# PROBABILISTIC APPROACH TO STRUCTURAL INTEGRITY ASSESSMENT AND LIFE-TIME PREDICTION

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The probabilistic approaches in assessing structural integrity reflect the fact that the fracture properties, crack sizes, and the fracture mechanics parameters K, J are random quantities or stochastic processes (time dependence). In assessing structural integrity, the risk (probability) of failure is calculated instead of the safety coefficient. The philosophy enables the reliability of the entire structure in question to be evaluated while accounting for the actual number of the critical locations (the size effect). In addition to the probabilistic failure risk evaluation models, also the elementary principles of application of the probabilistic model for the economic assessment of the system service life are presented.

## INTRODUCTION

The state-of-the-art methodologies of assessment of structural resistance to fracture have been based in calculating the safety coefficient in dependence on the crack length. By including also the extreme values or selected probability ranges, the dispersion of the material fracture properties can be accounted for (Milne et al. (1), Schwalbe and Cornec (2), Ainsworth et al. (3)). While the capacity of the methodology to provide for a detailed analysis of the respective effects of the different quantities is not doubted, it cannot address the problem of structural reliability assessment in terms of the potential occurrence of cracks of different lengths etc. Neither allows the use of the safety coefficients philosophy for assessing the reliability of the structure as a whole (the size factor), especially in cases when the structure consists of a large number of elements or critical locations (a welded piping system, gas- or oil-lines).

By assuming the structural behaviour being governed by concurrent joint effects of numerous factors which are random in their nature (material properties, occurrence of cracks of different sizes), probabilistic approaches and methods can be used to address the reliability problems (Provan (4)).

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However, with long-term exposure to the service loads also the fracture characteristics tend to degrade, especially so if the material is subject to elevated temperatures (such as temper embrittlement). Therefore, in-service degradation of the fracture characteristics can cause these parameters to decline even below the critical level, thus causing the structure to fail even if the initial state had been fully acceptable. Hence also the issues of assessing the service lives of systems in view of sudden loss of integrity must be addressed.

## FRACTURE PROPERTIES AS STOCHASTIC PROCESS

In modelling the fracture properties the probability distribution function shall be assumed to be given as a function of time t

$$P[K_C(t) \leq K] = F_{KC}[K; t] \tag{1}$$

The merits of such a model of fracture properties in terms of practical applications can be seen for example from Fig.1 in which the experimental results for the Charpy-V energy tests are plotted in dependence on the Larson-Miller parameter P. The test specimens were annealed at several different temperature levels for different exposure times. Following the temperature exposure, the Charpy-V energy was measured at 20°C and estimated using the relation proposed in (5)

$$\log(KCV) = A_1 + A_2P + A_3P^2 \tag{2}$$

where $P = T(\log t + A_4)$ and t is the exposure time at temperature T.

Since there exists a relation between the Charpy-V energy and fracture toughness, relation (2) can be expected to hold also for fracture toughness predictions as demonstrated in paragraph below.

## PROBABILISTIC MODEL OF FRACTURE

Considering the monotonous decrease of the fracture properties governed by equation (2), relation (1) can be interpreted as the conditional

$$P[K_C(t) \leq K] = P[t_C \leq t | K] \tag{3}$$

probability (under given K - Fig.2) of sudden fracture occurring before time t, i.e. where $t_C$ denotes the time of sudden fracture as a random variable.

Since K is a function of the crack size and stress, a more general formula to calculate the risk of fracture and the time prediction can be derived. The basic principles of fracture mechanics are used for the probabilistic model, with the stress intensity factor or the J-integral used to characterise the stress field in the crack tip (Milne et all (1)) being considered to be the main variables controlling the process. If K or J reach their critical values $K_C$ or $J_C$, sudden fracture of the material occurs. Using the language of probabilities, the above phenomenon can be formulated as follows. As the event **B** we denote the situation in which the critical value of the fracture characteristics at time t is exceeded, i.e. sudden fracture

$$\mathbf{B} = \{K_C(t) \leq K\} \text{ where K is assumed to be random quantity.} \quad (4)$$

