STATUS REVIEW OF THE J PLUS T STRESS FRACTURE ANALYSIS METHOD

J D G Sumpter* and J W Hancock†

Two parameter fracture mechanics techniques use J with a second parameter added to index crack tip constraint. An early suggestion for the constraint index was the elastic T stress. This has the advantage of not requiring an elastic-plastic finite element analysis for its determination. This paper shows that, in spite of its simplicity, the T stress does provide the basis for a comprehensive fracture design method which incorporates both test piece and structural constraint variations. Comparisons are made with more accurate analyses based on Q. Conclusions are substantiated by numerical and experimental data.

INTRODUCTION

Fracture toughness test and analysis standards are based on the assumption that the deformation field at a crack tip can be characterised by a single parameter, J. It is assumed that the material has a unique toughness which can be measured in the laboratory and used directly for critical defect size prediction in the structure.

This has remained the orthodox fracture mechanics approach since the early 1970s although a certain amount of contrary evidence has been available. For instance, the early numerical work of Larsson and Carlsson (1) questioned the accuracy of single parameter fracture mechanics. Observed toughness changes with test geometry have also been defined loosely in terms of high and low constraint, but there has been no obvious way of quantifying the trends into a useable fracture design package.

* Defence Research Agency, Dunfermline, KY11 5PW, UK
† Glasgow University, Glasgow G12 8QQ, UK

In the last few years this picture has changed completely. The breakthrough has come through increased computing power, which has enabled crack tip stress distributions to be examined in detail. It is now clear that J only provides a single parameter characterisation of the crack tip stress field for a very limited range of highly constrained loading configurations and deformation levels. Moreover, a number of practical schemes have been proposed to quantify crack tip constraint and hence predict the variation of J_c with changes in geometry and deformation level. J_c is used here to denote J at final cleavage fracture with no imposed plasticity limitation. All the analyses and experimental data assume plane-strain. The constraint being indexed is thus in-plane constraint and takes no account of thickness effects.

The first parameter to be suggested as an index of in-plane constraint was the elastic T stress. The significance of the elastic T stress to plasticity development was first pointed out in (1) and more fully explained by Rice (2); but these authors did not identify the potential of the parameter to provide a measure of crack tip stress state well into plastic yielding. This came much later through the work of Bilby et al (3), Harlin and Willis (4), and Betegón and Hancock (5).

Many other constraint indexing schemes are now being suggested. The most notable are: the Q stress - O'Dowd and Shih (6); a geometry and loading dependent correction factor developed by Dodds, Anderson and Kirk (7); and most recently, the J-A_2 theory of Chao, Yang, and Sutton (8).

These approaches can all be classified by the generic title of two parameter fracture mechanics. J is still used to quantify the severity of plastic deformation (or size scale of the crack tip stress-strain field), but a second parameter (T, Q, A_2) is added to index the effect of geometry and deformation level on crack tip tensile stress.

If crack tip tensile stresses are reduced, higher J values can be achieved without fracture. In terms of Q, it has been proposed (6) that the near crack tip stresses, σ_Q, takes the form

$$\sigma_Q = \sigma_{SSY} + Q\sigma_o \qquad (1)$$

where σ_{SSY} is the peak crack tip stress under very small scale yielding (SSY) and σ_o is the yield stress. Under SSY the term σ_{SSY} is wholly defined by J, and Q is zero. As plasticity develops Q becomes negative, ie the crack tip stresses reduce below the value which would apply in J dominated SSY. In general Q is a function of geometry, work hardening, and deformation level. For instance, in the centre cracked tension geometry Q quickly reaches -1; whereas, in deeply cracked bending, Q remains close to zero well into general yield.

Determination of Q requires a very detailed elastic-plastic finite element analysis. The most attractive feature of T is that it can be determined from an elastic finite element analysis. Although T is an elastic parameter, defined as the second term in the elastic crack tip stress field expansion, a case for its continued usefulness after extensive yielding can be made by an empirical route involving elastic-plastic finite element analysis and experimental testing.

