THE FRACTAL GROWTH OF PORES IN SINTERED STEELS UNDER FATIGUE LOADING

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The growth of fatigue defects in sintered steels is described with the help of fractals. The idea of fractal evolution and fractal sieve space localization are presented. Theoretical models are compared with experimental observations for sintered steels. Results confirm fractal growth of defects.

INTRODUCTION

Generally any physical process in materials can be observed at all of three distinct length scales. Moreover each level relates to its own specific degrees of freedom. The whole samples, their deformations and hysteresis loops are observed at macroscopic scale. At the opposite limit we view the single atoms, particles and their motion. However there is also an intermediate range of magnitude related to defect structures in materials. Thus grain boundaries, dislocations, small cracks correspond to mesoscopic scale.

Under external loading, the input energy (measured in terms of hysteresis loop) flows down into deeper levels. Finally at microscale we obtain a heat outflow. However some part of incoming energy (cold work) becomes stored at mesoscopic level entailing the defects growth. The final fatigue damage of sample occurs when defect approaches macroscopic scale. The dynamic of the whole fatigue process comes from the specific mesoscopic degrees of freedom.

In real materials the growing defects structures are extremely complicated and we apply fractals to model them. Then fractal measures and dimensions

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play the role of mesoscopic degrees of freedom. The two different ways of evolution are generally possible:
- the process with fixed fractal dimension (we called it the *nonfractal evolution*),
- the process in which both fractal characteristics, measure and dimension, change (we will refer to it as *fractal growth*).

Defects maintain their initial structure growing under constant dimension condition. In contrast the fractal deformations of defects effects an initial structure. Even if the initial and final states coincide we pass through different states moving along the above two paths. Therefore the dynamics of growth can be completely different for fractal and nonfractal deformations of defects.

The microscopic observations of growing defects structures suggest fractal deformations. The computer simulations of simplified stochastic models of composites also confirm fractal behavior (Kozaczewski and Rybaczuk (1)). The fractal model of evolution gives reasonable results in LCF regime in metals (Rybaczuk (3)).

It is well-known and confirmed by enormous number of observations that the growing defects (mainly cracks) become well-localized in some area. On the other hand fractals don't distinguish any length scale. Therefore localization cannot be understood in terms of single fractal with given, constant dimension. In contrast fractal deformations in above sense correspond to some characteristic fractal measure. Then the space localization runs according to a very general mechanism of *fractal sieve*. The verification of this mechanism in sintered steels is the main purpose of this paper.

Ordinary physical quantities have fixed physical dimension. Consequently fractal measures aren't physical quantities since they change their physical dimensions for labile fractal dimension. Such new case requires some new mathematical technique not presented here. The multiplicative derivative $\pi f(x)/\pi x$ is the one (among others) print of involved methods:

$$f : \mathbb{R} \longrightarrow \mathbb{R}_+, \qquad \frac{\pi\, f(x)}{\pi\, x} = \lim_{\epsilon \to 0} \left\{ \frac{f((1+\epsilon)x)}{f(x)} \right\}^{\frac{1}{\epsilon}}, \tag{1}$$

where $\mathbb{R}, \mathbb{R}_+$ denote the set of real, and real positive numbers suitably.

FRACTAL DEFECTS ENERGY AND SPATIAL LOCALIZATION

Let's denote by $\mathcal{E}$, ν_D the defect's energy and measure (D is the fractal dimension of the defect) suitably. Assuming linear dependence we write:

$$\mathcal{E} = a(D)\nu_D + \{other,\ neglected\ terms\}, \tag{2}$$

where $a(D)$ denotes some factor with physical dimension $(energy) \times (length)^{-D}$. In such way we generalize the usual expressions for dislocation's core energy or surface energy. The both quantities, energy $\mathcal{E}$ and fractal measure ν_D are

extensive physical quantities. The only one intrinsic quantity $a(D)$ describes all properties specific for given material. Moreover it is widely known that any smooth deformation of defect may change its energy maintaining fractal dimension. All such effects as well as interactions between defects are included into other terms in simplified expression (2) and neglected here.

