EXPERIMENTAL AND NUMERICAL CURVED CRACK GROWTH INVESTIGATIONS FOR BEAMS UNDER LATERAL FORCE BENDING

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The Modified Virtual Crack Closure Integral (MVCCI)-method has proved to be a highly effective and versatile numerical procedure for the fracture analysis of various crack problems in linear elasticity. In this paper it is shown that the MVCCI-method also can readily be utilized for computer aided curved crack growth simulations. For specimens, subjected to lateral force bending, the comparison of computationally predicted and experimentally obtained crack trajectories show excellent agreement in all cases considered.

INTRODUCTION

From failed structures and components it is known that cracks frequently originate and extend in regions characterized by complicated geometrical shapes and unsymmetrical loading conditions. Hence the developing crack paths are found to be curved and standard solutions for coplanar cracks do not apply. The computer aided simulation and prediction of such non-coplanar crack trajectories still is a challenging problem of fracture analysis. It is known that only through the comparision with related experimental findings one can decide on the significance of the computer aided crack growth simulation.

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CRACK GROWTH SIMULATION

According to Richard (2) the direction φ of crack propagation depends only on the stress intensity ratio $\lambda = K_{II}/K_I$ for several mixed-mode criteria, and for others a further dependence on Poisson's ratio ν is found. For small SIF-ratios $0 \le |\lambda| \le 0.1$ practically the same values for φ are given by all criteria (Theilig (2))

$$\varphi = -2\,\lambda\,. \tag{1}$$

This orientation will result in the condition of local symmetry at the crack tip ($K_{II} = 0$). For the evolution of the crack path after initiation under monotonic loading the generalisation of the local symmetry criterion can be regarded as the best physical basis. Thus, it can be stated that a continuously growing crack forms a curved path which has a pure normal separation at the moving crack tip at any crack tip position.

The monotonic crack growth process in reality can be divided into a number of finite simulation steps Δa. At each step the propagation direction is changed by an angle $\Delta\tau$ (Fig. 1a). Under the condition $K_{II}(a,p) = 0$, the change of the slope can be interpreted as the consequence of $\Delta K_{II} \neq 0$ due to a virtual tangential crack extension Δa. According to Eq.(1) one gets

$$\Delta\tau = -2\,\Delta K_{II}\;/\;K_I\,. \tag{2}$$

Therefore in the case of proportional loading systems the analysis of a crack path can be carried out by a gradually straight extension in combination with a finite change of direction (Fig. 1b). Due to the finite crack extension, the calculation of K_I and ΔK_{II} is necessary after each step Δa. This is effective by using the FEM. According to Fig. 1b it holds

$$\tau_j = \tau_{j-1} - 2(\Delta K_{II}\;/\;K_I)_{j-1}\,. \tag{3}$$

The width Δa can be chosen according to the condition

$$|\Delta K_{II}\;/\;K_I| \le 0{,}1. \tag{4}$$

From these relations the need for an efficient numerical mode separation technique in conjunction with a step by step finite element crack path analysis can be seen. Particularly with respect to this requirement the MVCCI-method has proved to be highly advantageous, because it delivers the separated energy release rates G_I, G_{II} simultaneously with good accuracy and without any additional effort.

According to Buchholz (3) for a LSE-discretisation, as it is forming the actual crack tip in Fig. 2, the following FE-representation of Irwin's crack closure integral relations can be given

$$G_I(a) = \frac{1}{t} \lim_{\Delta a \to 0} \frac{1}{2\Delta a}\left[F_{y,i}(a)\,\Delta u_{y,i-1}(a) + F_{y,i+1/2}(a)\,\Delta u_{y,i-1/2}(a)\right], \quad (5)$$

$$G_{II}(a) = \frac{1}{t} \lim_{\Delta a \to 0} \frac{1}{2\Delta a}\left[F_{x,i}(a)\,\Delta u_{x,i-1}(a) + F_{x,i+1/2}(a)\,\Delta u_{x,i-1/2}(a)\right]. \quad (6)$$

