Statistical Fracture Analysis in SiC_p-Si_3N_4 Composites

P. Lipetzky+ and W. Kreher*

Crack advance in multi-phase composites is influenced not only by the externally applied loads, but also the internal residual stresses arising from thermal expansion mismatch and the strength of individual components. The influence of internal stresses is pronounced in brittle materials due to their sensitivity to local stress concentrations. The material being modelled in the current investigation is a SiC-reinforced Si_3N_4 composite with a random microstructure and a range of volume fractions, particle sizes and interfacial toughnesses. The variable with the strongest influence on the apparent crack resistance is interfacial toughness.

INTRODUCTION

The analysis of crack advance and specimen failure necessarily involves the analysis of the stress state that exists in the neighborhood of a crack tip which arises from either internal or external sources. In ceramic materials a common source of internal stress is the thermal expansion anisotropy or mismatch in any one or more of the components. For ceramic composites these effects can be particularly important due to relatively high processing temperatures, combined with the fact that stress relaxation occurs primarily through limited visco-plastic flow. Calculations have shown, in fact, that thermal stresses can arise which are in the range of material strength, so thermal stresses can greatly influence the fracture behavior of the composite.

In order to more closely approximate actual material behavior this investigation considers random, rather than deterministic residual stress calculations. Previous statistical analyses of crack propagation have been extensive but have not focussed on the influence of random residual stresses and second phase particles on crack path. For example, Chudnovsky

+ MPI für Metallforschung, D-70174 Stuttgart, Germany
* MPG, AG Mechanik het. Festkörper, D-01069 Dresden, Germany

and Kunin have addressed the problem of a crack growing in a brittle solid which contains randomly fluctuating strength properties [1]. Although their work statistically analyzes crack advance, they considered no influence of second phase particles on the crack deflection mechanisms. In the present work, the effects of fluctuating random internal stresses and variable interfacial toughness on the behavior of cracks in a ceramic composite are examined.

MODEL AND METHOD

Crack propagation in this composite is influenced by both random and deterministic quantities, thus an appropriate model must also account for these factors. The model used in the current investigation, shown schematically in Figure 1, is developed for the simplified case of a through, planar crack which crosses many microstructural features in a two-phase composite as is typically encountered in a fracture toughness test. The residual stresses that arise in the material generate internal stress intensity factors (SIF's) both of mode I and mode II opening denoted by K_I and K_{II}, respectively. In addition to the randomness of the stresses or driving forces, a weak particle-matrix interface and variable particle position correspond to the case of a random level of crack resistance. Distinct values are given to the fracture resistance, Γ, of the matrix, particle and interface, generating possible $\Gamma(\theta)$ curves where θ is the angle measured from the main crack plane. Total energy release rate and fracture resistance curves are then compared for many possible combinations of stress and geometry in order to determine the probability and nature of crack extension. The fracture criterion that will be used for the remainder of this work is therefore based on the inequality,

$$G(\theta) \geq \Gamma(\theta) \tag{1}$$

where G is the energy release rate. Other criteria can also be applied.

A critical aspect of this investigation is the proper determination of the random stresses that arise in the unloaded material due to the differential thermal expansion. There are many deterministic field calculations in the literature based on FEM and BEM results, however considering the random microstructure of composite materials, it is necessary to apply a direct statistical theory. This type of analysis was initiated primarily by Beran [2,3]. Scientists working in this field often must assume simplified forms of the microstrucural correlation function and elastic field interaction. However, a more general model is available which takes into account some microstructural information as well as global information contained in the measured or calculated effective elastic constants. This is the Maximum Entropy Formalism which can be used for the determination of average component stresses and fluctuation of the internal stress or strain field [4,5]. We shall use the Entropy method to calculate the stress distributions here because of its non-deterministic nature and its link to measurable mechanical properties.

