A COMPARISON OF CLASSICAL CONTINUUM MECHANICS WITH A DISLOCATION MODEL TO DESCRIBE CYCLIC CRACK TIP PLASTICITY

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In the threshold regime of metals the fatigue crack growth rate is well below the predictions of the Paris relation, even if crack closure is taken into account. The deviation might be a result of the discrete nature of plasticity. To reveal differences between a continuous and a discrete description of plasticity we have developed a dislocation model. In this study our model is adopted to describe cyclic crack tip plasticity at a stationary mode I fatigue crack. Our results are compared to predictions of Rice's slip line model.

INTRODUCTION

It is generally recognized that the controlling parameter for fatigue crack growth is the cyclic plastic deformation at the crack tip. To get reliable predictions of the crack growth rate it is necessary to connect $\Delta CTOD$ (cyclic crack tip opening displacement) with the global loading (usually characterized by the far field stress intensity factor ΔK_{global}). At present relation derived by Rice [1] by means of continuum mechanics are commonly used. However, the applicability of these equations are limited: First, the size of the plastified region has to be small compared with overall specimen dimensions (i.e. crack length, specimen width, distance between the crack tip and the points of force application). Secondly the plastic zone size must be large in comparison to microstructural parameters (e.g. grain size, separation distance of slip bands, width of PSB etc.). Under service conditions the applied global loading is usually small, i.e ΔK_{global} is in order of the threshold value, and no difficulties arise with the upper limit, but $\Delta CTOD$ is often not much larger than the Burgers vector $(3 \cdot 10^{-10} m)$ of a lattice dislocation. One might guess that in this case the plastic deformation at the crack tip and, hence, the fatigue crack growth rate is significantly influenced by the microstructure, crystal anisotropy and the discrete nature of plasticity. There are some investigations concerning the influence of microstructure (e.g. [2]) and crystal anisotropy (e.g. [3]) but the discrete nature of plasticity has not been paid proper attention to so far. We

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have developed a discrete dislocation model which enables the investigation of a discretized plastic strain field.

The differences between the discrete description of plasticity and a continuous one are highlighted by a comparison of the discrete dislocation model with its continuous counterpart, the slip line model of Rice [1]. Crack tip plasticity was also modeled as motion of discrete dislocations by Li [4], Dai and Li [5], Pippan [6] for mode II and mode III cracks. Atkinson and Kay [7], Atkinson and Kanninen [8] and Lakshmanan and Li [9] considered dislocation arrangements at mode I cracks. These authors did, however, not reveal the fundamental differences between the discrete and a continuous description of plasticity.

DISLOCATION MODEL

The fundamentals and the procedure of the discrete dislocation model are explained in detail elsewhere [10]. Because of space restriction they are not repeated here. However, we recapitulate some important points in order to keep the readability of this study.

- In our simulation plasticity is described as motion of distinct edge dislocations. For the present study the dislocation model was adopted to describe a stationary mode I fatigue crack. The purpose of the investigation is to reveal differences between a discrete and a continuous description of crack tip plasticity. In our opinion the differences are the easier seen the simplier the model is. For that reason we developed a model which is equal to Rice's slip line model but with one distinction. Rice [1] described plasticity in terms of classical continuum mechanics, whereas plasticty is discretized in our investigations. Thus, we consider an unbounded body, which is cut along the negative x_1-axis. Plasticity is permitted on two planes symmetrically to the crack propagation plane.

- The dislocations are originated at the crack tip. Dislocation generation at internal sources (e.g. Frank-Read source) is not allowed in our simulations.

- A crack tip is shielded by dislocations, i.e. there is a difference between the far field K_{global} and the crack tip field k_{local}. The plastic flow (i.e. the motion of the dislocations) is controlled by K_{global}, whereas dislocation generation is solely determined by k_{local}. A pair of dislocations is emitted, when k_{local} reaches a critical value k_{e} [11].

