GENERALIZED PLASTIC STRIP MODEL FOR MODE I CRACKS

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The paper suggests an approach to plane elasto-plastic crack problems by using plastic strip model. Plastic zone is modelled by three inclined slip strips at the tip of a crack. It is assumed that the plate material is perfectly elasto-plastic and on the slip lines the yield criterion is obeyed. Thus, the problem of the development of plastic strips in a cracked plate is reduced to the boundary value problems of the elasticity theory for a region with branched cut. The singular integral equation method is used to solve this problem.

INTRODUCTION

The generalized plane stress state is realized when a thin plate is deformed by forces acting in its plane. Experimental investigations of plastic deformations of thin plates with opening mode cracks indicate that at first, when loading level is low, narrow plastic zones appear on the crack prolongation. When the loading is increased at the crack tip the secondary system of the very thin symmetric slip strips suddenly appears in the planes inclined at an angle of about 50^0 to the crack line. The primary and secondary plastic strips have different physical interpretation. The primary plastic strips appear on the plane of the maximal tensile stresses. In this case the slip occurs in the planes of maximal shear stresses, the planes are inclined at an angle of 45^0 to the plate surface. The secondary slip strips appear on the planes perpendicular to the plate

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surface with maximal shear stresses acting on the planes. Plastic strips will be modelled by means of the lines of displacements discontinuity; on the lines corresponding to the secondary slip strips only the jumps of the shear displacements are different from zero, whereas the primary plastic strips will simulate only the normal displacements jumps. Then the problem on the plastic strips development in a thin cracked plate is reduced to solving the elasticity theory problem for a branched cut as it is proposed by Panasyuk and Savruk (1), Panasyuk et al (2).

Integral equations of the problem. For the first time the plastic strips model was used by Leonov and Panasyuk (3), Dugdale (4), Leonov et al (5) papers for solving the problem of the elastic-plastic equilibrium of an infinite perfectly elastic-plastic plate with a straight crack under applied at infinity tensile stresses p directed perpendicularly to the crack line. In this case narrow plastic strips on the crack prolongation were substituted by conventional cuts with tensile stresses being equal to the yield stress σ_Y on their faces.

It can be shown that this model satisfies the Treska yield criterion only when external forces are small enough. Let the crack faces be stress-free, and at infinity the plate be tensiled by the stresses p and q acting perpendicular and parallel to the crack line respectively. The maximal shear stress in the plane is determined by formula

$$\tau_{max}=|\sigma_Y\sqrt{l^2-l_0^2}\, z(z-\bar{z})/(\pi\sqrt{z^2-l^2}(z^2-l_0^2))+1/2(p-q)|.$$

where $2l_0$ is the crack length, $l-l_0$ is the plastic strip length.

The curve on Fig.1. shows the relationship between quantities of the stresses p and q when τ_{max} reaches the yield stress in shear $\tau_Y=\sigma_Y/2$ and plastic deformation should appear. It appears at a short distance from the crack tip. The increase of the external loading leads to the extension of the plastic zone. Once this zone reaches the crack tip, the new slip strips can appear.

Let us assume that three plastic strips emanate from the crack tip. When the plate material is perfectly elastic-plastic and on the slip lines (contours L_k, $k=\overline{1,6}$) (Fig.2.) the Treska yield criterion is obeyed the following boundary conditions take place:

$$
\begin{aligned}
&N_0^{\pm}+iT_0^{\pm}=0, \quad t\in L_0; \\
&N_n^{\pm}+iT_n^{\pm}=\sigma_Y=2\tau_Y, \quad t\in L_n \ (n=2,5); \qquad (1) \\
&N_n^{+}=N_n^{-}, \ v_n^{+}=v_n^{-}, \ T_n^{\pm}=-\tau_Y, \quad t\in L_n \ (n=1,4); \\
&N_n^{+}=N_n^{-}, \ v_n^{+}=v_n^{-}, \ T_n^{\pm}=\tau_Y, \quad t\in L_n \ (n=3,6).
\end{aligned}
$$

Here N_n and T_n are the normal and tangential components of stresses and v_n is the projection of the displacement vector on the 0_ny_n axis, these values being specified on contours L_n $(n=\overline{0,6})$ related to the local coordinates system $x_n0_ny_n$. The 0_nx_n axes $(n=\overline{1,6})$ are directed along the contours L_n forming angles α_n with the 0_0x_0 axes, points 0_n $(n=\overline{1,6})$ being located at the centres of contours L_n. The values with indices + or - refer to the left or right faces of the cut, $t_n=x_n+iy_n\in L_n$.

The boundary value problem (1) of the plane theory of elasticity for an infinite plate with a branched crack is reduced to solving the system of singular integral equations

$$
\frac{1}{\pi}\begin{Bmatrix}\mathrm{Re}\\ \mathrm{Im}\end{Bmatrix}\Bigg\{\int_{-l_0}^{l_0}\Big[K_{n0}(t_0,x_n)+L_{n0}(t_0,x_n)\Big]g_0'(t_0)dt_0+
$$

$$
+\int_{-l_1}^{l_1} i\Big[K_{n1}(t_1,x_n)-L_{n1}(t_1,x_n)-K_{n3}(t_3,x_n)+L_{n3}(t_3,x_n)+
$$

$$
+K_{n4}(t_4,x_n)-L_{n4}(t_4,x_n)-K_{n6}(t_6,x_n)+L_{n6}(t_6,x_n)\Big]g_1'(t_1)dt_1+
$$

