INVESTIGATION ON THE CONSTRAINT EFFECT BASED ON ASYMPTOTIC STRESS AND DEFORMATION FIELD AT THE STEADY STATE GROWING CRACK TIP

Tianmin Guo[1]

What controls the steady state growing crack? We will try in this paper to answer this question basing on the asymptotic stress and deformation field. The investigation assumes mode I plane stress and plane strain, small deformations, quasi static case, elastic plastic material with linear hardening and J_2-flow plasticity theory. As the result to describe this problem we present a parameter W/δ^2, which is radius-independent and a material constant in the near field at the steady state growing crack.

INTRODUCTION

The investigation and analysis of cracked structures are the task of fracture mechanics. Evaluating those structures the global fracture criterions, which are based on the crack resistance behaviour, are used. In order to evaluate the crack resistance behaviour the stress and deformation field at the actual crack tip must be known. In linear elastic fracture mechanics the so-called stress intensity factor K, which is the amplitude factor for stress and deformation field, is used as a global fracture mechanics parameter [1] basing on the energy release rate G [2] and on the relation between K and G [1]. The elastic plastic fracture mechanics for a stationary crack uses the J-integral as a global fracture mechanics parameter. The J-integral has been derived in [3] from a energy balance for a cracked structure and is at the same time the amplitude factor in the stress and deformation field, the so-called HRR-solution [4,5]. So it is justified as a criterion for crack initiation in an elastic plastic material. But the initiation values K and J are strictly speaking

[1] Bundesanstalt für Materialforschung und -prüfung (BAM)
Under den Eichen 87, 12205 Berlin, Germany

not material constants and depend on loading and geometry conditions, which are in discussion under the concept "constraint effects" in recent years. This is the T-stress [6] and the Q-stress [7]. One of the main reasons is that in in-plane-problems the hydrostatic stress in the ligament in a bending stressed specimen is greater than in a tension stressed specimen, so that the strain in the ligament is constrained and the crack opening stress in a bending stressed specimen is greater than in a tension stressed specimen, whereas the reverse results will be expected from the theory [4,5]. This problem is intensified for the steadily growing crack in an elastic plastic material, where the J-integral has two essential problems. The first is that the J-integral is path-dependent [8] and the other is that the experimentally obtained J-integral as a crack resistance curve depends on loading and geometry conditions [9, 10]. In order to explain this geometry or constraint effects an additional parameter, which is defined as hydrostatic stress σ_m over effective stress σ_e in the ligament [11], has been introduced. But this parameter depends on the radius r to the crack tip so that it cannot be defined simply. Also it is not clear, in which form this parameter or the T-stress and the Q-stress is related to the experimentally obtained J-integral in dependence of the specimen geometry. The question is whether it is possible in a special case of a steady state growing crack to find a control parameter, which can describe the crack resistance behaviour of a steady state growing crack by formulation and definition of a parameter basing on the asymptotic stress and deformation field, and how this parameter can be physically plausible and simply defined. For this purpose we carry out investigations and assume mode I plane stress and plane strain, small deformations, quasi static case, linear hardening material and J_2-flow plasticity theory.

A FRACTURE MECHANICS PARAMETER OF A STEADY STATE GROWING CRACK BASING ON THE ASYMPTOTIC STRESS AND DEFORMATION FIELD

Let us now consider a steady state growing crack. The stress and deformation field at the steady state growing crack are not changing during the crack growth. For an elastic plastic material with linear hardening it has the following asymptotic solution form

$$\begin{aligned} \sigma_{ij}(r,\theta) &= A\ \sigma_0\ r^{-s}\ \tilde{\sigma}_{ij}(\theta) \\ \varepsilon_{ij}(r,\theta) &= A\ \varepsilon_0\ r^{-s}\ \tilde{\varepsilon}_{ij}(\theta) \\ u_i(r,\theta) - u_i^* &= A\ \varepsilon_0\ r^{-s+1}\ \tilde{u}_i(\theta) \end{aligned} \qquad \textbf{(1)}$$

where σ_0 is the yield stress, ε_0 the yield strain, s is the field singularity and $\tilde{\sigma}_{ij}$, $\tilde{\varepsilon}_{ij}$ and $\tilde{u}_i$ are the angular functions. They depend only on the material behaviour and are determined by the field equations [12,13,14,15]. u_i^* is the rigid body displacement, A is the amplitude factor and depends on loading and geometry conditions.

