ON THE YIELD ZONE AT THE TIP OF INTERFACE CRACK WITH CONTACT SURFACES

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Approximate plastic zone shape at the interface crack tip is evaluated by equating Mises equivalent stress with the yield strength of materials. An exact and asymptotic elastic solutions obtained under the "contact zone" model are used for equivalent stress determination.

INTRODUCTION

In composite materials interface fracture are common and usually determine the overall strength of materials. This kind of fracture is mainly initiated by the crack which occurs between two medias. Well-known results (see bibliography in the paper by Shih and Asaro (1)) clearing main aspects of the problem have been obtained for elastic materials. The classical oscillatory (Rice (2)) and "contact zone" (Comninou (3)) models was utilized for these investigations.

Determination of the plastic zone shape at the tip of interface crack were approximately obtained in

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numerical (1) and analytical (Sywicz and Parks (4)) manners. Classical model of interface crack was applied. But less attention was paid to the plastic zone determination under the "contact zone" model. The purpose of the work reported here was to examine this problem.

AN EXACT SOLUTION OF ELASTIC PROBLEM

We consider an interface crack $c \leqslant x \leqslant b,\ y=0$ between two dissimilar half-planes $y \geqslant 0$ (μ_1, $\varkappa_1$) and $y<0$ (μ_2, $\varkappa_2$). Remote ($x^2+y^2 \to \infty$) external load $\sigma_y=\sigma>0, \sigma_{x1}, \sigma_{x2}, \tau_{xy}=\tau$ satisfy the continuity conditions at the interface. We assume that the crack surfaces are traction free for $x \in [c,a]=L_1$ and are in frictionless contact for $x \in (a,b)=L_2$ $(a<b)$.

Using Muskhelishvili potential and combined Dirichlet-Riman boundary problem formulation leads to the following exact solution of the problem (see details in the paper by Loboda (5)):

$$\sigma_y - i\tau_{xy} = F(z) + \gamma F(\bar{z}) + (z-\bar{z})\overline{F'(z)} \qquad (1)$$

$$\sigma_x = \frac{4}{1+\gamma} Re\left[F(z)\right] - \sigma_y \qquad \text{for } Im\, z>0$$

where $F(z) = P(z)\,X_1(z) + Q(z)\,X_2(z)$

$$X_1(z) = i\, e^{i\varphi(z)} / \sqrt{(z-c)(z-b)}, \quad X_2(z) = e^{i\varphi(z)} / \sqrt{(z-c)(z-a)}$$

$$\varphi(z) = 2\bar{\varepsilon}\, \ln \frac{\sqrt{(b-a)(z-c)}}{\sqrt{(b-c)(z-a)} + \sqrt{(a-c)(z-b)}}, \quad \bar{\varepsilon} = \frac{1}{2\pi}\ln\gamma, \quad \gamma = \frac{\mu_1+\mu_2\varkappa_1}{\mu_2+\mu_1\varkappa_2}$$

and $P(z)=C_1 z + C_2$, $Q(z) = D_1 z + D_2$; real coefficients C_1, C_2, D_1, D_2 are determined by the remote load.

The main stress intensity factors

$K_2 = \lim\limits_{x \to b+0} \sqrt{2(x-b)}\ \tau_{xy}(x,0)$ can be found in the form

$$K_2 = -\sqrt{(b-c)/2}\ \sigma \left[\delta \cos\beta - \sin\beta - 2\bar{e}\sqrt{1-\lambda}\ (\cos\beta + \delta \sin\beta)\right]$$

where $\delta = \tau/\sigma$, $\lambda = (b-a)/(b-c)$, $\beta = 2e\ln\dfrac{\sqrt{b-a}}{\sqrt{b-c}+\sqrt{a-c}}$

To satisfy additional conditions $\sigma_y(x,0) \leqslant 0$ for $x \in L_2$ and $v^+(x,0) - v^-(x,0) \geqslant 0$ for $x \in L_1$ (signs + and - relate to the upper and lower half-planes) we obtain equation $f(\lambda,\bar{e},\delta)=0$ with respect to λ which for $\lambda_0 \leqslant 0.01$ gives the following asymptotic solution:

$$\lambda_0 \cong 2/\cosh\left[2 - \tan^{-1}(\delta^{-1})/\bar{e}\right].$$

Assuming in the formulas (1) $z-b = r e^{i\theta}$, the following asymptotic expressions at the singular points $z = b + i0$ for $y>0$ was found:

$$(1+\gamma)(\sigma_y - i\tau_{xy})\Big|_{z\to b} = \frac{K_2}{\sqrt{2r}}\Big\{\Big[(\gamma-1)\sin\frac{\theta}{2} + \sin\theta\cos\frac{3\theta}{2}\Big] - i\Big[(1+\gamma)\cos\frac{\theta}{2} - \sin\theta\sin\frac{3\theta}{2}\Big] \qquad (2)$$

$$(1+\gamma)\,\sigma_x\Big|_{z\to b} = -\frac{K_2}{\sqrt{2r}}\Big[(m+3)\sin\frac{\theta}{2} + \sin\theta\cos\frac{3\theta}{2}\Big].$$

PLASTIC ZONE SHAPE DETERMINATION

For isotropic elastic solids, the Mises equivalent stress is

$$\sigma_i^2 = p(\sigma_x^2 + \sigma_y^2) + q\,\sigma_x\sigma_y + 3\tau_{xy}^2 \qquad (3)$$

with $p=1$, $q=-1$ for plane stress and $p = \nu^2 - \nu + 1$ $q = 2\nu^2 - 2\nu - 1$ for plane strain (ν is Poisson's ratio of the solid). Using formulas (1) and equating σ_i in (3) with the material yield strength σ_{ys} we obtain for $\operatorname{Im} z > 0$ the following equation

$$\sigma_i = \sigma_{ys} \qquad (4)$$

Taking into account that in (1) parameter Z near the right crack tip ($x=b$, $y=0$) can be expressed in the form $z = b + r e^{i\theta}$ relation (4) for each θ can be considered as equation with respect to r.