$$+\int_{-l_2}^{l_2}\Big[K_{n2}(t_2,x_n)+L_{n2}(t_2,x_n)+$$

$$K_{n5}(t_5,x_n)+L_{n5}(t_5,x_n)\Big]g_2'(t_2)dt_2\Big\} = \tag{2}$$

$$=\begin{bmatrix}-p,\ n=0;\ 2\tau_Y-p,\ n=2\\ -\tau_Y+(p-q)\sin\alpha\,\cos\alpha,\ n=1\end{bmatrix},\quad x_n\in L_n$$

for determination the unknown functions $g_k'(t_k)$ $(k=\overline{0,2})$. Here

$$K_{nk}(t_k,x_n) = \frac{e^{i\alpha_k}}{2}\left[\frac{1}{T_k-X_n} + \frac{e^{-2i\alpha_n}}{\overline{T}_k-\overline{X}_n}\right],$$

$$L_{nk}(t_k,x_n) = \frac{e^{-i\alpha_k}}{2}\left[\frac{1}{\overline{T}_k-\overline{X}_n} - \frac{T_k-X_n}{(\overline{T}_k-\overline{X}_n)^2}\,e^{-2i\alpha_n}\right],$$

$$T_k=t_ke^{i\alpha_k}+z_k^0,\quad X_n=x_ne^{i\alpha_n}+z_n^0,$$

$$z_k^0=l_0+l_k\exp(i\alpha_k)\ (k=\overline{1,3}),\quad z_k^0=-l_0+l_k\exp(i\alpha_k)\ (k=\overline{4,6}),$$

$$\alpha_1=\alpha,\ \alpha_2=0,\ \alpha_3=-\alpha,\ \alpha_4=\alpha-\pi,\ \alpha_5=-\pi,\ \alpha_6=\pi-\alpha.$$

The unknown functions $g_k(t_k)$ are expressed in terms of the jumps of the displacement vectors on the contours L_k

$$g_k(t_k)=(E/4)(u_k^+-u_k^-),\ k=0,2,\quad g_1(t_1)=(E/4)(v_1^+-v_1^-),$$

where u_n is the projection of the displacement vector on the O_nx_n axis.

To define the plastic strips lengths $2l_1$, $2l_2$ we have two conditions

$$g_1'(l_1)=0,\quad g_2'(l_2)=0, \tag{3}$$

orientation angle α will be obtained by considering that slip strips L_1, L_3, L_4, L_6 are oriented so that their length l_1 takes the maximal value: $l_1(\alpha)\to\max$.

Numerical results. The numerical solution of the system of integral equations (2) with the specified parameters

l_1, l_2, α was found by the mechanical quadratures method.

The numerical results show, that if new slip strips are small they are inclined at an angle of about 60^0 to the crack line. The increase of the tensile stresses leads to the decrease of this angle, and its magnitude is about 45^0 when $(p-q)/\sigma_Y \to 1$. The influence of the lateral slip strips development and the loading acting parallel to the crack direction on the crack opening displacement $\Delta = \pi E(v_0^+ - v_0^-)/(8 l_0 \sigma_Y)$ is shown in Fig.3 (solid lines). Dotted lines in this figure correspond to the solution obtained in the framework of the δ_c-model as it is described in reference (5).

SYMBOLS USED

E =Young's modulus
$2l_0$ =crack length
$2l_1$ =secondary plastic strip length
$2l_2$ =primary plastic strip length
x, y =Cartesian coordinates
z =$x+yi$
α =orientation angle of the secondary plastic strip
Δ =dimensionless crack opening displacement

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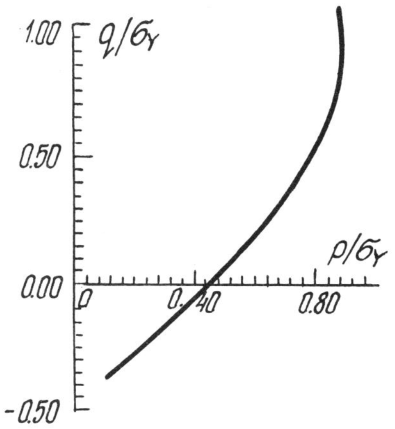

Figure 1. Relationship between parameters q/σ_Y and p/σ_Y wherein the secondary yielding appears in the vicinity of crack tip

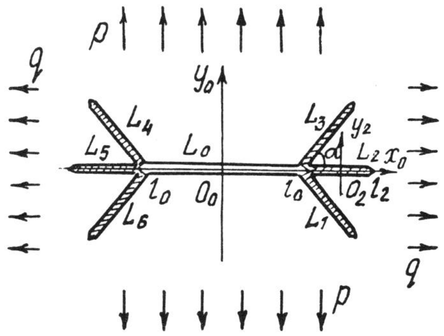

Figure 2. Straight crack with plastic strips in an infinite plate

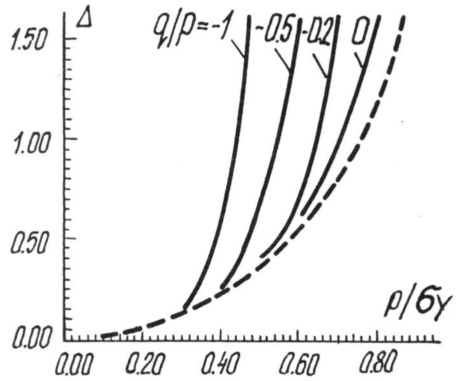

Figure 3. Crack opening displacement versus loading level p/σ_Y