The numerical results from a FEM-calculation [16] show: 1st the stresses and strains at the actual crack tip are not changing after a certain crack growth , 2nd the stresses and strains at the growing crack tip have approximately the asymptotic solution form (1) and 3rd the amplitude factor alone describes loading and geometry conditions. So we have all informations of stresses and deformations at the growing crack tip. Now, the question is which fracture mechanics parameter controls the steady state growing crack. If we use the amplitude factor as a controling parameter, we have two problems. 1st: the amplitude factor has different values for various loading and geometry conditions. 2nd: the amplitude factor of a steady state growing crack tip field is dependent on a normalized condition and is only a relative value. Therefore, the amplitude factor is not in a position to describe the steady state growing crack. Certainly we can replace the amplitude factor by the fracture mechanics parameters as the near-tip-J-integral, the crack tip opening displacement CTOD or the crack tip opening angle CTOA [17], but all those parameters depend on the structural geometries as well as on the radius to the actual crack tip.

There is still remaining the question, what controls the steady state growing crack. A fracture mechanics parameter must be independent on loading and geometry conditions and the radius to the crack tip. It should depend on material behaviours only. According to the asymptotic solution form (1) the angular functions of stresses and deformations seem to fulfil these conditions. Therefore, we will use the angular functions as a tool to find a controling parameter. For that purpose we consider two mechanics values, the strain energy and the displacement on the crack flank.

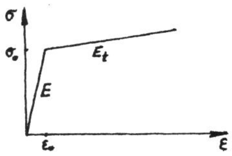

Fig. 1 linear hardening material

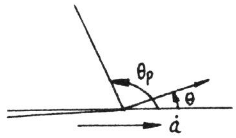

Fig. 2 crack tip field

Let us first show the strain energy density

$$dw = dw_e + dw_p \tag{2}$$

In the loading zone it is

$$w = \int dw = \frac{1+\mu}{3E}\sigma_e^2 + \frac{3(1-2\mu)}{2E}\sigma_m^2 + \frac{\omega}{2E}\sigma_e^2 - \frac{\omega\sigma_0^2}{2E} \tag{3}$$

where E is the elastic modulus, μ is the Poisson's ratio and $\omega = E/E_t - 1$, E_t is the tangent modulus. If we consider the asymptotic solution form (1) it becomes

$$w_1(r,\theta) = A^2\sigma_0\varepsilon_0 r^{-2s}\left[\frac{(1+\mu)}{3}\tilde{\sigma}_e^2 + \frac{3(1-2\mu)}{2}\tilde{\sigma}_m^2 + \frac{\omega}{2}\tilde{\sigma}_e^2\right] - \frac{\omega}{2}\sigma_0\varepsilon_0 \tag{4}$$

The last term in (4) is a constant and can be here neglected. In the unloading zone the strain energy density is

$$w_2(r,\theta) = A^2\sigma_0\varepsilon_0 r^{-2s}\left[\frac{(1+\mu)}{3}\tilde{\sigma}_e^2 + \frac{3(1-2\mu)}{2}\tilde{\sigma}_m^2 + \frac{\omega}{2}\left(\frac{\sin\theta_p}{\sin\theta}\right)^{2s}\tilde{\sigma}_e^2(\theta_p)\right] \tag{5}$$