FINITE ELEMENT STUDIES

Karstensen, Nekkal and Hancock (9) have recently re-analysed a range of geometries to determine the extent to which predictions based on T provide a good approximation to Q derived from a full elastic-plastic analysis. Results were compared with near crack tip stresses obtained from an elastic stress intensity, K, and T stress boundary layer analysis. The near crack tip stress (at a distance $2J/\sigma_o$ from the crack tip) based on the modified boundary layer formulation (mblf) is denoted σ_T,

$$\sigma_T = \sigma_{SSY} + Q_T\sigma_o \qquad (2)$$

where Q_T is a function of T for a given work hardening exponent.

$$Q_T = a_1\left(\frac{T}{\sigma_o}\right)^2 + a_2\left(\frac{T}{\sigma_o}\right)^2 + a_3\left(\frac{T}{\sigma_o}\right)^3 \qquad (3)$$

where a_1, a_2, a_3 are empirically determined constants (6).

Figure 1 shows results for a single edge crack in bending (SEB) plotted in terms of $Q\text{-}Q_T$ and P/P_L, where P is the applied load, and P_L is the plastic limit load. The analyses have been performed for a work hardening exponent, n, equal to 13. The notable features of these plots are: firstly, that $Q\text{-}Q_T$ is close to zero until well after plastic limit load; and secondly, that $Q\text{-}Q_T$ is an easily expressed function of load virtually independent of a/W.

Figure 2 shows similar results for the centre cracked tension (CCT) geometry. In this case there is a much greater effect of a/W. The trends with increasing deformation level are divergent, suggesting that fracture prediction by T stress analysis will be conservative at large a/W, but unconservative at small a/W. Although these results are less encouraging than those for the SEB geometry, they still show the possibility of estimating Q from Q_T, and hence T, at any a/W and P/P_L. The results also suggest that for structural geometries, where P/P_L is usually less than 1.0, it would be an acceptable approximation to take $Q\text{-}Q_T$ equal to zero, and to conduct structural analysis directly from the T stress.

EXPERIMENTAL DATA

Figure 3 shows data on mild steel previously reported by Sumpter and Forbes (10). Figure 4 shows these same data reanalysed using Q from (6). Specimens tested were three point bend (3PB) with a/W between 0.03 and 0.8 and centre cracked tension (CCT) with a/W between 0.6 and 0.8. Specimens had thickness, B, of 25 mm, with the ligament, W-a, varied to be equal to, or less than, B. All tests were conducted at −50°C and ended in cleavage with no prior ductile crack growth. Material yield strength at -50°C was 290 MPa. Further details are given in (10) and by Sumpter (11).

Figures 5 and 6 show more recent data from tests in progress by the same authors. The material here is a high strength weld metal with yield of around 700 MPa. The test temperature was -30°C. Specimen geometries are similar to those for the mild steel, but the effect of specimen configuration on J_c is now even more pronounced. Average toughness for the deeply notched bend specimens was .04 MN/m, similar to that for the mild steel. A valid K_{IC} was obtained in one test. All failures in the deeply notched bend specimens initiated by cleavage. As a/W was reduced, and toughness increased, the failure mode changed to tearing followed by cleavage. The onset of tearing occurred at around 0.15 MN/m and the tearing resistance curve appeared to steepen with reducing a/W. The highest 3PB J_c was 0.9 MN/m, corresponding to 2.6 mm of tearing (from a 2.1 mm fatigue crack), followed by cleavage. Three out of the four CCT specimens were fully ductile (no cleavage). The fourth failed by cleavage preceded by 0.85 mm of crack growth indicating an even higher tearing resistance curve for the CCT than for the small a/W 3PB specimens in accord with the observations of Hancock, Reuter, and Parks (12).

The data in figures 5 and 6 have been plotted at a J_c corresponding to final cleavage instability (ignoring the effect of prior stable tearing) with the exception of the three fully ductile CCT specimens which have been plotted at J_m (stable maximum load). Some of the agreement in J_c between the 3PB and CCT specimens must thus be regarded as fortuitous; but nevertheless, both Q and T accurately index the trend of increasing resistance to fracture with reducing constraint.

Tensile tests were conducted to determine true stress-strain curves for both materials. The experimental curves were fitted with the relationship

$$\frac{\epsilon_p}{\epsilon_o} = \alpha \left(\frac{\sigma}{\sigma_o}\right)^n \qquad (4)$$

where ε_p is plastic strain, ε_o is yield strain, and α is a constant. Best fit values of α and n were

mild steel	α	=	6.67, n	=	4.3
weld	α	=	0.95, n	=	10.8

An attempt was also made to determine the cleavage stress of both materials using static Charpy tests - Sandström and Bergström (13). In both cases the cleavage stress was deduced to be approximately three times the yield stress at the test temperature. Shih and O'Dowd (6) have proposed a fracture locus.

