pattern obtained with the model described above is shown in Fig. 7.

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