SPECIFYING THE REQUIREMENTS TO A PLANE SPECIMEN WITH A CRACK IN ORDER TO REALIZE BRITTLE FRACTURE

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A requirement to the thickness of a cracked specimen, analogous with the empirical plane strain condition, has been obtained proceeding from the condition of elastic strain. The stress σ_{zz} parallel to the crack front is formed in layers close to the free surface of a specimen, and the stress distribution conforming to the solution of the theory of elasticity is brought about in the central part of the specimen. The Tresca condition is not violated due to the presence of σ_{zz}.

INTRODUCTION

Brittle fracture of a smooth specimen takes place at a stress, the value of which is equal to the brittle fracture strength σ_f of a material and less than the yield stress σ_Y in accordance with the Ludwik-Yoffe model. Increasing the temperature leads to a drop of σ_Y and an increase of σ_f. Brittleness temperature T_B is determined by the condition of equality of these two properties. The brittleness temperature may increase with creation of a triaxial stress state, induced, in particular, by a crack. It is assumed that the possible maximum value of the third stress $\sigma_3=\sigma_{zz}$ (parallel to the crack front) is determined by the condition of plane strain near the crack tip as follows:

$$\sigma_{zz}=\nu(\sigma_{xx}+\sigma_{YY}) \tag{1}$$

Hence, in accordance with Trescas' condition, the maximum value of tensile stress near the crack tip

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cannot exceed the value:

$$\sigma_{YY}^{max} \leqslant \sigma_Y/(1-2\nu) \tag{2}$$

for a material exhibiting an ideal elastic-plastic stress-strain curve. This means, that if the brittle fracture stress σ_f exceeds the value of $\sigma_Y/(1-2\nu)$ beyond the temperature T_1, then brittle fracture cannot occur in principle by enlargement of the body size. Such a conclusion is in contradiction with the practice of fracture and, in particular, with an empirical plane strain condition . According to this condition, the required specimen thickness B should be:

$$B \geqslant \beta\left(\frac{K_{Ic}}{\sigma_Y}\right)^2 \text{, where } \beta=2.5, \tag{3}$$

to meet the criterion of brittle fracture $K_I=K_{Ic}$. As follows from eq.(3), the enlargement of thickness, as shown by Francois and Krasowsky (1), should lead to infinite increase of the brittleness temperature .

The main premise of the present work consists in the fact, that brittle fracture is possible only at elastic strain in a material. The objective of the work is to show, that conditions of type (3) ensure sufficiently high stresses σ_{zz}, which provide stress and strain distributions close to those obtained by theory of elasticity.

A MODEL OF QUASI-ELASTIC STRESS DISTRIBUTION NEAR THE CRACK TIP

Consider a cracked body of sufficient size (Fig.1). The situation near the crack tip is determined by the K_I value and σ_Y for non-hardening materials. An area S of elevated strain originates near the crack tip with the boundary Γ for each K_I value. This region, tapering off towards the direction of the z axis, undergoes tension in the same direction, induced by the rigid part of the body, resulting in the stresses σ_{zz}.

It is assumed, that, near the crack tip, the region S is divided into 3 parts by the two planes parallel to the free surfaces of the body. The third principal stress σ_{zz} is built up in the two outer symmetric layers of thickness αB, owing to the action of the shear stress

$\tau_z=\sigma_Y/2$ on their boundaries. Here, stresses acting in the plane normal to the crack front are assumed to be equal to zero. There is an action of the normal stresses σ_{zz} in the central layer of the region S. Integral F_z of σ_{zz} is determined by the action of the outer layers and takes the value:

$$F_z = 0.5\ \sigma_Y\ \alpha B\Gamma, \tag{4}$$

where Γ is the length of the boundary of region S. The stresses σ_{xx}, σ_{YY} and τ_{xY} are somewhat higher than elastic solutions due to the redistribution of the stresses from the two outer layers into the central part of region S.

In accordance with an elastic solution, the maximum value of the principal stress σ_1 is equal to:

$$\sigma_1(r,\theta) = \frac{K_I}{\sqrt{2\pi r}}\ (\cos\frac{\theta}{2} + 0.5\sin\theta) \tag{5}$$

The boundary Γ between the region S and the rest rigid part of the body is determined by the locations where the stresses σ_1 are equal to the yield stress. Since the stress σ_1 is redistributed from the two outer layers of region S into the central one, then the condition for determining the boundary Γ is as follows:

$$\sigma_1(\Gamma) = (1-2\alpha)\sigma_Y \tag{6}$$

The stress σ_1 exceeds σ_Y inside the region S. In order to retain the elastic strain it is nessesary to require that the third principal stress σ_{zz} is not less than difference between $\sigma_1/(1-2\alpha)$ and σ_Y in each point of the central layer of the region S. Taking an integral of this condition over the area S gives:

$$\int\int_{(S)} \sigma_1 dS = (1-2\alpha)\sigma_Y\ S + F_z(1-2\alpha) \tag{7}$$

This equation serves for determining α, i.e. the thickness of the outer layers in which the normal stresses σ_{zz} are originated.