If we consider a circle volume with unit thickness and the symmetry condition, we can integrate the above equations (4) and (5) and get the strain energy

$$W = 2\int_0^r [\int_0^{\theta_p} w_1(r,\theta)d\theta + \int_{\theta_p}^{\pi} w_2(r,\theta)d\theta]\, rdr \tag{6}$$
$$= A^2\sigma_0\, \varepsilon_0\, r^{-2s+2}\, \tilde{W}$$

with

$$\tilde{W} = \frac{1}{1-s}[\int_0^{\pi}(\frac{1+\mu}{3}\tilde{\sigma}_e^2 + \frac{3(1-2\mu)}{2}\tilde{\sigma}_m^2)d\theta + \int_0^{\theta_p}\frac{\omega}{2}\tilde{\sigma}_e^2 d\theta + \int_{\theta_p}^{\pi}\frac{\omega}{2}(\frac{\sin\theta_p}{\sin\theta})^{2s}\tilde{\sigma}_e^2(\theta_p)d\theta] \tag{7}$$

$\tilde{W}$ depends only on material behaviours. In the strain energy W the whole near field is considered. The second value is the displacement on the crack flank and follows

$$\delta = u_y(r,\pi) - u_y(r,-\pi) = 2\, A\, \varepsilon_0\, r^{-s+1}\, \tilde{u}_y(\pi) \tag{8}$$

It describes the steady state growing crack form and is an important value. We square the equation (8) and get

$$\delta^2 = 4\, A^2\, \varepsilon_0^2\, r^{-2s+2}\, \tilde{u}_y^2(\pi) \tag{9}$$

Dividing the strain energy (6) by the equation (9) it follows

$$\frac{W}{\delta^2} = \frac{E\, \tilde{W}}{4\, \tilde{u}_y^2(\pi)} \tag{10}$$

It is very interesting that the left side of the equation (10) is the loading response and the right side is the material response. This parameter depends not on the amplitude factor and is radius-independent in the near field at a steady state growing crack.

CONCLUSION

In this paper a fracture mechanics parameter has been presented basing on the asymptotic stress and deformation field at a steady state growing crack in an elastic plastic material with linear hardening. This parameter is radius-independent and a material constant in the near field at a steady state growing crack. It can control the steady state growing crack in a linear hardening material at least for both limit cases, the plane stress or the plane strain state. The question remains open whether this parameter can be extended to control also the steady state growing crack in a power-law hardening material. According to the investigation in [18] it should be also approximately describe the steady state growing crack in a power-law hardening material. All those must be answered by experiments and numerical investigations with FEM.

REFERENCE

[1] Irwin, G.R.: Journal of applied mechanics 24, 1957, pp. 361-364

[2] Griffith,A.A.: Solids Phil Trans. Roy. Soc. London, A221,1992,pp. 163- 198

[3] Rice,J.R.: Journal of applied mechanics, 35, 1968, pp. 379-386

[4] Rice,J.R. & Rosengren,G.F.: Journal mech. phys. solids 16, 1968, pp. 1-12

[5] Hutchinson,J.W.: J. mech. phys. solids 16, 1968, pp. 13-31

[6] Hancock,J.W., Reuter,W.G. & Parks,D.M.: ASTM symposium on constraint effects in fracture, Indianapolis 1991

[7] Shih,C.F.: ASTM symposium on constraint effects in fracture,Indianapolis 1991

[8] Brocks,W. & Yuan,H.: VDI report 18, No 82, 1990

[9] Garwood,S.J.: ASTM STP 677, 1979, pp. 511-532

[10] Aurich.D.: BAM-report 174, Berlin, 1990

[11] Aurich,D. & Sommer,E.: Steel research 59, 1988, pp. 358-367

[12] Amazigo,J.C. & Hutchinson,J.W.: J. mech. phys. solids 25, 1977, pp. 81-97

[13] Castaneda,P.P.: J.mech. phys. solids 35, 1987, pp. 227-268

[14] Brocks,W. , Guo,T. & Fricke,S.: Proceeding of 9th ECF II, 1992, pp.821-828

[15] Guo,T. & Brocks,W.: Proceeding of 25th DVM-Arbeitskreise Bruchvorgänge, 1993, pp. 415-425

[16] Guo,T., Brocks,W. & Veith,H.: submitted for publication in 10th ECF, 1994

[17] Guo,T.: Report of institut for mechanics in Ruhr-university Bochum, No 85, 1993

[18] Guo,T. & Brocks,W.: Proceeding of 8th ICF in Kiev, 1993, in press