$$\frac{J_c}{J_{co}} = \left(1 - Q\frac{\sigma_o}{\sigma_c}\right)^{n+1} \tag{5}$$

For the mild steel the predicted elevation of J_c at Q = -1.4 is 5 which agrees well with the observed value; for the weld at the same Q the predicted elevation is 65 compared to the observed value of 20. The comparison for the weld is obviously affected by the occurrence of tearing at negative Q. The predicted elevation of J_c is so high as to suggest that cleavage could not occur at Q = -1.4. Finite element studies by O'Dowd, Shih, and Dodds (14) have shown that tearing elevates crack tip stresses at negative Q and makes cleavage more likely.

DISCUSSION

It is clear that geometry and loading have a major effect on J_c and that constraint indexing procedures derived from finite element studies can play a major role in explaining and predicting the observed trends. It is now possible to propose fracture analysis procedures to:

(i) Categorise material dependence on in-plane constraint by testing a range of laboratory specimen (most simply 3PB with a/W between 0.05 and 0.5).

(ii) Index structural constraint by finite element analysis or handbook solutions.

(iii) Determine the J_c appropriate to any particular structural flaw and load level.

The authors of this paper were early advocates of the use of elastic T stress as a constraint indexing parameter. The usefulness of the T stress was deduced through empirical correlation with elastic-plastic finite element analysis. The present paper shows that T provides an excellent basis for simple Q estimation schemes. For structural analysis, where an infinite variety of crack configurations can occur (including three dimensional shapes), the use of an elastically derived constraint parameter has significant attractions. One possible approach is to construct the toughness locus in terms of J_c/Q, but to perform the structural analysis using T. Figures 1 and 2 suggest that $Q\text{-}Q_T$ will be near zero in the region below plastic limit load where most structures are operated.

CONCLUSIONS

Two parameter fracture mechanics techniques can provide an accurate prediction of the effect of in-plane constraint on cleavage toughness measured by J_c.

The effect of geometry on J_c is material dependent. The J_c constraint locus for the material of interest is best measured by testing a range of fatigue cracked specimens. This is most easily achieved by testing bend bars having $0.05<a/W<0.5$ with W-a equal to the thickness B.

Constraint in laboratory specimens is best indexed by an elastic-plastic parameter such as Q; but the elastic T stress is likely to retain a role in indexing structural constraint.

ACKNOWLEDGEMENTS

Thanks are due to Ms A Karstensen for computational data, Mr A Forbes for experimental work, and Mr C Lau for curve fitting the stress-strain data.

REFERENCES

(1) Larsson, S G. and Carlsson, A J., Journal of Mechanics and Physics of Solids, Vol 21, 1973, pp 263-277.

(2) Rice, J R., Journal of Mechanics and Physics of Solids, Vol 22, 1974, pp 17-27.

(3) Bilby, B A et al., "Size Effects in Fracture", Institution of Mechanical Engineers, London, UK, 1986.

(4) Harlin, G., and Willis, J R., Proceedings of the Royal Society of London, A415, 1988, pp 197-226.

(5) Betegón, C., and Hancock, J W., ASME Journal of Applied Mechanics Vol 58, 1991, pp 104-110.

(6) O'Dowd, N P., and Shih, C F., ASTM STP for 24th National Symposium on Fracture Mechanics, Philadelphia, 1993.

(7) Dodds, R H., Anderson, T L., and Kirk, M T., International Journal of Fracture, 1991, pp 1-22.

(8) Yang, S., Chao, Y J., and Sutton, M A., Engineering Fracture Mechanics Vol 45, 1993, pp 1-20.

(9) Karstensen, A D., Nekkal, A., and Hancock, J W., (unpublished work).

(10) Sumpter, J D G., and Forbes, A T., "Shallow Crack Fracture Mechanics, Toughness Tests and Applications", TWI, Cambridge, UK, 1992.

(11) Sumpter, J D G., "Constraint Effects in Fracture", ASTM STP 1171, Philadelphia, US 1993.

(12) Hancock, J W., Reuter, W G., and Parks, D M., ASTM STP 1171, Philadelphia, US, 1993.

(13) O'Dowd, N P., Shih, C F., and Dodds, R H., ASTM STP 1244, Philadelphia, US 1994.

(14) Sandström, R., and Bergström, R., Metal Science Vol 18, 1984, pp 177-186.