Deriving the equation of the boundary Γ of the region S from eqs.(5) and (6) and making calculations according to eq.(7) one obtains:

$$\frac{1}{12\pi}\ (K_I/\sigma_Y)^2\ 4.49 = \alpha(1-2\alpha)^2 B\ 2.19 \tag{8}$$

The lefthand part of eq.(8) takes the maximum value at $\alpha=1/6$. This is why quasi-elastic stresses distribution is possible if:

$$B \geqslant 0.735\ (K_I/\sigma_Y)^2 \qquad (9)$$

Since strains are elastic at brittle fracture, then substitution of K_{IC} instead of K_I in eq.(9) is possible. In such a form, it exist as a condition for valid determination of the brittle fracture parameter K_{IC}. Considerable difference between 0.735 and the well-known empirical value 2.5 may be explained mainly by the following reasons:

- the model assumes implicitly that σ_{zz} is the third principal stress and it does not take into account that maximum shear stresses in the (x,y) plane of the region Scentral layer can exceed half the yield strength. The construction of quasi-elastic distribution of stresses in this region, which meets the condition $\sigma_1-\sigma_2 \leqslant \sigma_Y$, will result in an increase in S and, as a consequence, in an increase in β in condition (9);
- ignoring of the nominal stress when specifying the stress state near the crack tip.

ACCOUNT OF NOMINAL STRESS

Valid determination of the brittle fracture criterion K_{IC} is concerned not only with the requirement to sufficient thickness of a body, but also with the similar requirements to the crack length a and the remaining ligament size w-a. This is concerned with the increase of the nominal stresses σ_n at short a and w-a (at the same K_I) and the situation near the crack tip is determined by σ_n as well as by K_I. Account of their effect on the maximum stress requires implementation of a numerical procedure for each particular body. A simplified analysis is possible taking into consideration only the stresses $\sigma_{yy}(x)$ acting along the crack line:

$$\sigma_{yy}(x) = \frac{K_I}{\sqrt{2\pi x}} + \sigma_n \qquad (10)$$

Making calculations similar to those mentioned above and taking into account that region S represents itself the section of the x axis and boundary Γ transforms into to two limit points of this section and with due account of nominal stress, one can obtain a similar condition for quasi-elastic stress distribution:

$$B \geqslant F(\sigma_n/\sigma_Y)\ \beta\ (K_I/\sigma_Y)^2 \tag{11}$$

where the β is equal to 2.15 and function $F(\sigma_n/\sigma_Y)$ is presented analytically in the following way:

$$F(y) = [(1-y)\sqrt{1-y^2}]^{-1} \tag{12}$$

The concept "nominal stress" is not quite clear. So, in real calculations the "reference" stress σ_{ref}, as proposed by Ainsworth (2), is taken as "nominal". The σ_{ref} determines the level of stresses acting in accordance with the limit state theory. Two-criteria analysis of the stress state of standard specimens containing a crack, as shown by Krasowsky at al (3), indicates, that σ_{ref} value ranges from $0.35\sigma_Y$ to $0.5\sigma_Y$ at $B = 2.5(K_{IC}/\sigma_Y)^2$, and for compact tension (CT) specimens σ_{ref} reaches the minimum value. This is why CT specimens are most preferable for the fracture toughness determination. In view of eq.(12), the function F(0.5) value is equal to 2.31. Division of 2.5 by 2.31 gives 1.08, i.e. the value close to 0.735, which is obtained by the maximum stress σ_1 calculation.

Enlargement of the specimen width from the standard value $w_o=2B$ to infinity may result only in limited shift of the brittleness temperatures at the same thickness B. In view of reference (2), this maximum shift ΔT may be estimated as $\Delta T=T_o \ln 2.31$, where T_o is a coefficient in an empirical presentation of the dependence $K_{IC}(T)$ in the form of $K_{IC}(T) = A\ \exp(T/T_o)$, where A = const. This conclusion is in good agreement with experimental observations .

CONCLUSIONS

Brittle fracture is realized under elastic strain condition. The well-known empirical requirement to the specimen thickness to ensure brittle fracture represents itself as a neccessary condition of elastic deformation. The obtained model results are in good agreement with the observed phenomena.

SYMBOLS USED

ν - Poissons' ratio;
K_I - stress intensity factor for a mode I crack;
S - region with elevated strain near the crack tip;
Γ - contour spreading over the region S;
r,θ - polar coordinates with origin at the crack tip.

REFERENCES

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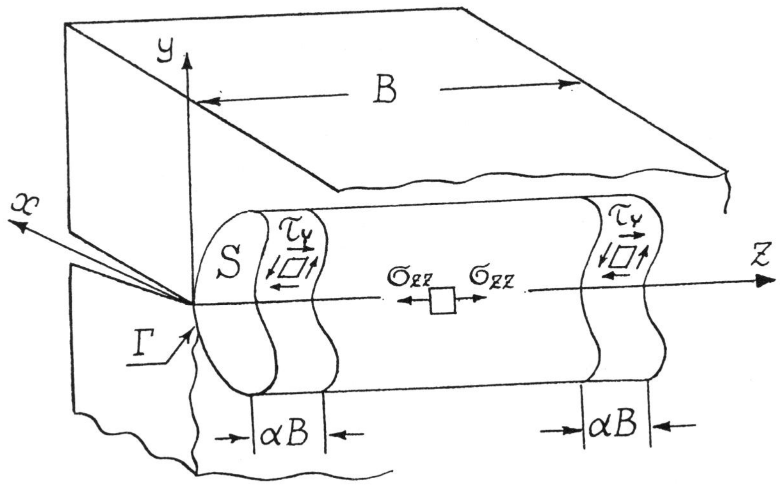

Fig.1. Region S location in a plane body with a crack.