A STOCHASTIC MODEL SIMULATING THE DEVELOPMENT OF IRREGULAR CRACK PATTERNS IN THERMAL FATIGUE

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The damage accumulation of an austenitic steel due to thermal fatigue loading is investigated. A stochastic simulation model has been developed on the basis of experimental results. The stochastic and fracture mechanics background of the model is presented and simulated and experimentally obtained crack patterns are compared using suitable characteristic quantities.

INTRODUCTION

One of the most complicated problems in fatigue-induced damage of metallic materials is the cyclic thermal shock. At the specimen surface an irregular pattern of multiple cracks is generated due to crack initiation, crack propagation, and crack coalescence. At an advanced stage of damage a closed network of cracks is formed. The formation of these typical crack patterns seems to be influenced by spatial interactions because there are both heavily and less heavily damaged regions.

It is the aim of the investigations to identify the various stages of damage evolution and to understand the underlying mechanisms. This is shown in three steps: (a) a quantitative description of experiments; (b) an evolution of a simulation model and evaluation based on to the experiments; and (c) the indication of some qualitative results of the simulation. From these investigations the possibilities and limitations of statistical analysis of this kind of experiments and simulations are evident.

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EXPERIMENTAL BACKGROUND AND MODELLING

The starting point for the current investigations are experiments [1] in which circular plates (diameter 150 mm, thickness 20 mm), made of the austenitic steel 1.4948, were cyclically heated and cooled. Heating started from 17°C, and 410°C was reached in about 4 min, cooling was achieved by pressurised water flow on the lower plate surface for nearly 5 s. Thermal loading resulted in an equibiaxial stress state at the specimen surface. The tests were interrupted after a certain number of load cycles and photographs of the specimen surface were taken. As the specimen surface had to be prepared to allow photographs to be taken, it was not possible to obtain crack patterns on the same specimen with different numbers of load cycles.

A visual impression of damage evolution is obtained from the set of surface crack patterns with increasing load cycles. Due to the variability of crack shapes, there is no deterministic correlation between crack growth rate and crack length. Therefore, a first important point of the investigations is the statistical characterisation of the surface crack patterns. We use well-established methods from stochastic geometry [2] to describe the typical geometrical features. Based on suitable statistical quantities which allow to distinguish between visually different crack patterns, both an assessment of the damage accumulation and a comparison between simulated and experimental surface crack patterns can be made.

Crack growth in depth direction was investigated which provided further information about the damage process. For this purpose, specimens with a given number of load cycles were selected and thin surface layers were removed step by step [3]. Successive planes of crack patterns became visible in this way and it was possible to determine the three-dimensional crack shapes. Thus, additional information is available about mechanisms of damage accumulation and crack coalescence.

On the basis of information derived from the experiments a stochastic simulation model is developed which describes the initiation, propagation, and coalescence of cracks at the specimen surface. At first, basic assumptions concerning both the geometrical nature and the mechanism of damage evolution are necessary. These assumptions must be confirmed or rejected by comparison of simulated with experimentally obtained crack patterns. The mechanism of the evolution of the irregular crack patterns should be determined by suitable variation of the physical model according to experiental data obtained. There is no straight forward procedure to identify the quantities which furnish the maximum of information on damage evolution.

ANALYSIS OF EXPERIMENTAL RESULTS

Digitizing of surface crack patterns

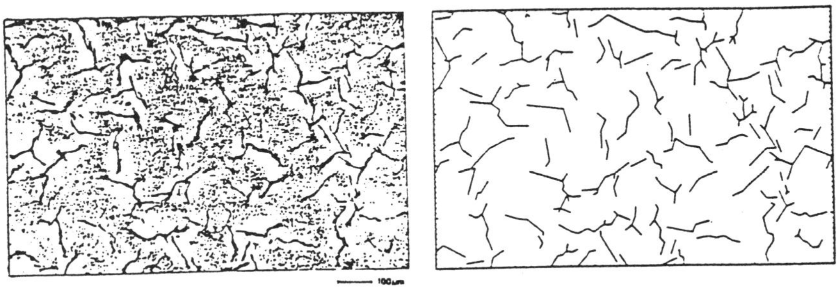

Figure 1. Crack pattern after 4500 load cycles. (a: observed via an optical microscope, b: after digitizing).