With the knowledge of the probability characteristics the stress distributions must obey it is possible to randomly select specific values to be used in the approximation of internal stress intensity factors. Given possible combinations of internal stresses, the internal mode I and II SIF for a planar crack of length a with tips located at $x = 0, a$ are calculated as [6]:

$$K_{I,II}(a) = \sqrt{\frac{2}{\pi a}} \int_0^a \sigma_{I,II}(x) \sqrt{\frac{x}{a-x}} dx \qquad (2)$$

where $\sigma_I = \sigma_{yy}$ and $\sigma_{II} = \sigma_{xy}$. Here, $\sigma(x)$ is the stress distribution that exists in the material along the crack line in the uncracked composite and $K(a)$ is the SIF at location a. The superpositioning of external tension approximately increases the SIF values according to the equation for an elastically homogeneous body, $K_I = K_I^i + K_I^e = K_I^i + \sigma_{yy}^e \sqrt{Ya}$ where a is the total crack length, K^i is the residual stress generated SIF, K is the total SIF and Y depends on the sample geometry ($Y = \frac{\pi}{2}$ for an internal crack). Subsequently each realization of the mode I and II stress intensity factors can be used to calculate an energy release rate distribution as a function of angle from the crack tip according to a homogeneous approximation function [7].

The knowledge of possible internal stress intensity factors and energy release rates leads to our ultimate goal of characterizing the probability and nature of crack growth. This problem is addressed using the Monte Carlo method, wherein the crack extension probability is calculated as the percentage of total realizations which satisfies the inequality, $G(\theta) \geq \Gamma$. Crack extension probability is then correlated with the applied load to determine the macroscopic toughness.

RESULTS

The data necessary for these calculations are the Young's modulus, Poisson's ratio, and α the coefficient of thermal expansion of the material components, SiC and Si_3N_4 [8,9,10]. From the Maximum Entropy Formalism of Kreher and Pompe it follows that the residual stresses can be described by Gaussian probability densities. The expectation values and standard deviations of the residual stresses in the matrix and particles are given in Table 1 [5,9]. The residual stresses in each crack segment

TABLE 1

Material	$<\sigma_{xx}>=<\sigma_{yy}>$	$\Delta\sigma_{xx}$	$<\sigma_{xy}>$	$\Delta\sigma_{xy}$
matrix	-83 MPa	82 MPa	0 MPa	45 MPa
particles	195 MPa	85 MPa	0 MPa	53 MPa

are therefore randomly assigned from within these distributions. The geometrical factors of the model are based on the two mean particle diameters of 20 and 50 μm and the volume fractions of 10, 20 and 30 %.

We must now examine the internally generated SIF because they necessarily alter the externally applied value. Results show that the average values of $K^i_{I,II}$ are -0.5 and 0.0 $MPa\sqrt{m}$ respectively, with standard deviations of roughly $1.5 MPa\sqrt{m}$. Recall that because the externally applied load is zero the randomness of the total stress intensity factors is a direct result of the many possible stress states that may exist in a random microstructure along a given crack.

Failure probability as defined above is the probability that a given realization of $K_{I,II}$ will result in an energy release rate greater than the critical value. It is understood to be a "failure criterion" because it corresponds to the onset of crack advance. Figure 2 shows the failure probability as a function of K^e_I for the cases $\Gamma_{int} = 70\frac{J}{m^2}$, $30\frac{J}{m^2}$ and $10\frac{J}{m^2}$. The maximum value of 70 is chosen to equal the fracture resistance of the components. Decreasing the fracture resistance of the interface necessarily influences the failure probability of the composite by opening avenues for crack advance at energy release rates lower than the critical values for the constituents. Figure 2 shows quantitatively that decreasing the crack growth resistance of the interface increases the probability of failure at a given load. The orientation of the interface also plays a role. therefore we assume that any position on the particle profile can be struck with an equal probability.

In order to predict directly measurable quantities we must now calculate the first order moment of the probability density which corresponds to the average macroscopic toughness of the specimen. First, the failure probability curves as shown in Figure 2 are used for the determination of the probability densities as a function of K_I^e and subsequently G_I^e. Then the average critical toughness, G_c is calculated. Figure 3 summarizes these results for various material combinations. Notice that a macroscopic G_I^e can be applied that is significantly higher than the critical energy release rate of the components prior to the onset of crack advance as a result of residual stresses. As shown in Figure 3, the average G_I^e at failure is a strong function of interfacial crack growth resistance until the interfacial toughness exceeds roughly 50% of the component value. Large particles result in composites which are able to sustain higher G^e prior to the onset of crack advance while volume fraction has only a small effect [11].