This equation was solved numerically for various parameters values and approximate forms of plastic zones were found. Alternatively, by using (2), (3), (4) an asymptotic formula for plastic zone form was found.

The results of $r_0(\theta) = r_p(\theta)/D$ determination which show the normalized size and form of plastic zone are shown in the figures 1-3. In this figures $r_p(\theta)$ is the distance of yield boundary from the crack tip; $D = 2\gamma(\beta - c)(1+4\bar{\varepsilon}^2)(\sigma^2+\tau^2)/[(\gamma+1)\sigma_{ys}]^2$.

The lower half-plane was assumed absolutely rigid and for upper half-plane plane strain state with $\varkappa_1 = 1.8$ was assumed. The solid and dashed lines relate to the exact and asymptotic solutions, respectively (asymptotic results for figures 2 and 3 are the same as for 1). Also shown, denoted by dash-dot lines, are results of reference (1) which were obtained with finite elements (FE) calculations for strain-hadening (Ramberg-Osgood n=3) materials and classical oscillatory model of the crack.

It is appropriately to note that the plastic zones related to the exact solution for the same values of σ/σ_{ys} and $\tau = 2\sigma$ are very near to the asymptotic plastic zone form shown in figure 1.

CONCLUSIONS

Plastic zone shape for interface crack under "contact zone" model was found. Exect elastic and asymptotic solutions at the crack tip was used. Obtained results were compared with FE calculations for strain-hardening materials and classical model of the crack.

SYMBOLS USED

$\mu_1, \varkappa_1, \mu_2, \varkappa_2$ = elastic constants of the materials

$\sigma_x, \sigma_y, \tau_{xy}, \sigma_i$ = components of stress tensor and equivalent stress

σ , τ = remote load

λ = relative contact zone length

$r_0(\theta)$ = normalized distance of yield boundary from crack tip

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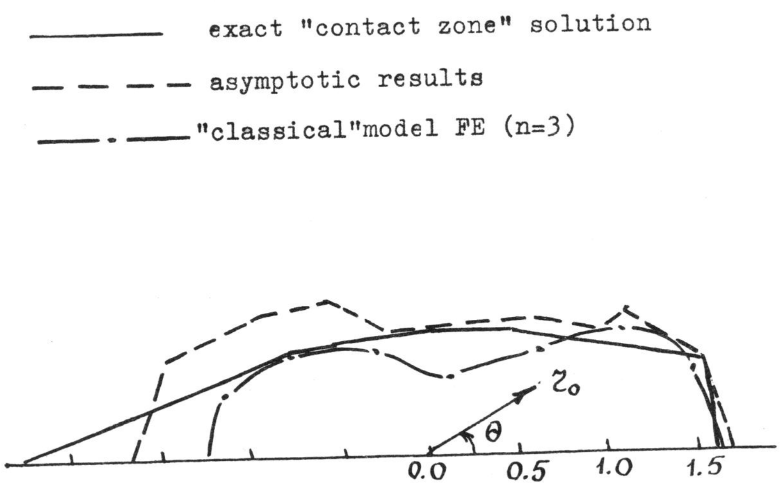
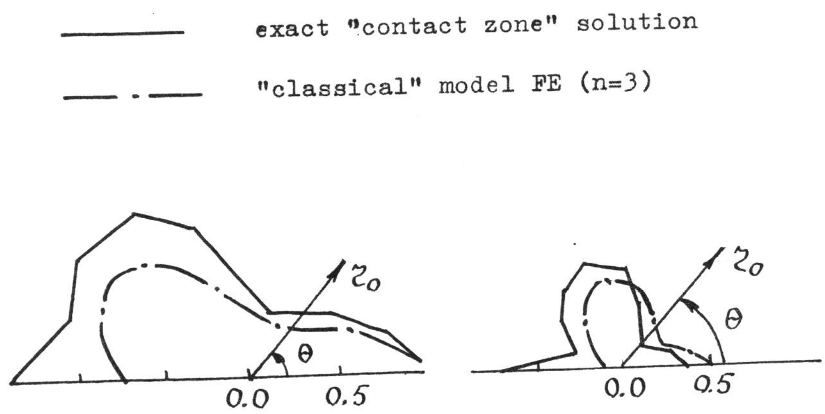


Figure 1 Plastic zone shapes for $\sigma/\sigma_{ys}=2\cdot 10^{-4}$



Figures 2,3 Plastic zone shapes for $\sigma/\sigma_{ys}=6\cdot 10^{-3}$ (left), $\sigma/\sigma_{ys}=0.2$