In Eqs. (5), (6) the strain energy release rates G_I and G_{II} are computed on the basis of the work to be done by the nodal point forces $F_{y,i}$, $F_{y,i+1/2}$ against the relative nodal point displacements $\Delta u_{y,i-1}$, $\Delta u_{y,i-1/2}$ in order to close the crack of length a + Δa by an amount Δa. Finally, with the relation

$$|\Delta K_{II} / K_I| = \sqrt{\Delta G_{II} / G_I} \quad (7)$$

all parameters necessary for a computer aided crack growth simulation are provided. The sign of ΔK_{II} must be determined from the crack opening displacement.

EXPERIMENTAL CRACK GROWTH TESTS

In order to evaluate the significance of the computer aided crack growth simulations a reliable and detailed data base from directly related crack growth experiments is required. Special specimens for lateral force bending have been designed (Fig. 3) in order to get an experimental basis. Notches have been attached at several positions from which cracks of characteristic shape initiated and extended during the fatigue tests. In Fig. 4 some of these experimentally obtained crack trajectories are shown and it is interesting to see in which way the cracks are curved and that the experimental scatter is narrow (Theilig (4), Kittelmann (5)).

Some of these detailed experimental findings have formed the basis on which an early approach to computer aided crack path simulation has been evaluated. In the following chapter recent results by the MVCCI-approach to these problems will be discussed (Theilig et al (6), Rossberg (7)).

RESULTS OF NUMERICAL SIMULATION OF CRACK GROWTH

The chosen FE-discretisations for the specimens are given in Fig. 5 together with mesh details, showing some of the simulated crack extension steps with $\Delta a = 1mm$ for crack initiation e.g. from the notch position at $\alpha = 40deg$. In Fig. 6 the computationally simulated crack trajectories for the notch position $\alpha = 40deg$. are shown together with the experimentally obtained scatterband for the related crack growth experiments. For a given crack increment of $\Delta a = 1mm$ an excellent agreement is found. The development of the related stress intensity factors versus crack length are given in Fig. 7. Clearly the implicit condition of local symmetry can be verified by $\Delta K_{II}(a)$ and from $K_I(a)$ actual fatigue lifetimes may be calculated.

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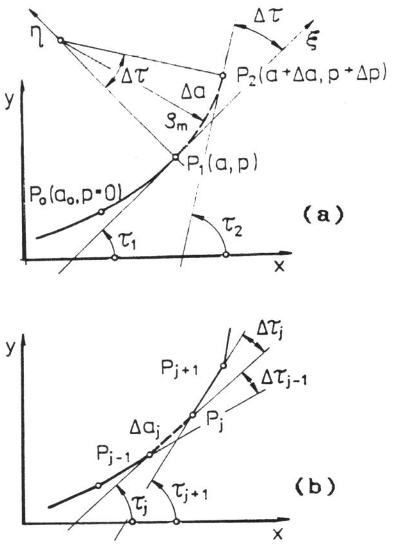

