STOCHASTICAL MODEL FOR THE CALCULATION OF GEARS

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The problem of determination of the service life of gearings is directly related to geometry of gears, loadings, materials and appropriate models for prediction of the crack propagation. If during the periodic inspection a defect is found, the following question arises: how long such a gear\gearing can still safely operate. To be able to answer this question we have developed a stochastic model for determination of service life of gears on the basis of the shape factors and tooth stress intensity factor Z obtained from a model for generation of actual loadings.

INTRODUCTION

Gearings mutually differing in shape and manner of loading are vital parts of all machines and devices. The most frequent loadings are random loadings of variable amplitude. To be able to determine the service life of such gearings it is very important to take into account those loadings as accurately as possible. For determination of the service life also the geometry of gears and properties of materials, from which the gears are made and which are known not to be constants, are important. The more precise modelling of these input parameters the more precise and reliable the results.

In practice the following basic problem arises when calculating the service life:
During design, the individual elements and the entire products are optimized particularly with respect to the

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service life. The basic requirement is that the service lifes of the individual elements are approximately equalized. In this case the model of occurrence of defects and the model of crack propagation are important.

TOOTH STRESS INTENSITY FACTOR

In our model we propose a new parameter by which we describe the fracture mechanics conditions in the tooth root where the defects, causing destruction, occur statistically most frequently as shown Figure 1. We must emphasize that this parameters not applied in general, but only for gears and for calculation of conditions in the tooth root, i.e., for calculation of their service life. We named that parameter the tooth stress intensity factor Z. The value of the factor Z is related to propagation of the plastic zone, deformation and orientation of grain in case of short cracks and stress intensity factors in case of long cracks. There are three forms of the factors Z:

If a = 0, i.e., if there is no crack, the factor Z is proportional to the stress concentration factor (1) occurring in the tooth root:

$$Z = f(\ ) \propto K_t\ \sigma$$

If $0 < a < \approx 10$ grains (according to Taylor (2)), the factor Z depends on local plasticizing of grain, grain size, shear module, grain orientation, shape factor:

$$Z = f\ (\ \epsilon,\ d,\ a,\ d - a,\ G,\ Y.....)$$

If a > 10 grains, such defect can be considered to be a long crack or a linear elastic fracture mechanics type crack and the following is obtained:

$$Z \approx f\ (K_{IC},\ K_{TH},\ K,\ Y(a/S),\ \sigma\ ....)$$

thus for the case of long cracks the tooth stress intensity factor can be written as follows:

$$Z = (\sigma_i - \sigma)\ \sqrt{\pi a}\ Y\left(\frac{a}{S}\right) \qquad (1)$$

where the shape factor (3) is:

$$Y\left(\alpha, \frac{C}{L}, \frac{S}{L}, \varphi\right) = Y_M(\alpha)\left(\cos\varphi - \frac{C}{L}\sin\varphi\right) - Y_{CT}(\alpha)\frac{S}{6\,L}\sin\varphi \quad (2)$$

When we want to write the tooth stress intensity factor in a more compact form for all three areas, the equation (1) obtains a more complex form (4):

$$Z = (0.2\sigma+e^2)\ln\sqrt{\pi a}\left[\frac{\sigma\, Y(a/S)\,\sqrt{\pi a}\,\,(\sigma Y(a/S)\,\sqrt{\pi a}-K_{th}}{K_{th}}\right] \quad (3)$$

which, in case of long cracks, is well approximated by the stress intensity factor.

STOCHASTIC MODEL OF CRACK PROPAGATION OF GEAR TOOTH

Now we will deal with the model of crack propagation on the gear tooth as a basis of the algorithm for calculating the service life of gears and gearings.

Statistical analysis of experimental results shows that the material parameters in the equations of crack growth are random variables. As an example let as take the modified Paris - Erdogan equation in stochastic form:

$$\frac{da}{dt} = f(\Delta Z)\;A(t) = C\;f(\Delta Z)\;[\mu+Y(t)] \quad (4)$$

Where μ = E(A(t)) and A (t) is stochastic time process, and Y (t) is a random process with mean μ. The two parameters C and m can be considered to be random functions, which have the following form :

$$C = C\;(x) = C_0 + C(x) \quad (5\text{ a})$$
$$m = M\;(x) = M_0 + M(x) \quad (5\text{ b})$$

where C_0 and M_0 are random variables describing random variation of mean value in different cases and C (x), M (x) are random fields describing the material inhomogeneity independently of C_0 and M_0.