The photographs of the surface crack patterns were manually digitized by approximating the cracks by piecewise straight lines and storing the nodal points of the segment system (see Figure 1). The digitized patterns are the database for statistical analysis. Nevertheless, there is some ambiguity in the result of the pattern-digitizing procedure. Differences may arise from the neglection of very short edges, from smoothing of cracks, and from separating cracks or closing the gaps between them, respectively. These effects have to be taken into account in the interpretation of the experimental results. It should be noted that this drawback in data acquisition also occurs if an image processing system is used.

Statistical characterisation of the surface crack patterns

An extensive discussion of the statistical characterisation of random crack patterns is given in [4].
First information about damage accumulation is obtained from the changes with time of quantities of descriptive crack statistics, namely:

- the number of cracks per unit area,
- the total length of all cracks per unit area,
- the average length of a single crack.

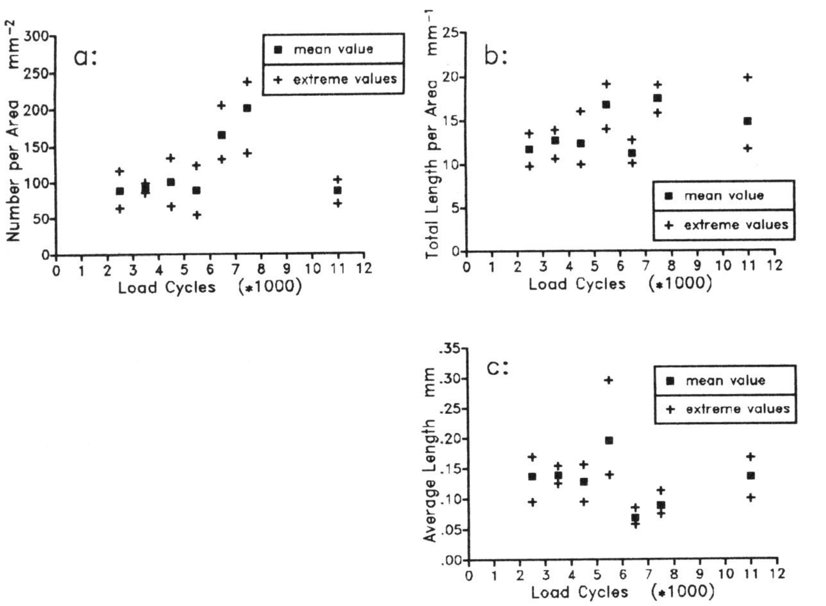

Figure 2. Quantities of descriptive crack statistics of experimental crack patterns. (a: number of cracks per unit area, b: total length of all cracks per unit area, c: single crack length).

The first two characteristics describe the global damage level, whereas the third one also includes local features of damage. The scatter of these characteristics contains additional information about the damage process.

In Figure 2 quantities of descriptive crack statistics for the experimental results are shown. For each number of load cycles the digitized crack patterns of different locations on the surface of a given specimen were analysed. Two features in the diagrams are remarkable: first, there is a large scatter in the quantities for a given number of load cycles, and second the evolution of the mean values with increasing number of load cycles does not create a uniform trend. The reason for this behaviour are different sources of scatter in the data. The scatter of the quantities for a given load cycle is influenced by the scatter of the material because they are obtained at different locations on the same specimen. The variation of the mean values which gives the trend for increasing load cycles on one hand comes from the scatter due to measurements on different specimens. But on the other hand it has also to be taken into account that the patterns according to the number of cycles from 2500 to 5500 and from 6500 to 11000 were manually digitized by two different persons. This could explain a certain shift in the data which can be observed between 5500 and 6500 load cycles.