It is also possible to calculate other experimentally observable quantities such as the amount of interfacial failure because as seen above, a crack can be classified clearly as interfacial or transparticle. Utilizing the above results, the calculated percentages of interfacial failure (PIF) as a function of Γ_{int} are plotted in Figure 4. In addition to the amount of interfacial failure, it is also of interest to determine the topology of the crack path as an indication of possible fracture surface roughness. The calculations show that roughness increases with increasing particle size and decreasing interfacial toughness. Roughness in all material combinations is on the order of the particle size.

DISCUSSION

The failure probabilities and subsequent calculations detailed above facilitate the understanding of the influence of interfacial toughness and residual stress fluctuations on local crack advance in a heterogeneous microstructure. As shown on Figure 3, significant global weakening does not occur until interfacial crack resistance decreases below $\sim 30 J \cdot m^{-2}$. In contrast, Figure 4 shows a significant increase in PIF as interfacial toughness decreases. In order to substantiate this result we will consider the amount of interfacial failure predicted by a different theory. Figure 4 shows the variation in the amount of interfacial fracture predicted here compared to that shown by Krell and Blank using an analytical model proposed by Cotterell and Rice [12,13]. Although the approaches are very different, the agreement is very good. This comparison is of particular interest because both of the models apply to a long, straight crack with a small increment of the crack tip extending out of the main crack plane.

Given the correlation between fracture surface roughness and K_{Ic} in some brittle materials, the results given here may reasonably be combined to predict an optimum composition for a given application. For example, increasing the amount of interfacial or intergranular fracture (crack deflection) has clear implications for fracture surface roughness and macroscopic crack resistance. Therefore the promotion of crack deflection may be necessary if higher toughness is desirable [14,15,16,17]. Alternatively decreasing Γ_{int} lowers the average G^e that a specimen can sustain before crack advance begins. These two factors must therefore be considered together due to their offsetting effects.

The final point of analysis for this model is in regard to the location of the crack tip which has been assumed to end in the matrix phase as shown in Figure 1. Physically, the crack tip may be in either phase as well as on the interface. However, when the crack ends in a particle it will see an increased K_I at its tip because of the tensile residual stresses. Therefore, upon loading the crack will start propagating into the matrix and become arrested because of the compressive residual stresses in the matrix phase. Thus a situation with the crack tip inside the particle will stably become the situation considered in our theory.

SUMMARY AND CONCLUSION

The stochastic analysis of crack growth presented above shows that macroscopic failure analysis of heterogeneous materials cannot be complete without considering microscopic crack advance as affected by both local and global variables. Internal stresses and their fluctuations as well as the toughness of individual components and interfaces have been shown to be important. This is most clearly demonstrated in the present results by the fact that the applied K or G necessary to advance a crack can be up to 30% higher than the critical values of the constituent materials due to residual stress effects. Furthermore, the effective toughness is at least as high as the component toughness until the critical energy release rate of the interface decreases to roughly less than 50 % of the component crack resistance. This result is directly attributed to the effects of residual compression in the matrix and lower energy release rates when a crack deviates from its original plane. This is not to say that residual stresses alone will create such observable changes, only that the influence is strong and complete analysis must include such effects. However, other potentially important factors such as crack bridging are being neglected. To further verify the results of this model deterministic FE calculations can be made to quantify the effects of a particle and interface on the energy release rate distribution at a crack tip. This should be done to compare the predictions of the present statistical theory with several deterministic microstructural arrangements.

ACKNOWLEDGEMENTS

Financial support from the Max-Planck-Gesellschaft is greatfully acknowledged. We also thank Drs. R. Dittrich and A. Krell for many helpful discussions.

REFERENCES

[1] Chudnovsky, A. and B. Kunin. (1987). A probabilistic model for brittle crack formation. J.Appl.Phys. (15.Nov.1987), 62, 4124-29.

[2] Beran, M. J. (1968). Statistical continuum theories (Vol.9 of Monographs in Statistical physics). London, New York: Interscience.

[3] Beran, M. J. (1980). Field fluctuations in a two-phase random medium. J. Math. Phys., 21, 2583-85.

