NUMERICAL ANALYSIS OF MODE I STEADY STATE CRACK TIP FIELD IN ELASTIC PLASTIC HARDENING MATERIALS

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Numerical results from finite element calculations on plane strain models for a compact tension specimen C(T) and a center cracked tension specimen M(T) are compared with the asymptotic solution for the crack tip field of a steady state growing crack taking linear and multilinear hardening material behaviours into account. The analytical solution can describe the stress and deformation field near the crack tip of a steady state growing crack as it is gained from FE calculations.

INTRODUCTION

Knowledge of the stress and strain field at a steadily growing crack tip in an elastic plastic material is essential for the application in engineering problems of fracture mechanics. In a special case of steady state crack growth an asymptotic stress and velocity field for an elastic plastic material with linear hardening has been presented by Amazigo & Hutchinson [1]. Considering a plastic reloading zone near a steady state growing crack flank a modified asymptotic stress and velocity field for the same material behaviour has been given by Ponte Castaneda [2] some years ago. The results from both calculations are nearly identical. Now, it is interesting to know, whether the asymptotic solution can describe a stress and deformation field in a real specimen or structure.

This paper refers to earlier publications [3, 4], where the asymptotic stress, strain and displacement fields at a steady state growing crack tip in a linear hardening material have been presented. The asymptotic solution has been compared with numerical results from the FE calculation for a C(T)-specimen. The angular functions of stresses and strains from both, asymptotic solution and numerical results, were in qualitatively good agreement. In this paper detailed numerical results of plane strain models for a C(T)-specimen and a M(T)-specimen are presented of which results have been obtained by numerical investigations with

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FEM for linear hardening material behaviour. On basis of the asympotitc solution form for a steady state growing crack

$$\begin{aligned} \sigma_{ij}(r,\theta) &= A\ \sigma_0\ r^{-s}\ \tilde{\sigma}_{ij}(\theta) \\ \varepsilon_{ij}(r,\theta) &= A\ \varepsilon_0\ r^{-s}\ \tilde{\varepsilon}_{ij}(\theta) \\ u_i(r,\theta) - u_i^* &= A\ \varepsilon_0\ r^{-s+1}\ \tilde{u}_i(\theta) \end{aligned} \qquad (1)$$

where σ_0 is the yield stress, ε_0 the yield strain, s is the field singularity, A the amplitude factor, $\tilde{\sigma}_{ij}$, $\tilde{\varepsilon}_{ij}$ and $\tilde{u}_i$ are the angular functions and u_i^* is the rigid body displacement, it is intended to answer the following questions numerically:

- whether the stress and strain field of a stable growing crack will become steady state in real specimens and structures,
- when the steady state phase of a growing crack will occur approximately,
- whether the amplitude factor A and the singularity s are constant for all stress and deformation components and
- how large is the zone where the asymptotic solution dominantes.

It is also interesting whether the numerical results from a FE calculation with a multilinear material behaviour can be characterized by the asymptotic solution assuming linear hardening material behaviour. Therefore, the numerical investigation with FEM was extended to a M(T)-specimen with a multilinear material behaviour (German standard steel StE460). In the investigations mode I plane strain, small deformations, quasi static case, isotropic material and J_2-flow plasticity theory were taken into account.

RESULTS AND DISCUSSION

The evaluations were performed for the German standard steel StE460 with youngs modulus of E=210000 MPa, Poissons ratio of ν=0.3, yield strength of σ_0=460 MPa and tangent modulus of E_T=6300 MPa. The FE networks for the specimens are shown in [6]. The minimun dimension of the elements in the ligament region was 0.05 mm.

Figs. 1, 2 ,3 and 4 show the stress and strain distributions in the ligament for various crack growth steps gained from FE-calculations. It is shown that the stress and strain distributions are reaching steady state after approximately 1 mm crack growth. According to the asymptotic solution form (1) the singularity s should be constant and identical for all stress and strain distributions. The results in Figs. 5 and 6 show that s values from stress and strain distributions of both types of specimens are qualitatively in good agreement and are nearly constant for the C(T)-specimen and the M(T)-specimen as follows from nearly the same slopes of the curves. s is about 0.14. Also the angular functions around the crack tip for the stress and strain distributions are very similar for both types of specimens as can be seen from Figs. 7 to 10. Only for the component $\tilde{\varepsilon}_{xy}$ of the C(T) specimen, Fig.7, a greater deviation is observed. The reason for the deviation is not clear. According to equations (1) the amplitude factor A should be a constant for the steady state growing crack. The results from Figs. 11 and 12 show approximately the same value for all stress and strain distributions but is influenced by the specimen geometry. So, for the C(T) specimen A is about 3 but for the M(T) specimen about 5.