Inserting the relative shifts of fractal variables we obtain from (2)

$$\mathcal{E}' = (1+\epsilon_\mathcal{E})\mathcal{E}, \qquad \nu_D' = (1+D\epsilon_\nu)\nu_D, \qquad D' = (1+\epsilon_D)D,$$

$$\epsilon_\mathcal{E} = D\epsilon_\nu + \epsilon_D \ln\left(\frac{\nu_D}{\mathcal{A}(D)}\right), \qquad \mathcal{A}(D) = \left(\frac{\pi\, a(D)}{\pi\, D}\right)^{-1}. \tag{3}$$

The *characteristic measures* $\mathcal{A}(D)$ define the position of mesoscopic level in any material. The increase of fractal dimension becomes analogical to the irreversible, Joule–Thomson process in gases (Rybaczuk (2)). The inversion curve is replaced by system of characteristic fractal measures. Moreover there is also some very general localization mechanism of defects related to $\mathcal{A}(D)$.

Mathematics says that any ball with finite, nonzero radius may include even infinite fractal with lower fractal dimension. However that is not possible in real physical systems because of discrete structure at microscopic, atomic scale. The fractal measure of maximal defect (with fractal dimension D_0) included into ball with radius ρ_0 is proportional to suitable power $\rho_0^{D_0}$. Let's write the characteristic measure $\mathcal{A}(D_0)$ in the same way, i.e., $\mathcal{A}(D_0) = l_0^{D_0}$. We consider the evolution of an isolated defect (i.e., $\epsilon_\mathcal{E} = 0$) growing with respect to measure and dimension solely. The evolution equation (3) gives

$$\rho' = \rho_0\left(1 + \frac{\Delta D}{D_0}\ln\frac{l_0}{\rho_0}\right) \tag{4}$$

where ρ' and D' ($\Delta D = D' - D_0$) denote the suitable ball radius and fractal dimension after small transformation of defect. Assuming $\Delta D > 0$ (an increase of fractal dimension) and $\rho_0 > l_0$ (large defect) we obtain $\rho' < \rho_0$. That means that defect becomes localized in smaller ball. The infinite shift of dimension ($\Delta D = \infty$) gives $\rho' = l_0$ (i.e., full possible localization). In some way the characteristic curve $\mathcal{A}(D)$ "attracts" defects.

In general the characteristic length l_0 can be different for labile dimension D. The effect described by the equation (4) resembles the *fractal sieve*. Only close packed defects with fractal measures equal $\mathcal{A}(D)$ pass through sieve without any changes in spatial localization ($\rho' = \rho_0$). Note however that the new defect with higher dimension D' arises from the whole preceding structure.

THE FRACTAL GROWTH OF DEFECTS IN SINTERED STEELS

The defects structure in such materials consists of pores involved by powder metallurgy technology. The direct fatigue experiments are difficult because

of brittlness of these materials. Under external loading the existing pores link themselves with small, thin bridges, but the input energy (measured in terms of hysteresis loop) is zero up to the accuracy of conventional methods. In turn the pores do not change during fatigue process. The only effect consists of growing bridges. The total fractal measure of pore surface and their volume depends on the magnitude of given sample. Therefore it becomes essential in what way the defects fractal dimension changes under external loading. According to (4), the fractal sieve mechanism can follow only from increasing dimension.

Figure 2 presents an observed structure of defects in the cross–section of a sample. All microscopic observations are done for the same sintered alloyed KA steel but the samples were prepared at different stages of fatigue process. The fatigue rupture results from the aggregation of small voids and microcracks (Dudziński et all (4)).

At first we seek for the growing structure of pores neglecting other defects not related to fatigue process. The linear size of observed pores varies from $5\mu m$ up to $25\mu m$. Moreover the distance between pores is of the same order as pores linear size. The volume of pores constitutes few percent but their surface can be enormous. The specific chemical and mechanical methods were applied to samples in course of preparation for direct microscopic observations.