In our code we used a model which randomizes the equation of crack growth ratio:

$$\frac{da}{dt} = q(a)\ X(t) \tag{6}$$

where X (t) is a random process.

CALCULATION OF SERVICE LIFE OF GEARING

On the basis of the model presented above it is possible to calculate the service life of the individual gear tooth. The service life of gear can be obtained from equation.

$$L_i = f(u)\ N^{-\frac{1}{e}} \tag{7}$$

The service life of gear drives is the sum of service lives of all components :

$$L_{GD} = \left(\sum_{i=1}^{n} L_i^{-e} \right)^{-\frac{1}{e}} \tag{8}$$

Weilbull's exponents are different for different elements and amount to e = 2,5 for gears, e = 1,5 for ball bearings and e = 10/9 for roller bearing etc., from which it is possible to calculate the relevant service life of the entire gearing L_{GP} for desired reliability different from 90 percent reliability.

Numerical example

In order to verify correctness of the presented model, we have compared the numerical results with experimental results (5). We have used the following data for gear material AISI 4130: K_{IC} = 2650 $Nmm^{-3/2}$, K_{th} = 286 $Nmm^{-3/2}$, m = 2,75 and C = $6,416*10^{-11}$.
and for geometry: module m = 10 mm, number of teeth z = 18 dimensions in critical cross section BxS = 20x15,7 mm according to Figure 1 .

CONCLUSION

Figure 2 shows a comparison of the results of crack propagation. The figure shows that by means of the presented model we have reached good accordance with experiment particularly in the area which is most interesting for technical practice, i.e., in the area of permissible subcritical cracks. Permissible crack length is $a_{perm} = 0.5\ a_c$ and for the presented case the critical crack length is approximatelly $a_c \approx 4$ mm. In the area where we approache the critical lengths and/or fracture impact strength of the material the deviation is slightly greater.

SYMBOLS USED

Z - Tooth stress intensity factors
a, u - General sign for crack length
K_t - Factor of the notch effect
σ - General sign for stresses
C, m - Material parameters
K_{IC} - Fracture toughness
K_{th} - Threshold stress intensity factor
N - Number of teeth

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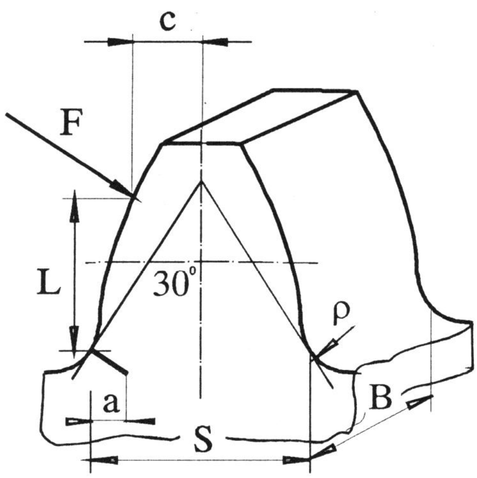

Figure 1: Critical cross section

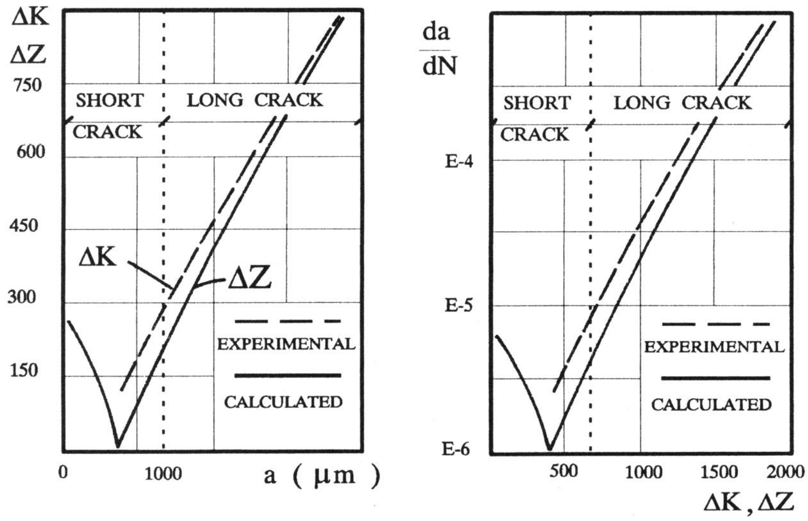

Figure 2: Comparision between experimental and calculated values of crack growth ratio