- A dislocation feels a Peach-Koehler force. For convenience is split into a slip and into a climb component (Note that only edge dislocations are emitted). Since in our simulation the dislocations are not allowed to change their slip plane the motion of the dislocations is controlled by the slip force. A dislocation is in rest when the slip force is less than the lattice resistance, otherwise the dislocation moves in the slip force direction.

- Due to the emission of a (positive) dislocation $CTOD$ increases.

These are the important points in the loading procedure. A resulting dislocation arrangement is depicted in Fig. 1a. During unloading some dislocations near to the crack tip are forced (by the other dislocations and due to the attractive force of the free surface) to return to the tip, where they emerge (i.e. in the simulation they are removed). Due to the emergence of the dislocations $CTOD$ decreases. Parallel with the reduction of K_{global} the local stress intensity k_{local} decrases. Once k_{local} becomes less than $-k_{\text{e}}$ "negative" dislocations are generated. For clarity, dislocations emitted in the loading sequence are called "positive". "Positive" and "negative" means that the sum of Burgers vectors[2] of a dislocation pair points into the positive and the negative x_2-axis, respectively. $CTOD$ decreases upon emission of negative dislocations. In our simulation a positive and a negative dislocation annihilate if their separation distance becomes less than $10\,b$, here b is magnitude of the Burgers vector.

Material Parameter: The elastic constants are those of iron, i.e. shear modulus $\mu = 80GPa$, Poisson ratio $\nu = 0.3$. The critical stress intensity factor $k_{\text{e}} = 0.5MPa\sqrt{m}$. The angle of inclination was chosen to minimize the energy required for dislocation emission $\alpha = 70.5°$ and the friction stress is in the order of the lattice resistance of bcc metals $\tau_0 = \mu/2000$.

RESULTS

In our simulations we have left the material properties (k_{e},τ_0) constant, K_{min} was always set equal to zero but we varied the maximum load K_{max} in order to study the differences between the two models (discrete dislocation model, slip line model) as a function of loading. A typical dislocation arrangement is shown in Fig. 1.

In Fig. 2 the alteration of the global loading in a single run (K_{max}=4 k_{e}) and the corresponding variation of the local stress intensity is shown. Note that our calculations are based on the static equilibrium and compatibility conditions. Consequently, time does not enter into our simulation and the time scale in Fig. 2 is given in arbitrary units. The diagram shows that k_{local} is always between[3] $+k_{\text{e}}$ and $-k_{\text{e}}$. The reason is the full shielding of the crack tip by

[2] The sign of the Bugers vector is determined by the RHSF convention

[3] It is interesting to note that due to the limitations of k_{local} the term "local driving force" is physically nonsensical. A further discussion of the significance of dislocation shielding on fatigue fracture theory is given elsewhere [12]

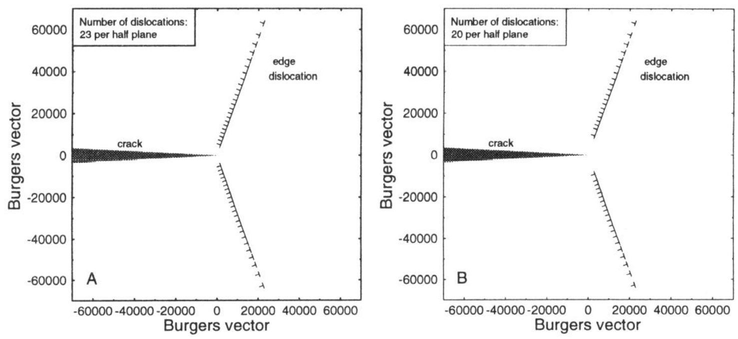

Figure 1: Discrete dislocation arrangement. a) at maximum load $K_{\max} = 1.5\,k_e$ b) at minimum load $K_{\min} = 0.0$

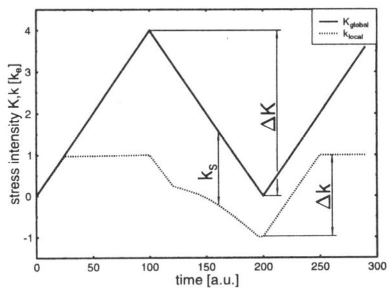

Figure 2: The behavior of K_{global} and k_{local} during one loading cycle.

those dislocations which are generated due to the cyclic loading.