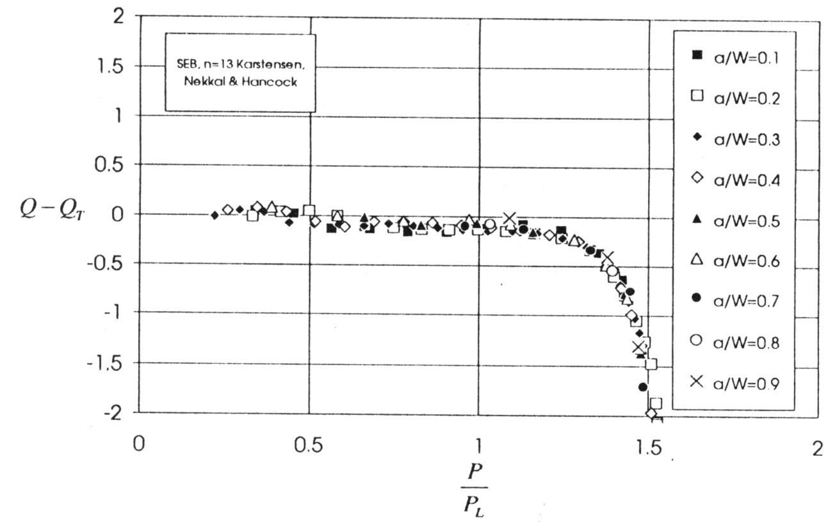

Figure 1 Difference between full field and modified boundary layer crack tip stresses for a single edge crack in bending

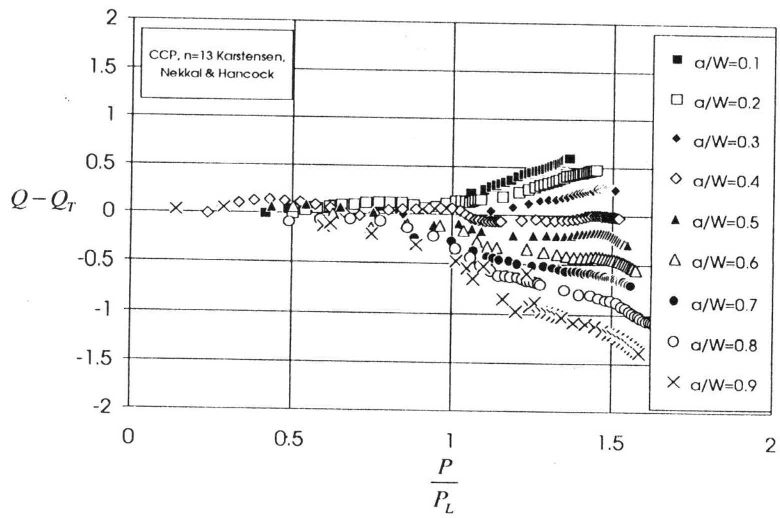

Figure 2 Difference between full field and modified boundary layer crack tip stresses for centre cracked tension loading

