SIMPLIFIED PROCESS ZONE FRACTURE CRITERIA AT CRACK TIPS

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Simplified process zone fracture criteria at crack tips under mixed-mode loading are proposed for engineering usage which cover the whole range of possible combinations of K_I and K_{II} in mixed-mode loading. Comparison with the well known maximum tangential stress criterion according to Erdogan and Sih shows that the latter might be non-conservative in the compressive K_I-range.

INTRODUCTION

Despite substantial progress in defining realistic fracture criteria based on microstructural phenomena at the crack tip, the conventional criteria based on parameters from linear-elastic fracture mechanics continue to be used by engineers. Especially, design and qualification codes refer to allowable equivalent stress intensity factors in cases of mixed mode crack or slit tip loading, or recommend even rougher approaches such as projecting the inclined crack into a plane normal to the first principal stress.

OBJECTION TO THE ERDOGAN–SIH CRITERION

One of several fracture criteria at crack tips used by engineers is the Erdogan-Sih hypothesis (1) which states that the crack will propagate in the direction of maximum tangential stress $\sigma_{t\max}$ at a critical value of that stress

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at some small, material-dependent distance from the crack tip. It has been shown by several authors that this hypothesis gives nearly the same critical loads as the hypothesis of minimum strain energy density S_{min} proposed by Sih (2).

Despite its applicability and plausibility an objection to the Erdogan-Sih criterion may be raised in loading cases which are not predominated by tensile K_I-loading. Pure shear K_{II}-loading, for example, results in maximum radial stresses σ_{rmax} which are two times the maximum tangential stresses σ_{tmax}. Therefore, the crack is supposed to propagate perpendicular to σ_{rmax} at a critical value of this stress in the loading case considered. Thus, the critical load is heavily reduced in relation to the σ_{tmax}-criterion. The Erdogan-Sih criterion might be non-conservative not only in cases of predominant K_{II}-loading but also in all loading cases with predominant compressive K_I-loading.

ORIGINAL PROCESS ZONE CRITERIA

The objection to the Erdogan-Sih criterion mentioned above induced the authors to propose a criterion which can be applied more generally with a higher degree of conservatism. This "process zone criterion" (3) distinguishes between the crack-initiating process zone near the crack tip and the fracture criterion applied within this zone. Two versions of this criterion are available, one for brittle initiation of fracture (i. e. low-stress brittle fracture initiation) and the other one for "ductile" initiation of fracture (i. e. fatigue crack initiation):

- Brittle initiation of fracture occurs in a process zone in the direction of maximum dilatational strain energy density at a critical value of the first principal stress there (σ_{1b}-criterion). The initiated crack propagates perpendicularly to this stress.
- "Ductile" initiation of fracture occurs in a process zone in the direction of maximum distortional strain energy density at a critical value of the principal shear stress there (τ_{1d}-criterion). The initiated crack propagates perpendicularly to the first principal stress.

The problem with these criteria is that the numerical procedures are complicated and the results at least partially multivalued. The brittle initiation of fracture criterion mentioned above has originally been proposed by Ritchie, Knott and Rice (4).

SIMPLIFIED PROCESS ZONE CRITERIA

The numerical problem with the process zone criteria may be removed by introducing the same type of stress to determine the position of the process zone and to define the critical condition within this zone, Fig. 1:

- Brittle initiation of fracture occurs in a process zone in the direction of maximum first principal stress at a critical value of this stress (σ_{1b}^*-criterion). The initiated crack propagates perpendicularly to this stress.
- "Ductile" initiation of fracture occurs in a process zone in the direction of maximum principal shear stress at a critical value of this stress (τ_{1d}^*-criterion). The initiated crack propagates perpendicularly to the first principal stress.

Crack closure effects are neglected both in the original and in the simplified criteria.

COMPARISON OF ORIGINAL AND SIMPLIFIED CRITERIA

The simplified criteria (σ_{1b}^* and τ_{1d}^*) are compared with the original criteria (σ_{1b} and τ_{1d}) in respect of critical stress intensities and in respect of process zone and cracking angles. The calculations were performed based on the derivations by Radaj and Zhang (3).

Comparing original and simplified criteria in the case of brittle fracture, Figs. 2 and 3, shows that stress intensity limit values and cracking angles are sufficiently close together in the tensile range and identical in the compressive range.

Comparing original and simplified criteria in the case of "ductile" fracture, Figs. 4 and 5, shows that stress intensity limit values and cracking angles are identical within line width in the tensile range. The stress intensity limit values deviate in the higher compressive range with conservative values from the simplified version. The process zone and cracking angles are completely identical in the compressive range. The angles from the tensile range may be transferred to the compressive range with changed signs designating a fictitious crack initiated perpendicular to the second principal stress which is larger in absolute value than the first principal stress in the compressive range. Actually no crack initiation will take place as long as both stress components are compressive. The original version of the criterion takes this into account.

COMPARISON OF CONVENTIONAL AND NEW CRITERIA

The stress intensity limit values according to the different conventional and new criteria are compared in Fig. 6. Not all the criteria are defined in the compressive range. The lettering of the curves of the conventional criteria reads as follows:

- σ_{tmax}, maximum tangential stress criterion according to Erdogan and Sih;
- S_{min}, minimum total strain energy density criterion according to Sih;
- S_V^*, maximum dilatational strain energy density criterion according to Radaj and Heib;
- G_{res}, resultant energy release rate criterion according to Irwin;
- G_{max}, maximum energy release rate criterion according to Hussain, Pu and Underwood.