The events $\mathbf{A}_i$ shall denote the stress intensity factor K at time t falling in the elementary interval $<z_{i-1}, z_i)$, i.e.

$$\mathbf{A}_i = \{K_i \in <z_{i-1}, z_i)\}\ ,\ z_{min}=z_o < z_1 <...< z_i <...< z_n = z_{max}. \quad (5)$$

It follows from the above definition of the events $\mathbf{A}_i$ that $\mathbf{A}_i \cap \mathbf{A}_j = \varnothing$ for $i \neq j$. Event **B** can thus be expressed as

$$\mathbf{B} = \bigcup_{i=1}^{n} (\mathbf{B} \cap \mathbf{A}_i).$$

Since $(\mathbf{B}\cap\mathbf{A}_i)\cap(\mathbf{B}\cap\mathbf{A}_j) = \varnothing$ for $i \neq j$, the probability of **B** can be written as

$$P(\mathbf{B}) = \sum_{i=1}^{n} P(\mathbf{B}\cap\mathbf{A}_i). \quad (6)$$

From the conditional probability theorem (Gnedenko (6)), the relationship

$$P(\mathbf{B}|\mathbf{A}_i) = \frac{P(\mathbf{B}\cap\mathbf{A}_i)}{P(\mathbf{A}_i)} \quad (7)$$

follows. By substituting relation (7) into relation (6), equation

$$P(\mathbf{B}) = \sum_{i=1}^{n} P(\mathbf{B}|\mathbf{A}_i).P(\mathbf{A}_i) \quad (8)$$

can be obtained for the probability of the event **B**.

The risk of occurrence of sudden fracture can be calculated using the above relation provided the probabilities $P(\mathbf{B} \mid \mathbf{A}_i)$ and $P(\mathbf{A}_i)$ are more closely specified. The event $(\mathbf{B} \mid \mathbf{A}_i)$ denotes the occurrence of sudden fracture with the critical values of $K_C(t)$ or $J_C(t)$ being less than $K_i$ or $J_i$ respectively at the time t, i.e.

$$\{\mathbf{B} \mid \mathbf{A}_i\} = \{ K_C(t) \leq K_i \} \text{ or } \{\mathbf{B} \mid \mathbf{A}_i\} = \{ J_C(t) \leq J_i \}. \quad (9)$$

From the above identity of events (9) also the equality of probabilities (Kampen (7)) follows

$$P(\mathbf{B} \mid \mathbf{A}_i) = P[K_C(t) \leq K_i]\ ,\quad P(\mathbf{B} \mid \mathbf{A}_i) = P[J_C(t) \leq J_i]\ . \quad (10)$$

The probabilities in relation (10) can be calculated from relation (1).

The probabilities $P(\mathbf{A}_i)$ of events $\mathbf{A}_i$ in relation (5) are the probabilities with which the stress intensity factor falls in the interval $<z_{i-1}, z_i)$. These probabilities can be calculated using the probability density functions for the crack size a(t) - g(a;t) and stress $\sigma(t)$ - $h(\sigma;t)$ respectively. For this purpose, both these processes can be assumed to be stochastically independent. The set $\Omega_i$ (j, k) = {$a_j$, $\sigma_k$ for which $K_i \in <z_{i-1}, z_i)$} is determined and the formula

$$P(\mathbf{A}_i) = \sum_{\Omega_i(j,k)}\sum g(a_j;t)\Delta a_j . h(\sigma_k;t)\Delta\sigma_k \tag{11}$$

can be found to hold for the probability $P(\mathbf{A}_i)$. After substituting (10) and (11) into (8), relation

$$P(\mathbf{B}) = \sum_{i=1}^{n} P[K_c(t) \leq K_i] \sum_{\Omega_i(j,k)}\sum g(a_j;t)\Delta a_j . h(\sigma_k;t)\Delta\sigma_k$$

$$P(\mathbf{B}) = \sum_{j}\sum_{k} P[K_c(t) \leq K(a_j,\sigma_k)].g(a_j;t).h(\sigma_k;t)\Delta a_j \Delta\sigma_k \tag{12}$$

can be derived which, for $n \to \infty$ assumes the form

$$P(\mathbf{B}) = \int_0^{s}\int_0^{\sigma_m} P[K_c(t) \leq K(a,\sigma)].g(a;t).h(\sigma;t)da d\sigma. \tag{13}$$