In Fig. 3a the development of the monotonic plastic zone size during the first loading is depicted . w is the maximum extent of the plastified region. The results are plotted in a $log(w)$–$log(K_{\text{global}}/k_e)$ diagram, i.e. the maximum extent of the plastified region in Rice's slip line model [1] appears as straight line with a slope equal to 2. For large K_{global} levels both the classical continuum mechanics and the discrete dislocation model lead to the same result. For small K_{global} (less than $3\,k_e$) large differences appear. They are caused by a lack of plasticity at K levels smaller than k_e. Analogously appears the change of δ_t ($= CTOD$) with increasing K_{global} in Fig. 3b. Again there is a comparison to Rice's solution [1].

The influence of the discretized description of plasticity on a cyclically loaded crack is depicted in Fig. 4. Fig. 4a shows the dependence of the $\Delta\delta_t$ on the

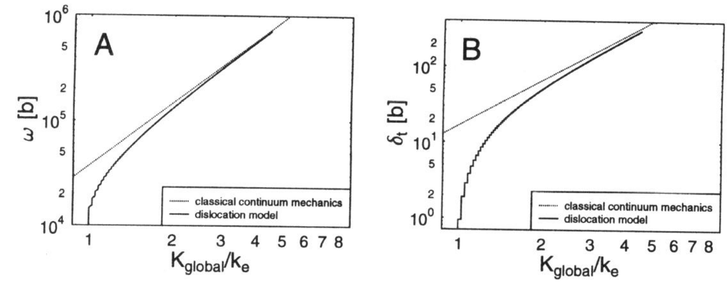

Figure 3: a) Monotonic plastic zone size and b) the crack tip opening displacement in units of Burgers vectors as a function of K_{global}

stress intensity range.

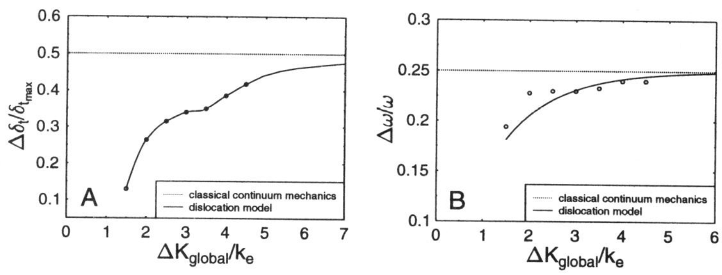

Figure 4: Comparison of the estimated a) cyclic crack tip opening displacement $\Delta\delta$ and b) cyclic plastic zone size with predictions of continuum mechanics.

We assume that at high K_{global}-levels ($> 10\,k_{\text{e}}$) the discrete dislocation model would predict $\frac{\Delta\delta_t}{\delta_{t\,\text{max}}} = 0.5$. Then both the dislocation model and the classical continuum mechanics lead to the same result.

In Fig. 4b the evaluated $\frac{\Delta w}{w}$ ratios[4] are compared to their "theoretical" value 0.25. The fitting line (—) which is shown in the diagram is based on the assumption that for high ΔK levels the dislocation model should approach asymptotically the classical continuum mechanics result.

CONCLUSION

[4]The cyclic plastic zone size Δw is the size of the region of cyclically moved dislocations.

The comparison of the discrete dislocation model and its continuous counterpart, Rice's slip line model, clearly demonstrated that:

- The plastic strains and, consequently, $CTOD$ are overestimated by classical theories; the deviations become insignificant at large K_{global} levels.
- The differences are even more pronounced at cyclically loaded cracks.

In the threshold regime of metals the fatigue crack growth rate is well below the predicted one by the Paris relation, even if crack closure is taken into account. In our opinion this is caused by the discrete nature of plasticity.

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