Figure 1 Step by step approach for crack path simulation

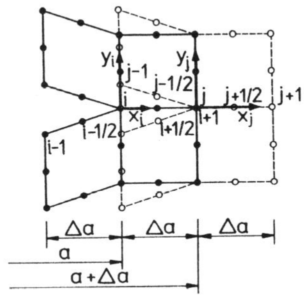

Figure 2 Numerical VCCI- and MVCCI-methods

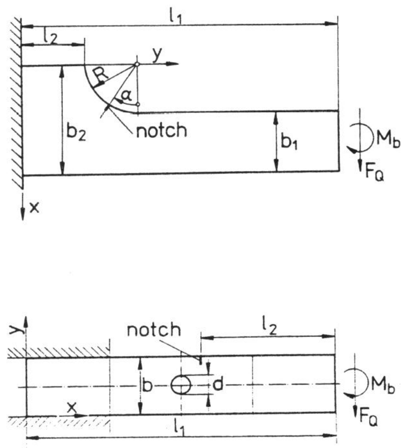

Figure 3 Specimens for lateral force bending

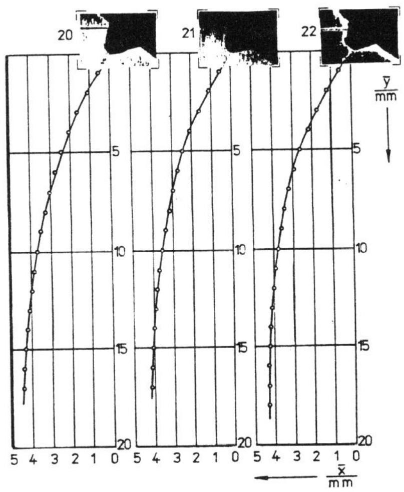

Figure 4 Experimentally obtained crack trajectories

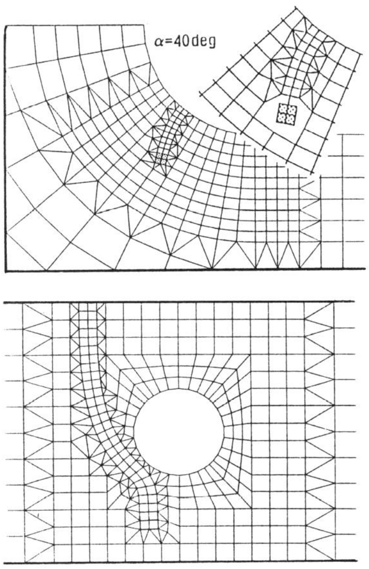

Figure 5 FE-models of the specimens

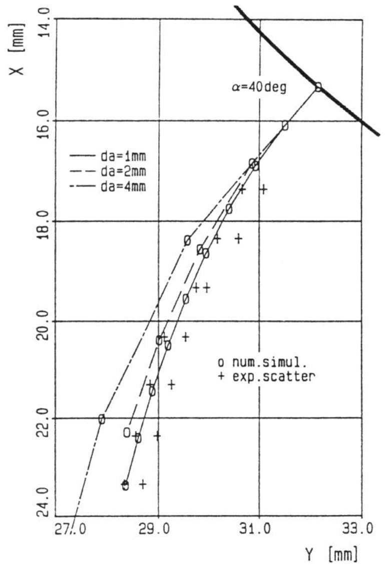

Figure 6 Comparison of crack trajectories (α = 40deg.)

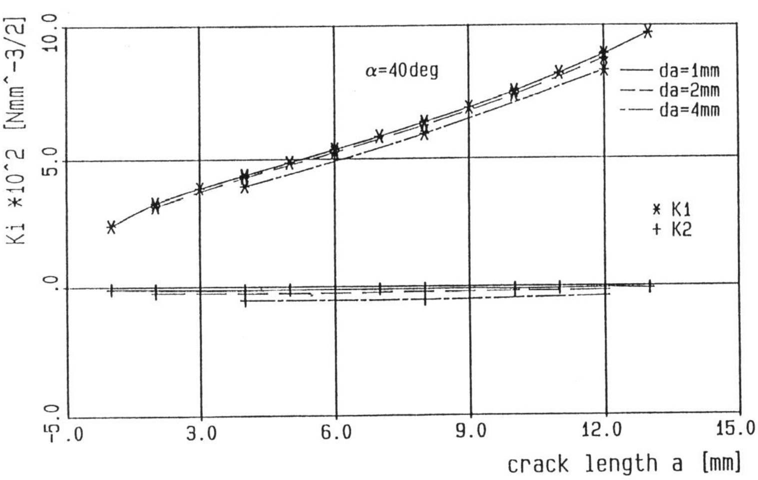

Figure 7 Computed stress intensity factors K_I and ΔK_{II} versus crack length (α = 40deg.)