Though this large influence of the digitizing procedure had not been anticipated, a small number of patterns were digitized by both persons for reasons of compatibility. It turned out that the digitizer of the patterns with a smaller number of load cycles often decided to disregard some of the small particles and to neglect small gaps between cracks (i.e. to treat them as one connected crack) which led systematically to a smaller number of cracks and to a larger value of the mean crack length in the statistical analysis. A check of the photographs showed that there is indeed some ambiguity in the characterisation which is an additional source of uncertainty in the interpretation of the results. It was not possible to decide if one of the two digitizers was definitely wrong which means that these difficulties would also arise if image analysis was used.
Taking this shift between 5500 and 6500 load cycles into account the figures indicate that similar tendencies in the quantities can be observed in both parts of the load cycles. These tendencies are: There is an incubation period until about 4500 load cycles during which the number of cracks slowly increases and the mean crack length is approximately constant. Here, crack initiation and crack coalescence should be of nearly the same magnitude in the damage process. After that period, the average length of the cracks increases whereas the total length of all cracks is nearly constant. These tendencies indicate that the dominant process of damage evolution in that period is crack coalescence which corresponds to the decrease of the number of cracks per area for a high number of load cycles.

Crack growth in depth direction

Additional information about the damage evolution can be obtained from the investigation of crack growth in depth direction. In Figure 3 the histogram of the maximum depth of cracks for different numbers of load cycles is shown. The increasing extension of the crack patterns in depth direction with increasing

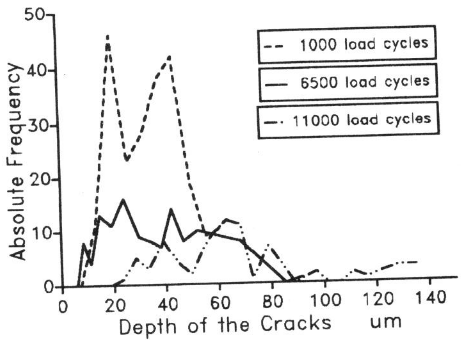

Figure 3. Histogram of the extension of the cracks in the depth.

number of load cycles due to crack growth can be observed. Damage evolution in depth direction is found to be more pronounced than the increase in damage on the surface. The changes in the shape of the distribution of crack extension could be a consequence of crack interaction effects. No further information is available at present.

SIMULATION OF THE DEVELOPMENT OF IRREGULAR SURFACE CRACK PATTERNS

Basic assumptions

The characteristics of descriptive crack statistics are often used in fatigue analysis but they cannot describe the typical net-like characteristics of the crack patterns. Therefore, the statistical analysis was extended by suitable statistical methods describing random fields of cracks. We use the model of an incomplete random mosaic for this purpose. Starting from an underlying mosaic, the crack pattern is generated with some probability by successive failure of the edges of the mosaic. The so called DIRICHLET-tesselation is used as a specific planar mosaic because its statistical properties are known. On the basis of this model a simulation procedure is developed. The underlying tesselation can be determined from experimental results. The simulation algorithm consists of two main parts:

The first part involves the generation of a geometric structure defining potential crack paths which can be considered as 'weak lines' of the material. A direct correlation between the pattern of 'weak lines' and the grain structure is not possible because metallographic observations show that crack growth is both transgranular and intergranular.

The second part of the model which influences the form of the crack patterns is the physical model for damage accumulation. Only a certain failure probability for the edges of the mosaic is determined by this physical model in order to take into account the stochastic nature of the damage process. Then the formation of crack patterns is simulated by successive failures of the edges according to the calculated probability.

Simulation procedure

From these basic assumptions a stochastic simulation procedure consisting of three main steps has been developed.

The **first step** consists in the generation of the random planar DIRICHLET-tesselation which serves as the geometric structure defining potential crack paths. The intensity of the generating point process (i.e. the mean number of cells per unit area) is chosen in accordance with experimental data.

The **second step** consists in the simulation of the initiation of microcracks. At the beginning of the damage process only few cracks are initiated so that the distances between them are large compared with their lengths. Therefore, crack interaction can be neglected and the initiation of microcracks can be simulated by the independent random failure of the edges of the tesselation. Each edge *k* of the tesselation has the same probability

$$p_k = p_0 \tag{1}$$

of forming a microcrack. After this step a configuration of microcracks is present.