The analysis of FE calculations for the M(T) model using a multilinear hardening behaviour demonstrates that s values evaluated from the various stress and strain distributions agree very well, Fig 13. The angular functions are very similar to those gained from linear hardening material behaviour, Figs. 14 and 15, and A values in Fig. 16 determined from the various stress and strain distributions agree also very well.

The comparison of asymptotic solutions for crack tip field of a steady state growing crack in [6] with results of FE calculations on plane strain models of a C(T) and a M(T) specimen has demonstrated that the asymptotic solutions can describe the stress and strain field near the tip of a steady state growing crack for linear hardening material behaviour. For multilinear hardening material behaviour the stress and strain field at the crack tip can also be characterized by the parameters of the theory for the asymptotic stress and strain field of a steady growing crack.

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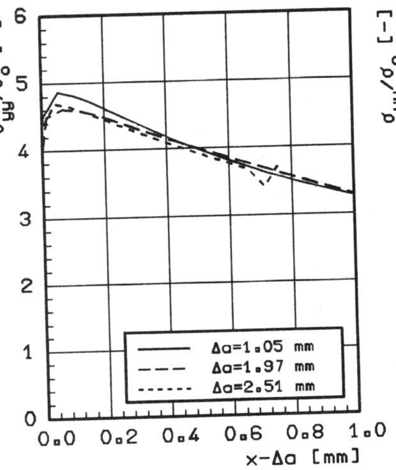