The computer picture analysis system determines the contour of any observed pore as the one shown in the Figure 3. We divide the contour into small pieces. Next the suitable program estimates the box–counting dimension for the whole contour and independently for each fragment to verify how stable are numerical calculations. (Estimation of fractal dimensions are based on extrapolations of some functions.) Such procedure is repeated for many pores and bridges in samples at different stages of fatigue process. Finally we determine the mean fractal dimension for both structures.

The above research program has been presented in Figure 1. The box 8 and partially 9 contains the already formed bridge. For comparison we have also evaluated fractal dimension of other structural defects (the box 22) not related to pores and fatigue process. The last black box depicts the mean fractal dimension for the whole cross–section of the sample.

The pore surfaces have dimensions $\simeq 2.2$ whereas for growing bridges we obtain higher values $2.3 \div 2.35$ $(=1+(1.3 \div 1.35))$. Moreover the growth process has *fractal character*. The small unconnected defects appear close to bridges (see the boxes 1, 3, 4 and 8). At next stages, when bridges link together the fractal dimension remains approximately constant.

The just borne bridges have magnitude of the same order as distances between pores. Therefore the mean distance between pores defines the *mesoscopic level* in sintered steel. According to observation the linear size related to mesoscopic scale lies between $5\mu m$ and $25\mu m$.

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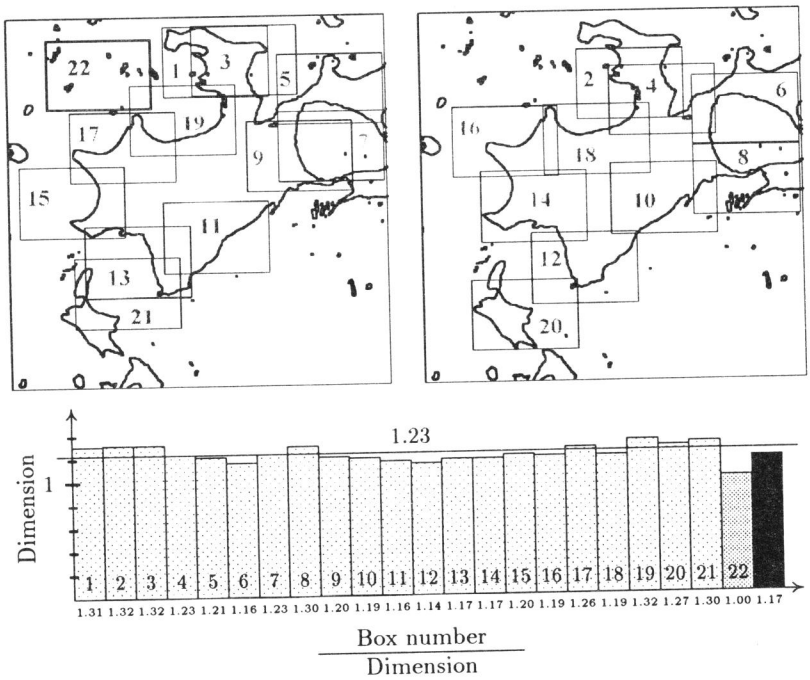

Figure 1 The local estimation of fractal dimension along pore contour

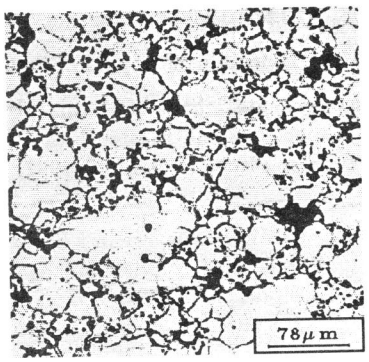

Figure 2 The observed structure of pores in sintered steel KA

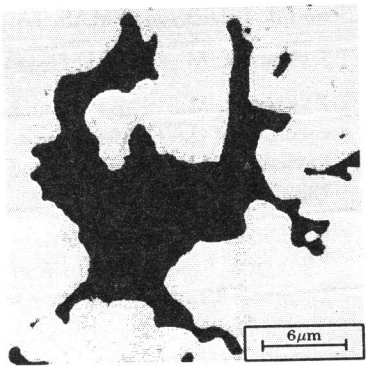

Figure 3 The extracted single pore