The **third step** of the procedure consists in the simulation of damage evolution. Here crack growth, crack coalescence and branching as well as initiation of new microcracks have to be modelled. This is done in a number of subsequent simulation steps.

Obviously, the failure of an edge may be influenced by the number of failed edges in a given environment which leads to the following scheme of simulation. For edges adjacent to existing cracks a loading parameter B is calculated by a fracture mechanics model. For isolated edges this loading parameter is set zero. Then the failure probability of the uncracked edges of the tesselation is assumed to depend on B by the equation

$$\begin{aligned} p_k &= (1 - p_{\min})\left(\frac{B - B_{\min}}{B_{\max} - B_{\min}}\right)^n + p_{\min} && \text{for } B > B_{\min} \\ p_k &= p_{\min} && \text{for } B \leq B_{\min} \end{aligned} \tag{2}$$

which is shown in Figure 4. The quantities $B_{\min}$, $B_{\max}$, $p_{\min}$, n are parameters of the simulation model. The development of the typical shape of the simulated surface crack patterns can be controlled by variation of these parameters.

Fracture mechanics model

According to the mosaic model, there are two possible edges at each end point of a crack along which crack propagation may occur, and there is one possible edge for crack propagation at a kink point of existing cracks. The failure probability of edges adjacent to existing cracks is calculated from the normal stress distribution σ_φ acting on the uncracked edges which is shown in Figure 5. We have to distinguish between two cases, namely edges at the end points and edges at the kink points of the cracks. In both cases, the stresses are singular but the shape of stress distributions is different. To obtain a general loading parameter B for all edges, a local average of the normal stress σ_φ ahead of the crack tip is calculated [5]:

$$B(\varphi) = \frac{1}{r_0} \int_0^{r_0} \sigma_\varphi (r, \varphi)\, dr \quad . \tag{3}$$

The length r_0 which determines the range of averaging is a further parameter of the simulation model. Variation of r_0 leads to a different assessment of edges adjacent to end points and kink points which affects crack branching. The integral in (3) is the tensile force acting on the uncracked edge over the distance r_0. A simplified fracture mechanics model has been developed to determine σ_φ for various crack shapes [6].

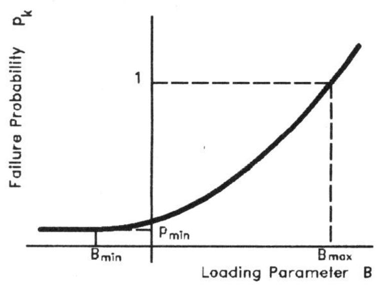

Figure 4. Relation between the loading parameter and the failure probability.

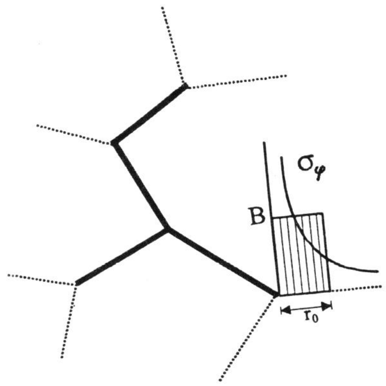

Figure 5. Normal stress distribution acting on the uncracked edges.

