CYCLIC PROPAGATION OF SUBSURFACE CRACKS

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The perturbation method developed by James Rice for somewhat circular tensile cracks is applied to the propagation of subsurface cracks. It is shown that the method allows for a cycle by cycle three-dimensional crack size and shape calculation under arbitrarily tensile loading in an economically feasible way.

INTRODUCTION

In order to perform crack propagation calculations in a three-dimensional linear elastic body usually a crack driving parameter such as the stress intensity factor K is determined for selected crack geometries. Since the current numerical methods (e.g. the Finite Element Method) are still quite expensive and time consuming for three-dimensional models, only a limited number of crack configurations can be examined and it is not uncommon to interpolate between the calculated values. Indeed, to be able to calculate the crack propagation due to an arbitrary load history, the knowledge of K for every crack size and geometry which establishes itself in the propagation process is needed. It is clear that due to the interpolation additional inaccuracies are introduced.

In the present article a perturbation method developed by James Rice is used to calculate the cycle by cycle crack propagation of a subsurface crack due to arbitrary loading. The method is based on the knowledge of the Green function for the K-solution along a circular crack in an infinite domain and is accurate to first

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order for small deviations from the circular form. However, it has been shown by Gao and Rice (1) that the method produces good results even for an elliptical crack with a ratio of the axes equal to two. After cycle extraction on the load history, the above technique is used to determine the K-solution for the starting crack geometry which together with an appropriate crack propagation law (e.g. Forman) leads to a new crack geometry. Using the perturbation method again to find the K-solution for the new crack front, this process is repeated for all successive cycles and produces a crack geometry history. Since the perturbation method yields the K-solution in a much shorter time than conventional procedures it is a convenient alternative to perform a cycle by cycle crack propagation calculation for up to a few thousand cycles.

BASIC THEORY

The governing equations can be found in the work of Gao and Rice (1). Consider a crack with a near circular shape described by $a(\psi)$ (Figure 1), under normal loading $p(r,\psi)$ where r and ψ are polar co-ordinates. Then the stress intensity factor at $\psi = \theta$ is given by

$$K(\theta)=K^{\circ}[\theta;a(\theta)]+\frac{1}{8\pi}PV\int_{0}^{2\pi}\frac{K^{\circ}[\phi;a(\theta)][a(\phi)/a(\theta)-1]}{\sin^{2}[(\phi-\theta)/2]}d\phi \tag{1}$$

where PV denotes the principal value and $K^{\circ}[\phi;a(\theta)]$ is the stress intensity factor at $\psi = \phi$ for a circular crack with radius $a(\theta)$ under the governing normal loading, i.e.

$$K^{\circ}[\phi;a(\theta)]=\int_{0}^{2\pi}\int_{0}^{a(\theta)}p(\rho,\chi)k(\phi;\rho,\chi;a(\theta))\rho d\rho d\chi \tag{2}$$

The function $k(\phi;\rho,\chi;a(\theta))$ is the K-factor at $\psi=\phi$ due to a unit compressive force at (ρ,χ) within a circular crack with radius $a(\theta)$:

$$k(\phi;\rho,\chi;a(\theta))=\frac{\sqrt{a(\theta)^{2}-\rho^{2}}}{\sqrt{a(\theta)\pi^{3}}[a(\theta)^{2}+\rho^{2}-2a(\theta)\rho\cos(\chi-\phi)]} \tag{3}$$