[4] Kreher, W. and W. Pompe. (1985). Field fluctuations in a heterogeneous elastic material - an information theory approach. J. Mech. Phys. Solids, 33, 419-45.

[5] Kreher, W. and W. Pompe. (1989). Internal stresses in heterogeneous solids. Berlin: Akademie Verlag.

[6] Rice, J. R., (1968). Mathematical Analysis in the Mechanics of Fracture. In Liebowitz, H. (Ed.) "Fracture - An Advanced Treatise",Vol II, Academic Press, New York, 1968, pp. 191 - 311.

[7] Hussain, M.A., S.L Pu and J. Underwood (1974). "Strain Energy Release Rate for a Crack under Combined Mode I and Mode II", Fracture Analysis, ASTM STP 560, ASTM, 1974, pp 2 - 28.

[8] Li, Z. and R. C. Bradt. (1989). Micromechanical stresses in SiC reinforced Al2O3 composites. J.Am.Ceram.Soc., 72, 70-77.

[9] Kreher, W. and R. Janssen. (1992). On microstructural residual stresses in particle reinforced ceramics. J.Europ.Ceram.Soc., 10, 167-73.

[10] Ziegler, G., J. Heinrich and G. Wötting. (1987). Relationships between processing, microstructure and properties of dense and reaction-bonded silicon nitride. J. Mat. Sci., 22, p3041 - 86.

[11] Lipetzky, P. and W. Kreher. (1993). submitted to Mech. Mat.

[12] Krell, A. and P. Blank. (1992). Inherent reinforcement of ceramic microstructures by grain boundary engineering. J. Europ. Ceram. Soc., 9, 309-22.

[13] Cotterell, B., and J. R. Rice. (1980). Slightly Curved or Kinked Cracks. Int. J. Frac., 16, 155-69.

[14] Green, D. J., P. S. Nicholson and J. D. Embury. (1979). Fracture of a brittle particulate composite, Part 1 Experimental Aspects, JMS, 14, 1413-1420.

[15] Freiman, S. W., G. Y. Onoda and A. G. Pincus. (1974). Mechanical Properties of $3BaO{\cdot}5SiO_2$ Glass Ceramics, J. Am. Ceram. Soc., 57, 8-12.

[16] Borom, M. P., A. M. Turkalo and R. H. Doremus. (1975). Strength and Microstructure in Lithium Disilicate Glass-Ceramics, J. Am. Ceram. Soc., 58, 385-391.

[17] Lange, D.A., H.M. Jennings and S.P. Shah (1993). Relationship between Fracture Surface Roughness and Fracture Behavior of Cement Paste and Mortar. J. Am. Ceram. Soc., 76, 589 - 597.

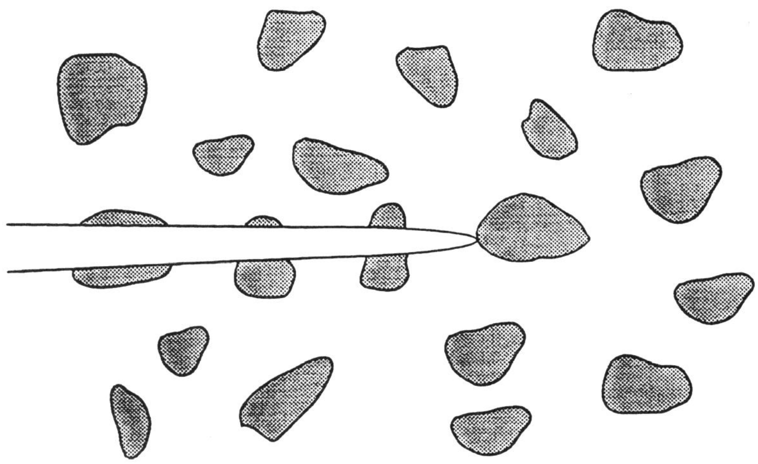

Figure 1 Schematic of the two-dimensional crack-particle model. A long, planar crack terminates at a particle-matrix interface.