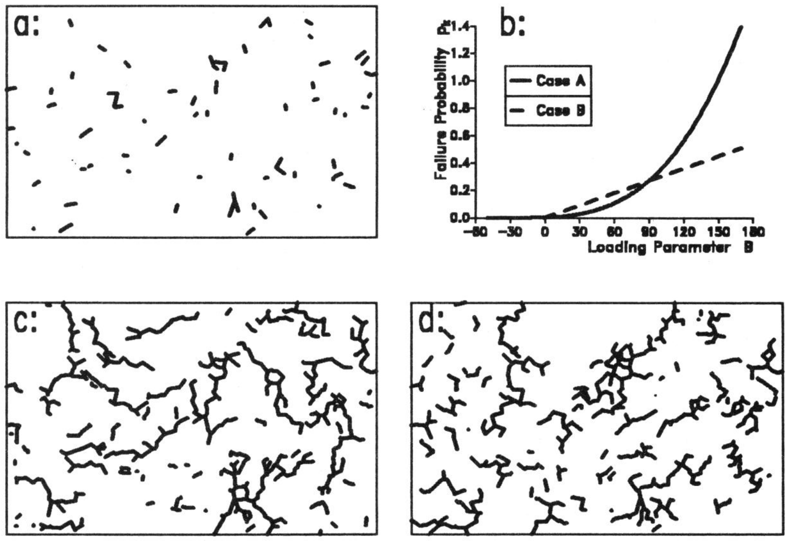

Figure 6. Simulated crack patterns. (a: crack pattern - initiation step, b: relation between loading parameter B and failure probability p_k, c: simulated crack pattern - case A - after 8 successive simulation steps, d: simulated crack pattern - case B - after 8 successive simulation steps).

DISCUSSION OF THE RESULTS OF SIMULATION

The geometric model of the DIRICHLET-tesselation for crack initiation and propagation sites together with the fracture mechanics model allow a simulation procedure to be established which should be able to reflect the main characteristics of the damage evolution process that can be observed in the experiment. Typical results are shown in this section.

Formation of irregular crack pattern

General trends of the damage accumulation process can be modelled by a suitable choice of the parameters which determine the relation between the loading parameter and the failure probability. The results for two different relations between the loading parameter B and the failure probability p_k are shown in Figure 6. The probability of independent edge failure during simulation of crack initiation and the integration length r_0 for the determination of B are equal for both cases.

A first qualitative assessment of the simulation model can be made by the investigation of the evolution of the quantities of descriptive crack statistics and the comparison with the experimental results. The simulated crack patterns after 8 successive simulation steps and the evolution of the quantities of descriptive crack statistics are shown in Figure 6 and Figure 7.

In both cases the tendencies of the mean values of descriptive crack statistics are nearly the same. The mean number of cracks shows a slight increase over a wide range of damage accumulation and a small drop in the last simulation step. Here, the principal tendency observed in the experiments can be reflected by the simulation results. Also the increase in the average crack length corresponds qualitatively to experimental observations. A different behaviour is found if the total length of all cracks is considered. Starting from a very low level of damage compared with the experimentally obtained values, the total length increases considerably with the simulation. In the last simulation step a value of the same order of magnitude as observed in the experiments was reached.

This means that the time scale for the present simulation process does not coincide with the analysed range of experimental observations. The simulation seems to describe a very early stage of damage. This conclusion is confirmed if the development with time of the shape of the crack pattern is considered which requires that additional quantities are used for the description of crack shapes. In the first simulation steps, the crack patterns consist of a large number of very small straight cracks as can be seen in Figure 6a.

Contrary to that, the experimental observations show that the typical shape of the crack pattern with branched cracks has developed after a comparatively low number of load cycles. Subsequent damage evolution consists of two main parts: a slow progress in the formation of closed networks of the surface cracks by crack coalescence and a more pronounced growth of the cracks in depth direction (see Figure 2, Figure 3).

This means that crack interaction effects will affect the development of crack patterns already in a very early stage. In the present stage of the simulation model, however, only a very crude modelling of interaction effects is included. Therefore, we think that the simulation algorithm is only able to reflect a beginning stage of the damage process, lets say up to about 1000 load cycles. This was a surprising result which gave an important hint regarding the improvement of the simulation procedure. To confirm this impression, it is necessary to investigate if the simulated crack patterns at the end of this early stage of damage exhibit the same typical net-like geometrical features as the experimental pat-