The integration needed to find $K^{\circ}[\phi,a(\theta)]$ can be accelerated by eliminating the $R^{-3/2}$ singularity at $(r,\psi)=(a(\theta),\phi)$. This can be done by subtracting a constant pressure term $p(a(\theta),\phi)$ from the total loading and dealing with this constant contribution separately. After changing the integration variables from (ρ,χ) to (R,α) one arrives at:

$$K^{*}[\phi;a(\theta)] = \int_{-\pi/2}^{\pi/2} \int_{0}^{2a(\theta)\cos\alpha} [p(R,\alpha)-p(R=0,\alpha=0)]\frac{\sqrt{2a(\theta)\cos\alpha - R}}{\sqrt{a(\theta)\pi^{3}}R\sqrt{R}} R dR d\alpha \tag{4}$$

$$+2\ p(r=a(\theta),\psi=\phi)\ \sqrt{a(\theta)/\pi}$$

APPLICATION TO THE CRACK PROPAGATION PROBLEM

The previous theory can easily be applied to crack propagation problems of subsurface cracks. It is assumed that the starting crack configuration is near circular and that any other boundaries are far enough from the crack not to influence the crack substantially. The loading is available in the form of a load history. The angle ψ, extending from 0 to 2π, is initially discretised into n intervals defining n points on the crack front characterized by $p_i(\psi_i,a(\psi_i))$. These are the points which are monitored during crack propagation. At the start of the calculation the linear elastic normal stress at the crack location in the uncracked body due to a reference load is determined, e.g. with the Finite Element Method. The relationship between the load and the normal stress on the crack faces, which has thus been established, is assumed to be linear. The stress must be determined in a region large enough to encompass the largest crack which is anticipated to occur during crack propagation.

At this point the interation process can start. Application of the rainflow method to the load history (see e.g. Downing and Socie (2)) yields a load cycle leading to a stress cycle (p_{min},p_{max}) at each point on the crack faces.By using equations (1) and (4) the stress intensity factors $(K_{min})_i$ and $(K_{max})_i$ can be determined in each crack point p_i. By using an appropriate crack propagation law, e.g. the Paris law or the Forman law, one obtains the crack extension Δa_i due to the extracted cycle. The new crack front is established by adding Δa_i to a_i in a direction locally normal to the crack front. At this point the next cycle can be extracted.

The advantages of this method include:
- only one stress calculation has to be performed at the start of the crack propagation process.
- no restrictions are imposed on the crack propagation law.
- the new crack front (size and shape) is obtained in a natural way (no a priori assumptions have to be made about the new crack front).
- the method is faster than the performance of consecutive Finite Element analyses.

However, the method only works if
- the crack shape remains near circular during the whole propagation process.
- any free boundaries are far enough not to influence the crack

EXAMPLE

Figure 2 illustrates the method for an originally circular crack in a stress field perpendicular to the crack faces which increases linearly in the y-direction (about a factor of three load increase over the crack). This field is typical for the hoop stress in a rotating disk with increasing values towards the bore of the disk. The calculations were made for one hundred (0,max) cycles and the new crack front was depicted every twenty cycles. As was to be expected the crack moves faster in the positive y-direction and changes into an egg shape.

CONCLUSIONS AND PROSPECTIVE RESEARCH

The perturbation method developed by J. Rice has been successfully applied to the problem of a propagating subsurface crack. Present research concentrates on the extension of the theory to half-circular surface cracks. It is interesting to notice that for the method to work near the free boundary of a surface crack a higher order behaviour of K (i.e. K=0 at the boundary) is necessary in order for equation (1) to make sence. Indeed, a $1/(\phi-\theta)$ singularity is integrable only provided the integration encompasses both sides of the singularity and by using the principal value technique. A higher order K near the free boundary would be in agreement with previous findings (e.g. Becker (3)).

SYMBOLS USED

$a(\theta)$	= local crack radius ([L])
[F]	= unit of force
$k(\phi;\rho,\chi;a(\theta))$	= K-factor at location ϕ due to a unit compressive force at position (ρ,χ) within a circular crack with radius $a(\theta)$ ($[L]^{-3/2}$)
$K(\theta)$	= K-factor at position θ of the current crack due to the current loading ($[F].[L]^{-3/2}$)
$K^{\circ}[\phi;a(\theta)]$	= K-factor at position ϕ of a circular crack with radius $a(\theta)$ due to the current loading ($[F].[L]^{-3/2}$)
[L]	unit of length
$p(\rho,\chi)$	compressive stress at location (ρ,χ) ($[F].[L]^{-2}$)
r	polar co-ordinate ([L])
R	integration variable ([L])