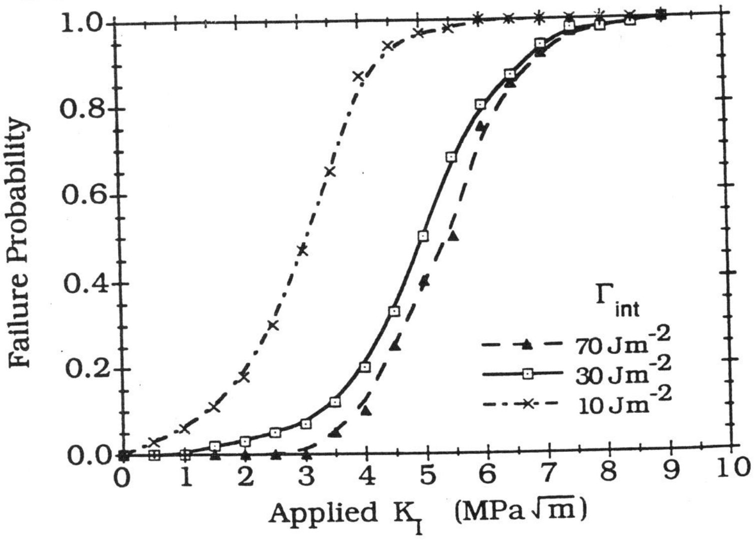

Figure 2 Failure Probability plotted as a function of applied K_I for a range of interfacial fracture resistance.

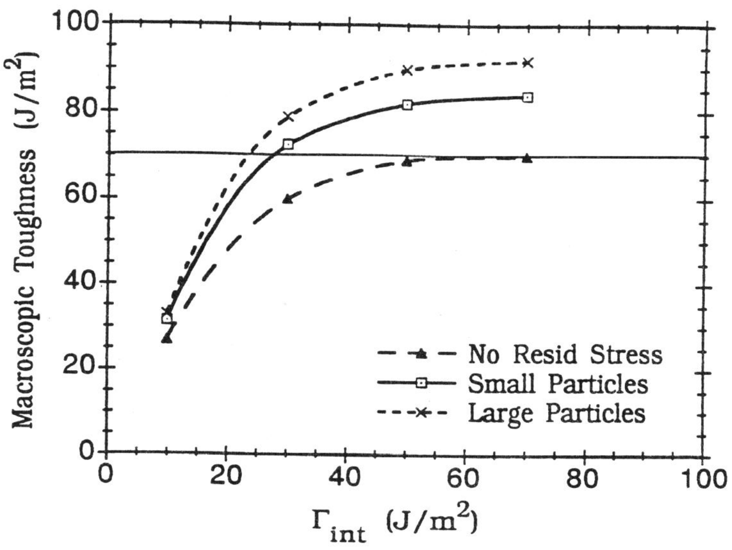

Figure 3 Average macroscopic toughness values plotted as a function of interfacial toughness. Component toughness equals 70 J/m^2.

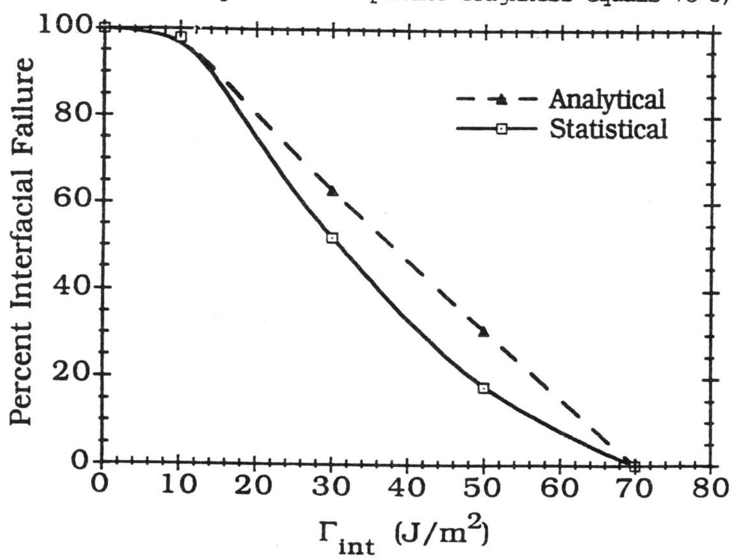

Figure 4 Predicted values of PIF plotted as a function of interfacial toughness. Results compare well to an analytical model.