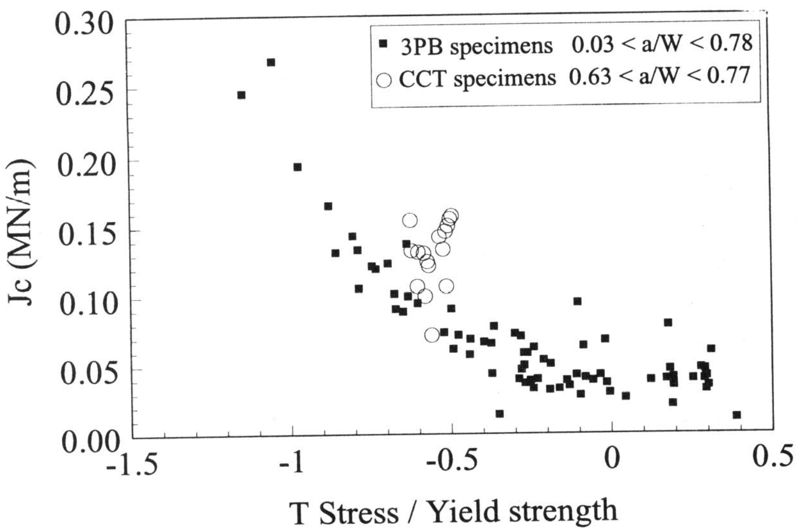

Figure 3 Experimental data for mild steel in terms of T stress

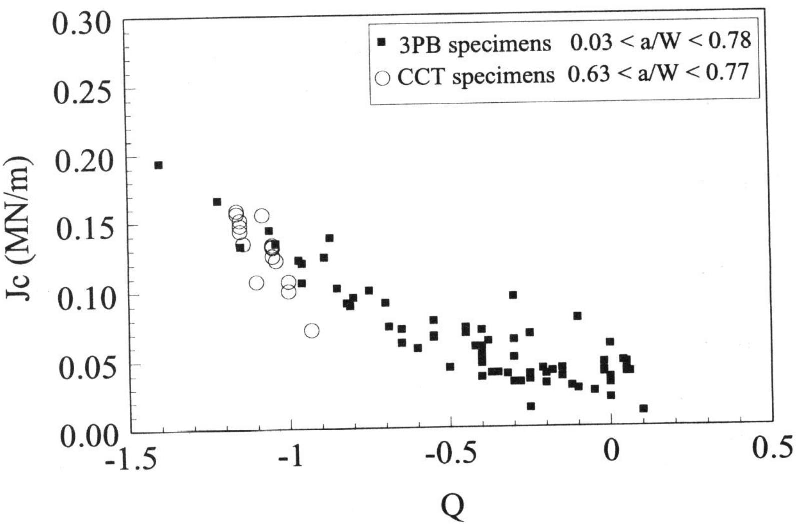

Figure 4 Experimental data for mild steel in terms of Q

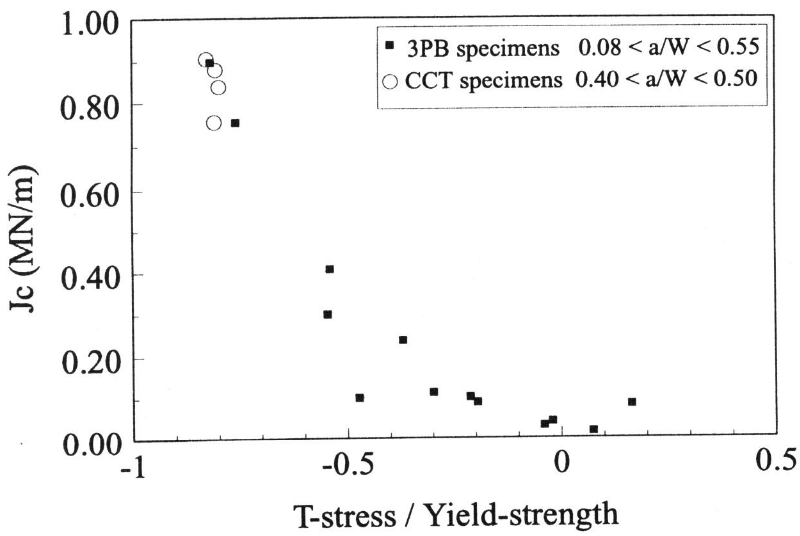

Figure 5 Experimental data for a high strength weld metal in terms of T stress

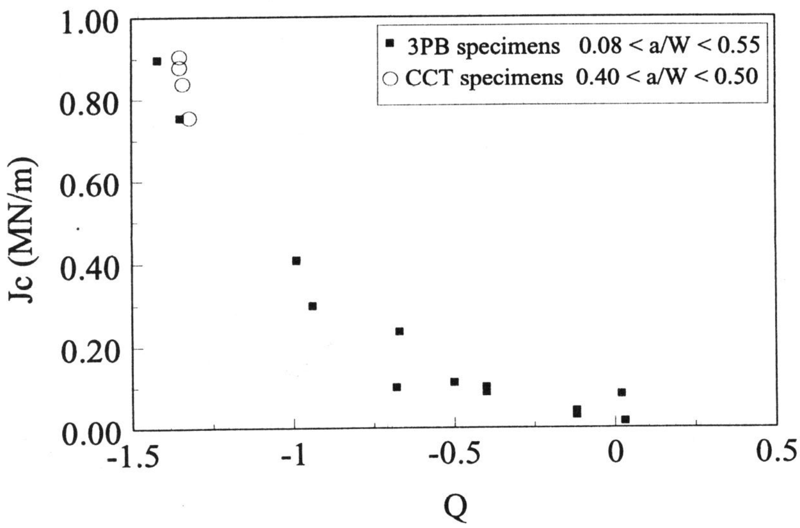

Figure 6 Experimental data for a high strength weld metal in terms of Q