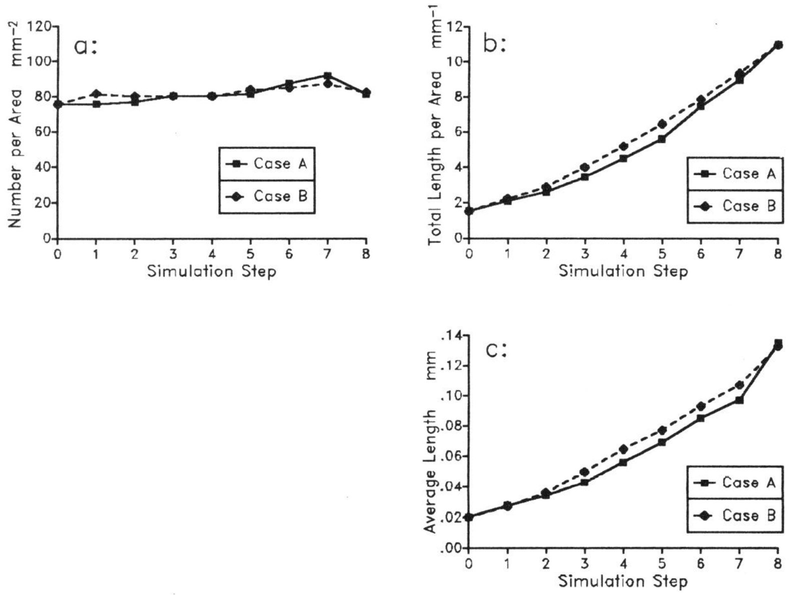

Figure 7. Average values of descriptive crack statistics of the simulated crack pattern. (a: number of cracks per unit area, b: total length of all cracks per unit area, c: single crack length).

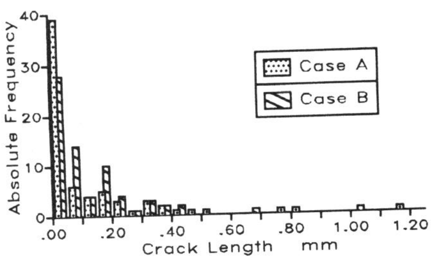

Figure 8. Crack length distribution of the simulated crack patterns.

terns after about 1000 cycles. This answer can not be given from the obtained mean values of descriptive crack statistics.

Assessment of different crack patterns

A very important point in the assessment of the simulation algorithm is the quantitative evaluation of the simulated crack pattern. This is demonstrated by the example of the two simulated crack patterns shown in Figure 6c,d. In both cases, the same 15.4 % fraction of cracked edges of the mosaic is attained and the evolution of the mean values of descriptive crack statistics with increasing simulation steps is not very different. Nevertheless, the crack patterns have some different geometrical features. This can be demonstrated if one looks at the empirical crack length distribution (see Figure 8). In case A there is a great number of very short cracks in addition to few long cracks. Compared with this, the crack length is more uniformly distributed in case B. This behaviour corresponds to the different relation between the loading parameter B and the failure probability p_k of an edge of the tesselation which is shown in Figure 6. In case A, the growth rate is very slow for initiated cracks which are very short because of the low level of failure probability for small values of the loading parameter B. But if crack linking leads to a long crack, the loading parameters rapidly increase for this crack which accelerates its subsequent growth. The opposite is true in case B. Here small cracks grow faster and the growth of long cracks is not so much favoured as in case A.
This is a typical example of how mechanisms of damage evolution can be found by comparing simulated with experimental crack patterns based on quantities which describe the crack shape. For a detailed analysis more refined statistical quantities than those shown here have to be used.

SUMMARY

Experimental results on the damage evolution of an austenitic steel under thermal fatigue loading were used to establish a combined stochastic and fracture mechanics model which allows to quantify damage accumulation and to identify different stages of damage evolution. It was shown how statistical quantities can be used to interpret the development of random crack patterns and to describe the damage process. The stochastic model was based on a random ge-

ometrical model describing sites of crack initiation and possible crack paths. The fracture mechanics model allowed to establish a loading parameter which controls random crack initiation and propagation.
With this model, surface crack patterns were generated, which exhibit typical characteristics of the experimental observed crack patterns, from a first optical impression. A quantitative comparison between experimental results and simulated crack patterns was performed by using quantities of descriptive crack statistics and more advanced quantities like statistical distributions of crack lengths.
As the investigations are still under way, this paper can only give a review of the current state of the art. The main result which can be stated at least qualitatively is that the onset of crack interaction starts at a very early stage and that damage propagation in the depth direction plays a more important role than anticipated.

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