α	integration variable (-)
θ	location along the crack front (-)
ρ	integration variable ([L])
ϕ	location along the crack front (-)
χ	integration variable (-)
ψ	polar co-ordinate (-)

REFERENCES

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(3) Becker, I., "Numerische Berechnung von Ecken- und Kantensingularitäten elastischer Felder für dreidimensionale Rißprobleme", Universität Karlsruhe, Dissertation, 1989.

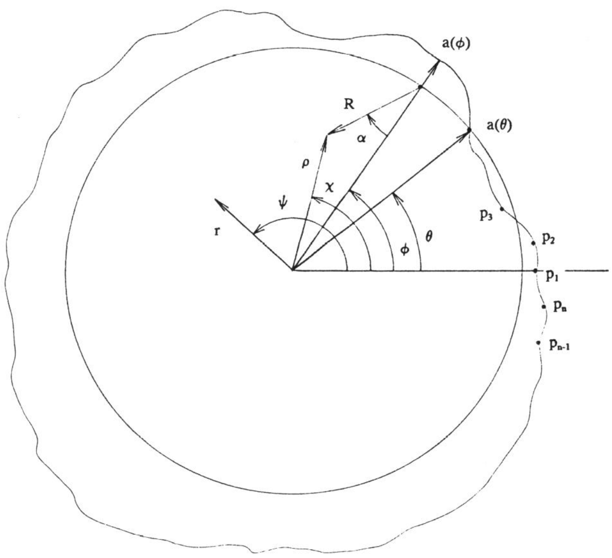

Figure 1 Crack configuration

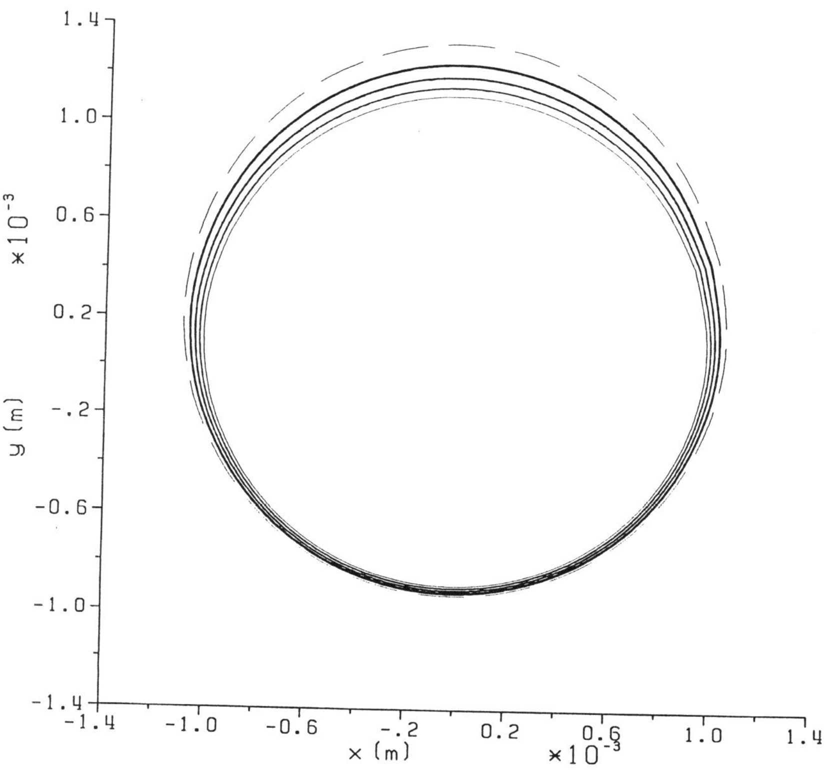

Figure 2 Crack propagation due to a non uniform load