There are major differences between the conventional criteria and the simplified process zone criteria which result from the different formal and physical contents of the criteria. The process zone criteria are more conservative than the conventional criteria. Whether or not a conventional criterion with higher limit values is applicable in definite cases should be decided on the basis of test results. Crack closure effects may justify higher values in the compressive range. Such test results are not readily available from the literature.

CONCLUSION

Theoretical considerations indicate that the Erdogan-Sih criterion might be non-conservative in the compressive K_I-range. Experiments are desirable to prove this statement.

REFERENCES

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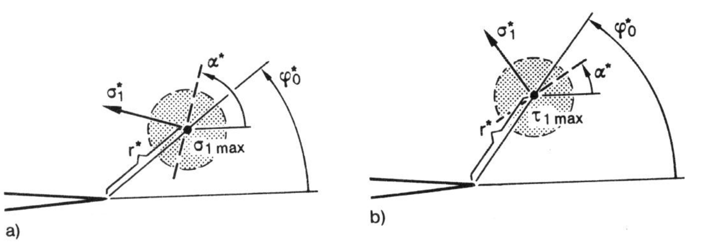

Figure 1: Fracture initiation in a process zone at distance r* from the crack tip in the direction (angle φ_0^*) of maximum first principal stress σ_{1max}^* (a) (brittle initiation) or maximum principal shear stress τ_{1max}^* (b) ("ductile" initiation), cracking perpendicularly to σ_1^* (angle α^*)

See next page for Figures 2 to 5

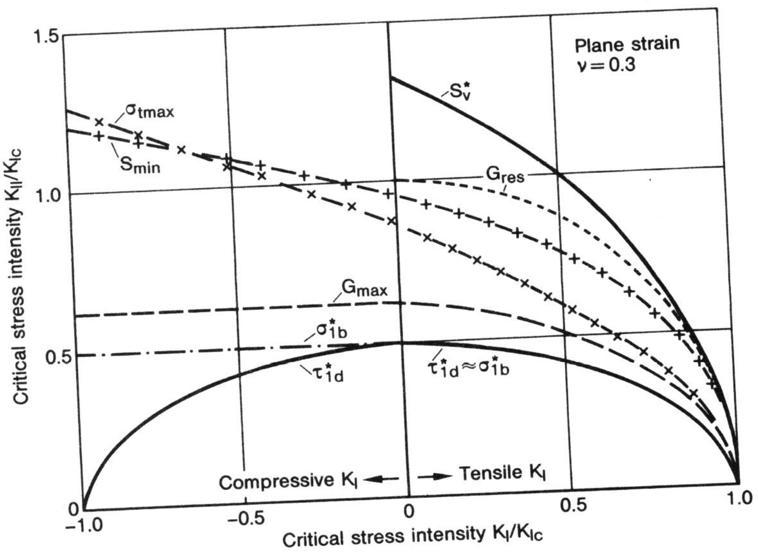

Figure 6: Critical stress intensities for crack propagation according to conventional and newly proposed criteria

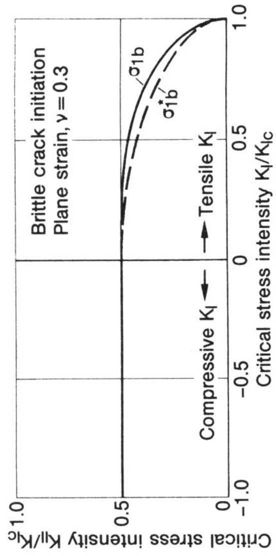

Figure 2: Critical stress intensities for brittle initiation of fracture

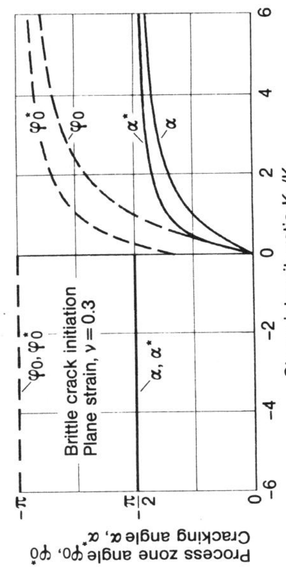

Figure 3: Cracking angles for brittle initiation of fracture

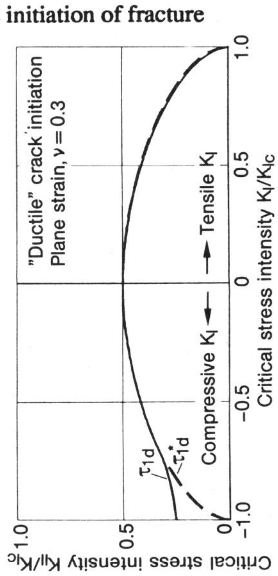

Figure 4: Critical stress intensities for "ductile" initiation of fracture

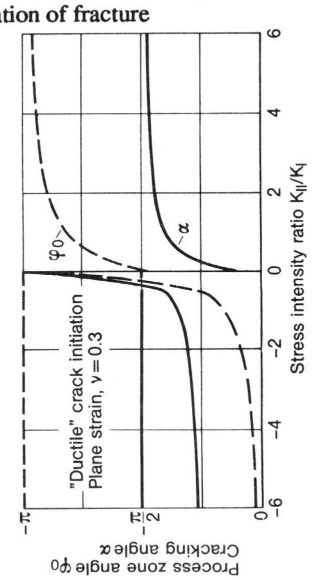

Figure 5: Cracking angles for "ductile" initiation of fracture