The crack growth and stress processes can be assumed to be processes increasing with time (e.g. due to material fatigue damage and corrosion respectively) and hence also the stress intensity factor shall be a growing random function - cf. Fig.3. In that case relation (13) can be interpreted as the risk of failure by sudden fracture before time t, i.e.

$$P[t_C \leq t] = P(\mathbf{B}). \tag{14}$$

Relation (13) can be further modified to allow the application of the two criterion.Under the two criterion, fracture occurs whenever any operating point defined by its co-ordinates $(S_r,K_r)$ falls above the limit curve G. Fracture occurs whenever the condition (event **B**)

$$\mathbf{B} = \{K_r(t) \geq G(S_r)\} \tag{15}$$

occurs (Provan (4)).In the above relation (15), G stands for the function defining the limit curve depending on $S_r$. $K_r(t) = K(a,\sigma)/ K_C(t)$ ; $S_r = \sigma/\sigma_f$ where $\sigma$ is the effective stress and $\sigma_f$ is the stress flow.

Relation (15) can be rearranged after substituting $K_r$, $S_r$ into

$$\mathbf{B} = \{K_C(t) \le K(a,\sigma)/G(\sigma/\sigma_f)\} = \{K_C(t) \le K^*\}. \qquad (16)$$

The events $\mathbf{A}_i$ will be defined analogously using relation (5).

$$\mathbf{A}_i = \{K^*_i \in \langle z_{i-1}, z_i)\}\ ,\ z_{min} = z_o < z_1 < \ldots < z_i < \ldots < z_n = z_{max} \qquad (17)$$

Conditional event $\mathbf{B}$ under given $\mathbf{A}_i$ can be expressed by relation

$$\{\mathbf{B} \mid \mathbf{A}_i\} = \{K_C(t) \le K^*_i\},\ \text{where}\ K^* = K(a,\sigma)/\ S(\sigma/\sigma_f). \qquad (18)$$

Consequently, the probability of $\{\mathbf{B}|\mathbf{A}i\}$ expresses the risk of sudden fracture before time t at given $K^*_i$. The unconditional probability - failure risk - is obtained analogously as described for relation (12) or (13). In addition, the material variables such as $K_C(t)$ and $\sigma_f(t)$ degrade with time monotonously. Stress can be assumed to grow in time due to effects like element wall thickness decrease due to corrosion. The crack size is always a quantity growing with time. Using the above assumptions, the trajectory of the point $(S_r,K_r)$ shall be a monotonously growing curve (see Fig.4). The risk of fracture, or the probability of fracture occurring before time t, can be calculated by integrating as

$$P[t_C < t] = \int_0^s \int_0^{\sigma_m} \int_0^{\sigma_{fm}} P\left[K_C(t) \le K(a,\sigma)/G(\sigma/\sigma_f)\right] g(a;t) hg(\sigma;t) f(\sigma_f;t) da\, d\sigma\, d\sigma_f \qquad (19)$$

where $g(a;t)$, $hg(\sigma;t)$ and $f(\sigma_f;t)$ are the respective probability density functions of the crack size a, effective stress and the stress flow at time t.

## PROBABILITY DISTRIBUTION FUNCTIONS OF THE CRACK SIZE

A number of different probability distribution or density functions have been proposed for the quantity such as the exponential or Weibull (8) functions all of which, however, can model one dimension of the crack only, while for more realistic modelling at least two dimensions - the length and the depth - are necessary. In case stochastic independence is assumed, the multi-dimensional distribution function can be relatively easily obtained as the product of the respective crack length and depth distribution functions. However, it has been demonstrated that rather than independent, crack lengths and depths tend to be statistically correlated quantities. In the paper, the truncated two-dimensional normal probability distribution with a correlation coefficient (Gnedenko (6)) was used.