Fig.1: stress in ligament

C(T)25-specimen
bilinear material with α=0.03

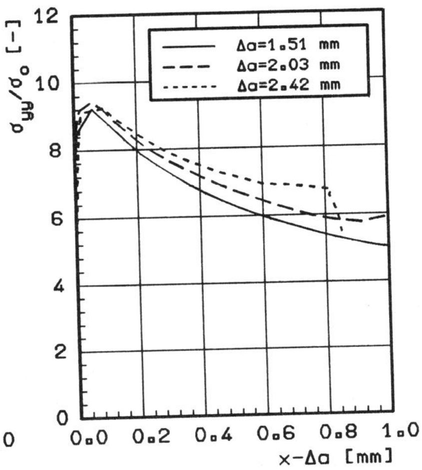

Fig.2: stress in ligament

M(T)25-specimen
bilinear material with α=0.03

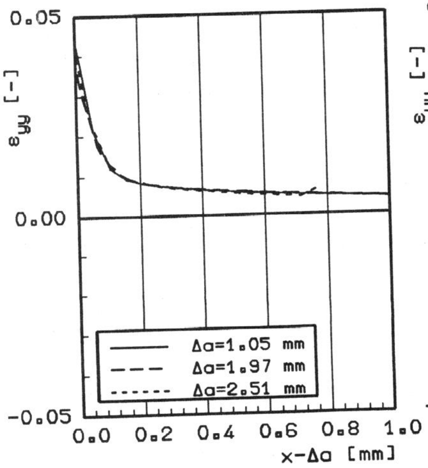

Fig.3: strain in ligament

C(T)25-specimen
bilinear material with α=0.03

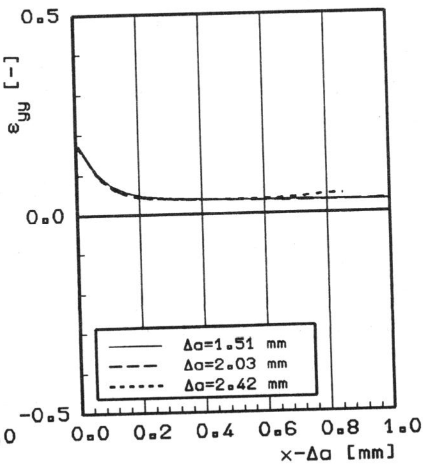

Fig.4: strain in ligament

M(T)25-specimen
bilinear material with α=0.03

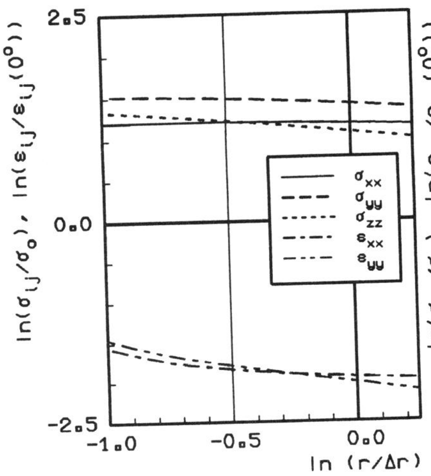

Fig. 5: evaluation of s

C(T)25-specimen
bilinear material with α = 0.03
Δa = 1.97mm, Δr = 0.5mm

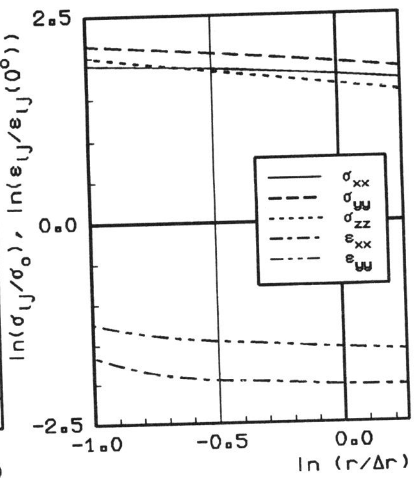

Fig. 6: evaluation of s

M(T)25-specimen
bilinear material with α = 0.03
Δa = 2.03mm, Δr = 0.5mm

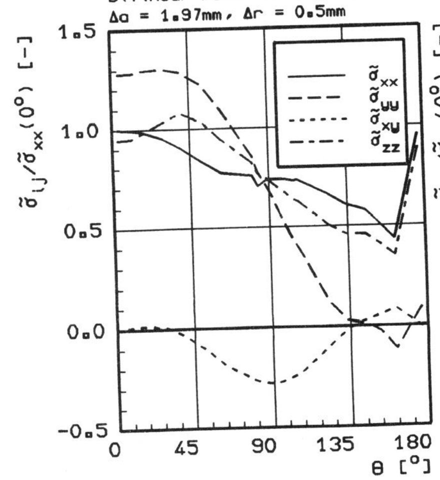

Fig.7:angular functions

C(T)25-specimen
bilinear material with α = 0.03
Δa = 1.97mm, r = 0.41mm

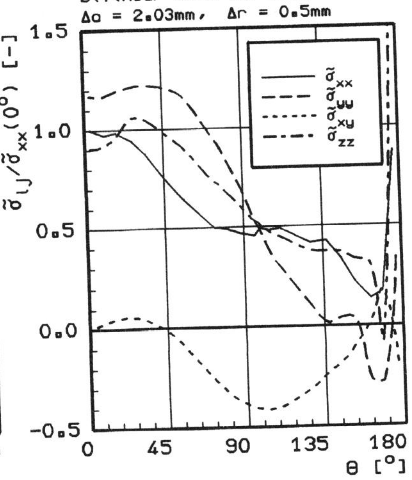

Fig.8:angular functions

M(T)25-specimen
bilinear material with α = 0.03
Δa = 2.03mm, r = 0.41mm

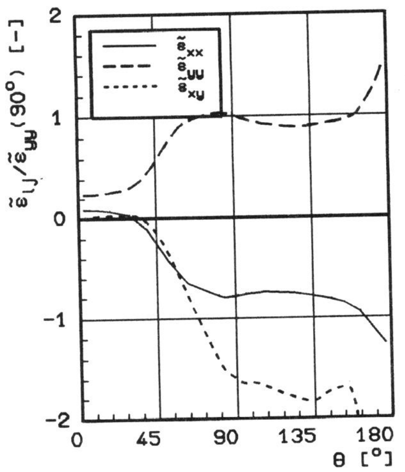

Fig.9: angular functions

C(T)25-specimen
bilinear material with α = 0.03
Δa = 1.97 mm, r = 0.41 mm

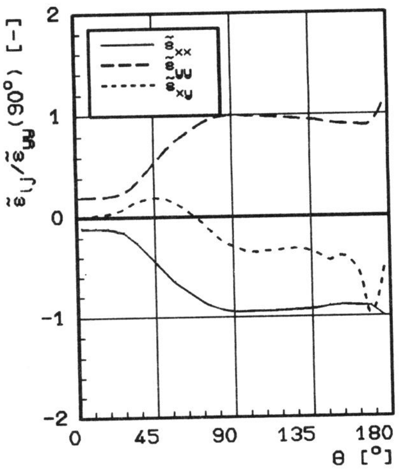

Fig.10: angular functions

[redacted](T)25-specimen
bilinear material with α = 0.03
Δa = 2.03 mm, r = 0.41 mm