## APPLICATION OF PROBABILISTIC METHODS FOR SYSTEM INTEGRITY EVALUATION

Using the relations defining the risks of failure of the different elements of the structure the risk of failure of the whole structure can be calculated as

$$P(\text{failure}) = 1-\prod_{i=1}^{m} [1 - P_i(\text{failure})] \qquad (20)$$

where P(failure) is the risk of failure of the whole structure; $P_i$(failure) is the risk of failure of i-th element of the structure and m is the total number of elements.

The above probabilistic methods were used to calculate the risk of failure of a piping system transporting hydrogen contaminated by $H_2S$. The system comprised 16 welds between the elbows and the straight pipe sections. The Weibull distribution function was used for the $K_{ISCC}$ (Mode I threshold stress intensity factor for Stress Corrosion Cracking). The degradation of the mean values of $K_{ISCC}$ was modelled using relation (1) obtained from experimental findings published in (9). The regression curve showing the dependence of $K_{ISCC}$ on the Larson-Miller parameter is presented in Fig.5. The standard deviation was calculated to be 0.125 of the mean, i.e. corresponding to the lower tolerance limit of 0.75 of the mean (9). The minimum values were calculated to be 0.625 of the mean value which corresponds to the reduction of the mean by three times the standard deviation.

The truncated two-dimensional normal distribution function was used to model the crack size distribution. The relative mean crack depth (longitudinal semi-elliptic cracks growing from the inner surface of the circumferential weld) was 0.23 times the wall thickness. The relative mean crack length was 0.69 of the wall thickness. The results obtained from the failure risk calculation performed for the whole piping system (for the correlation coefficient of 0.7) for different service times are presented in Fig.6.Inevitably, the issue of the acceptable risk level has to be dealt with, however. One possible alternative is to define the failure risk in view of the economic effects. The function

$$C(t) = Z + I(t) - O(t) - R(t).H \qquad (21)$$

can be introduced, where the symbols have the following meanings:
Z - the initial investment for the system erection, I(t) - the earnings from the system operation over time t, O(t) - operating costs over time t, R(t) - the risk of failure over time t, H - the costs incurred due to the failure.

The optimation of the function C(t) (its maximum) in time t could also serve as an indicator for the risk level definition.

## CONCLUSIONS

1. In a system which is safe as-designed, the deterioration of material properties results in a significant increase of the failure risk with service time.
2. With increasing the crack length/depth correlation coefficient, the calculated failure risk can grow by a factor of up to five.

## SYMBOLS USED

| | |
|---|---|
| $a(t)$, $\sigma(t)$ | = crack size and stress at time t |
| $K_C(t)$, $\sigma_f(t)$ | = fracture toughness and flow stress at time t |
| $K(a,\sigma)$ | = stress intensity factor as a function of the crack length a and stress $\sigma$ |
| $F_{KC}(K;t)$, $P[K_C(t)\leq K]$ | = probability distribution function of the fracture properties considered a stochastic process $K_C(t)$ |
| s | = wall thickness |
| $\sigma_m$, $\sigma_{fm}$ | = upper limit of the effective and flow stresess |
| $g(x;t)$, $h(\sigma;t)$, $f(\sigma_f;t)$ | = probability density function of the stochastic process a(t), stress $\sigma(t)$ and stress flow $\sigma_f(t)$ respectively |
| $P(t_C \leq t \mid K)$ | = risk of failure occurring before time t at a given value K of the stress intensity factor |
| $P(t_C \leq t)$ | = risk of failure occurring before time t |

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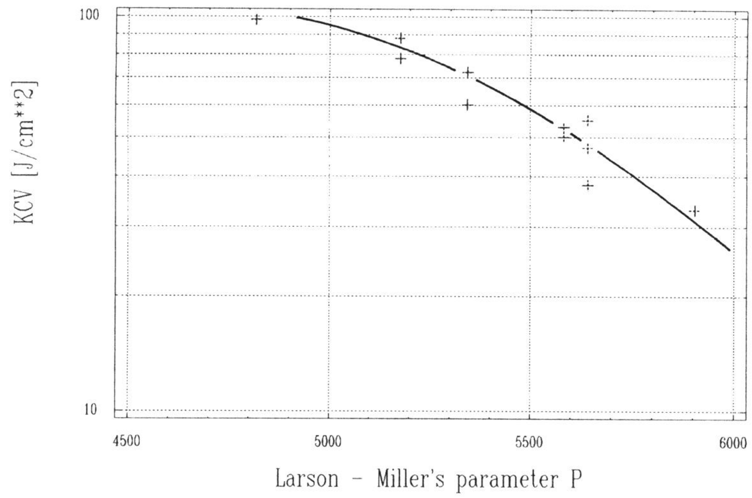


Fig.1 The effect of time-temperature exposition on notch toughness of Cr18Ni11Ti steel

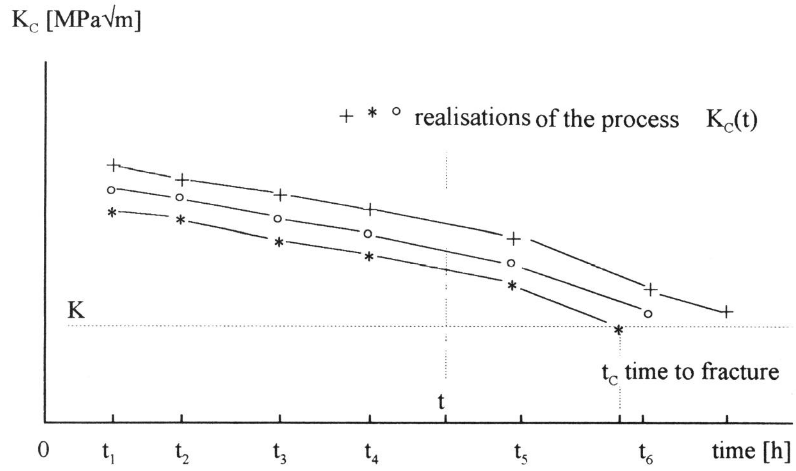


Fig.2 The shape of the $K_C(t)$ process in the time and intersection with K level