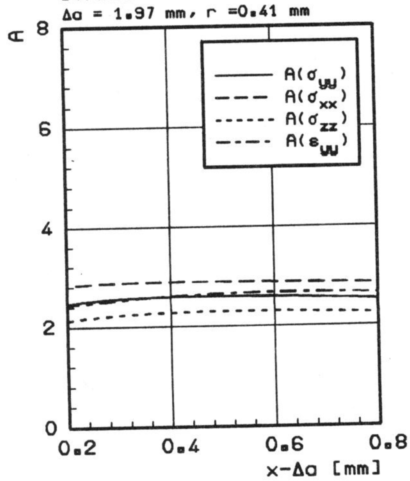

Fig.11: amplitude factor

C(T)25-specimen, Δa=1.97 mm
bilinear material with α = 0.03

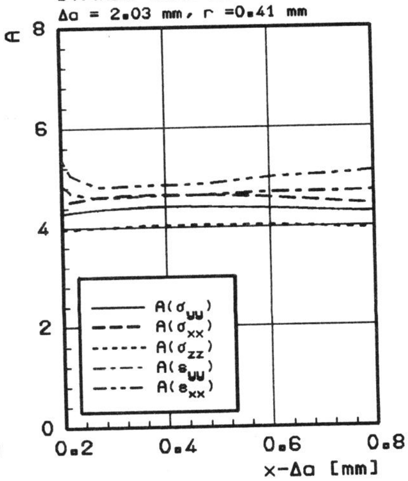

Fig.12: amplitude factor

M(T)25-specimen, Δa=2.03 mm
bilinear material with α =0.03

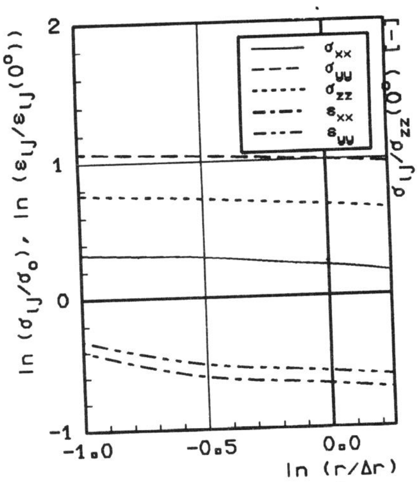

Fig.13:evaluation of s

M(T)25-specimen
multilinear material StE 460
Δa = 2 mm, Δr = 0.5 mm

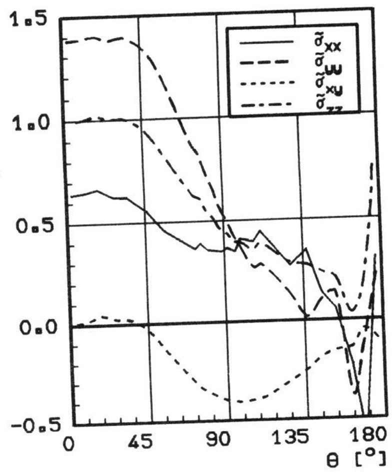

Fig.14:angular functions

M(T)25-specimen
multilinear material StE 460
Δa = 2 mm, r = 0.41 mm

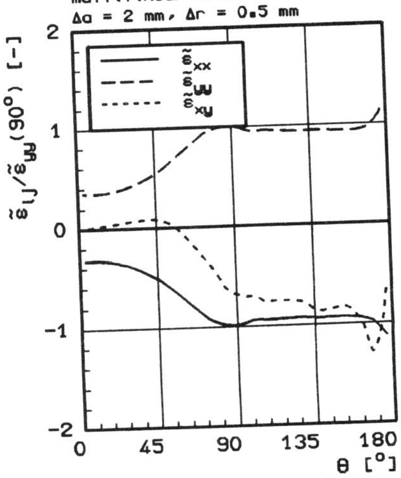

Fig.15:angular functions

M(T)25-specimen
multilinear material StE 460
Δa = 2 mm, r = 0.41 mm

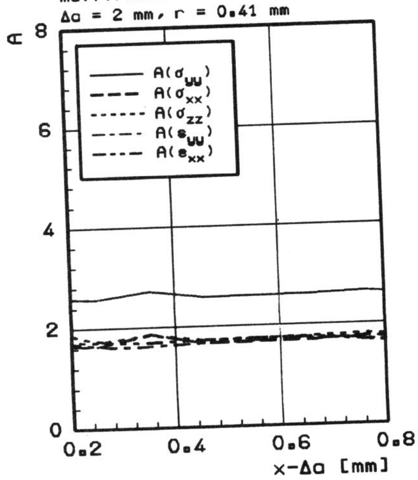

Fig.16:amplitude factor

M(T)25-specimen, Δa=1.76 mm
multilinear material STE 460