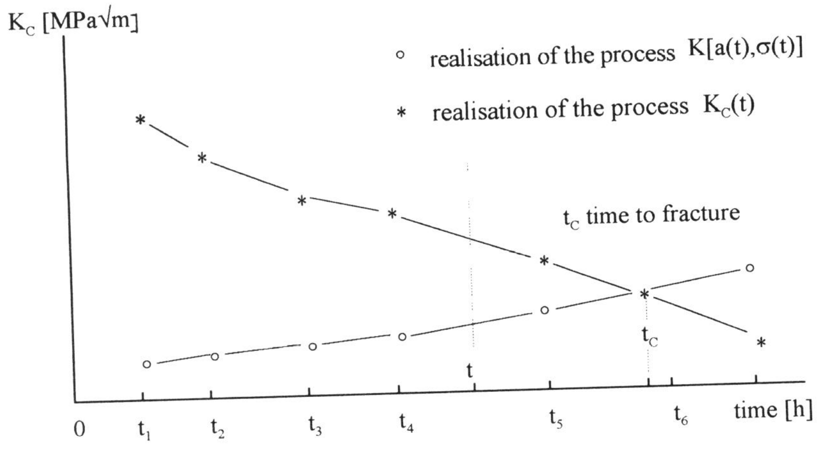


Fig.3 Characteristic shapes of processes $K_C(t)$ and $K[a(t),\sigma(t)]$

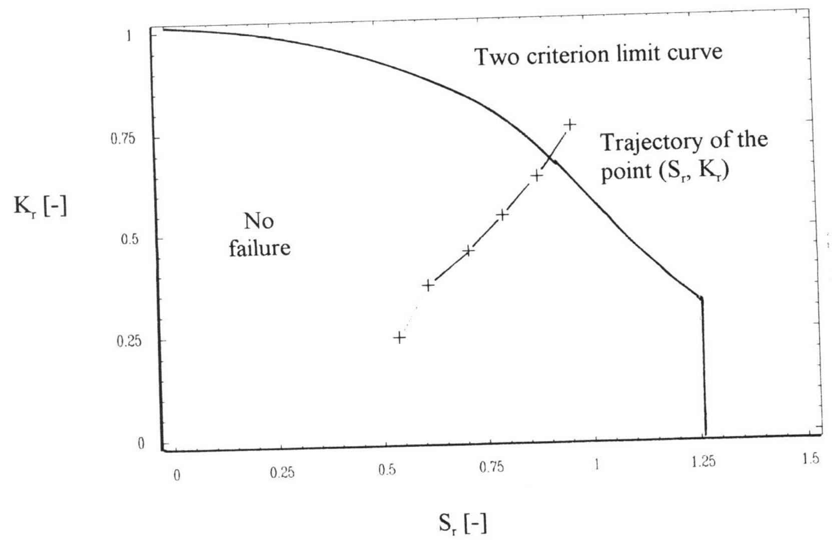


Fig.4 Trajectory of the points $(S_r, K_r)$ due to material damage and crack growth

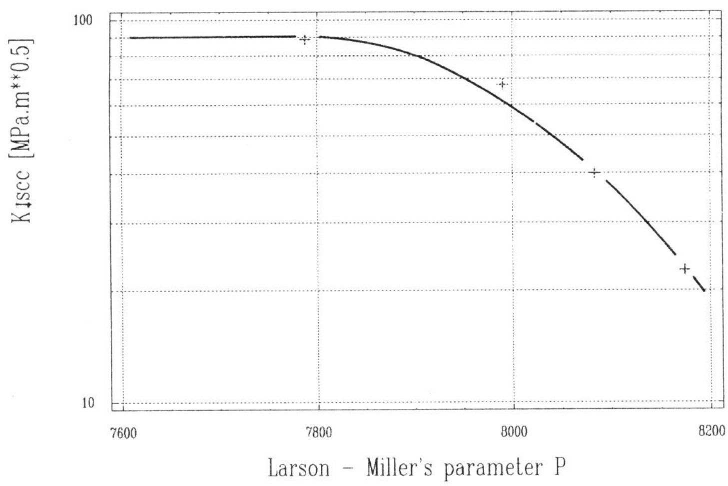


Fig.5 The effect of time-temperature exposition on $K_{ISCC}$ of Cr2.25Mo1 steel

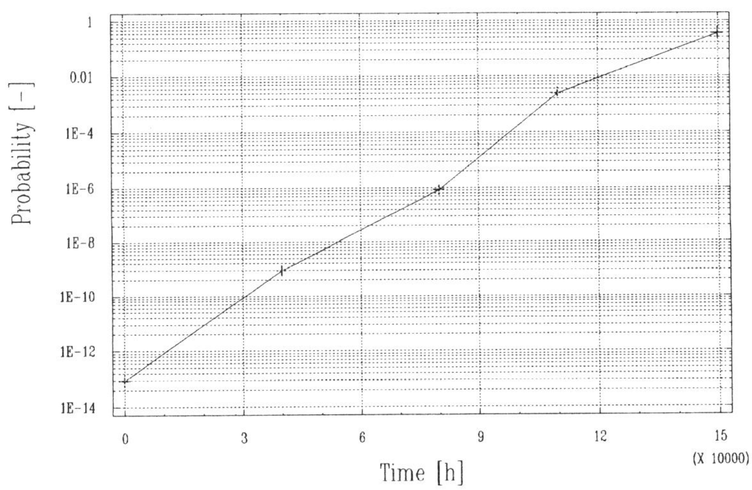


Fig.6 The failure risk growth with service time