MATHEMATICAL MODEL OF CRACK FORMATION IN BRITTLE MATERIALS

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A mathematical model of crack growth which is based on analysis of physical mechanisms of fracture is presented. The crack growth is modeled as a result of formation of microcracks in front of crack tip and joining of these microcracks to the crack. On a basis of a model of crack growth as a fractal cluster formation, fractal dimension of formed crack is determined. Relationship between crack velocity and size is deduced.

INTRODUCTION

To design new high-strength materials , one need mathematical models of fracture which allow s for physical mechanisms of crack growth and the crack structure.

The task of development of integrated macro- and microtheory of fracture has been set by T.Yokobori (1). Approaches of theory of fractals came into wide use recently in modelling fracture.This approach shows considerable promise for development of general theory of fracture.

The paper seeks to develop a mathematical model of crack growth which allows for physical (dislocation) mechanisms of crack growth , and structure of crack.

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MECHANISM OF CRACK GROWTH

The crack growth proceeds as follows (Finkel (2),Panassyuk et al (3)). Under loading, one or more microcracks are formed in front of crack tip. Then, the microcrack adds to the crack. That leads to crack growth. The formation of microcracks proceeds most commonly through the mechanisms of accumulation of dislocations under load or locked shearing (i.e. accumulation of dislocations in the vicinity of inclusions or other barriers). So one can suppose that the crack grows provided at least one pileup of dislocations in front of crack tip is available, and this pileup can turn into microcrack under load. So, the crack growth is discrete process, which is caused by the interaction between the crack and pileups of dislocations. An angle between the plane in which the crack lies and the direction of crack propagation by the mechanism outlined above is a random value. The interval between two consecutive increments of crack size depends on random distribution of dislocations and their pileups throughout the material, and is random value as well. So, the process of crack growth is a random process.

Taking into account the mechanism of formation and growth of cracks described above, as well as random character of this process one can consider cracks as fractal clusters which are formed as the result of joining small microcracks.

EQUATION FOR CRACK GROWTH

Assume that a crack which has been formed as the result of successive joining N microcracks with size a. One can write

$$dN / dt \propto q, \tag{1}$$

where t - time. Determine the value q which is supposed to be equal to a probability of formation of microcrack in front of crack tip. If there is a pileup of n dislocations some distance away from the crack tip, microcrack is formed provided that the stress in this point is equal to

$$\sigma_{cr} = k_o \gamma / nb, \tag{2}$$

where n - amount of dislocations in pileup, b - Burgers vector (Finkel (2)). The stress distribution in front of crack tip in elastic material is described as follows (3)

$$\sigma_c = K_I \, f(\theta) / \sqrt{2 \pi r}, \tag{3}$$

where K_I - stress intensity factor, $K_I = \sigma \sqrt{\pi l}$, l - linear size of crack, r - distance between a point and crack tip , θ - the angle between the line r and the plane in which the crack lies , f (θ) - given function of θ (3). Suppose that the pileups from n dislocations are randomly distributed throughout the material. Then the distance between the crack tip and nearest pileup can be considered as being distributed by Poisson's law with a parameter $\lambda = (\rho / n)^{1/3}$, where ρ - density of dislocations in the material. So the probability of microcrack formation in front of crack tip is equal to a probability that the stress in a point at a distance r from the crack tip exceeds σ_{cr}.

A linear size (radius) of fractal cluster (in this case, crack) and theü number of microcracks from which it has been formed are related by the following formula (Smirnov (4)):

$$N = (1 / a)^{D} \tag{4}$$

With eqs.(1) -(4) , we obtain a relationship between crack velocity and crack size

$$dl/dt = (a^{D} / D)\, l^{1-D} [1 - \exp (-\lambda_1 \sqrt{l})], \tag{5}$$

where $\lambda_1 = \lambda \sigma f(\theta) / \sigma_{cr}$. When the crack is relatively small, this equation, with the approximate formula $1 - \exp(-x) \approx x$, reduces to the following form

$$dl/dt = (\lambda_1 a^{D} / D)\, l^{1.5 - D} \tag{6}$$

When the crack is large , we have

$$dl/dt = (a^{D} / D)\, l^{1 - D} \tag{7}$$

From eqs.(5) and (7) follows that when the crack becomes rather large (i.e. when $\exp (-\lambda_1 \sqrt{l}) \rightarrow 0$) , the crack velocity ceases to depend on the load.

FRACTAL DIMENSION OF CRACK

To determine the fractal dimension of crack one can use approach developed by Turkevich and Scher (5). These authors have shown that the fractal dimension of cluster is determined by features of process of cluster growth.

One can write an equation for cluster growth as follows (5)

$$dl / dN = p(N)\, a \tag{8}$$

To determine the function p (N) consider following alternatives: the crack can propagate rectilinearly (in this case , $p(N) = 1$),and can branch, kink, propagate curvilinearly, etc. Just the branching, kinking , curvilinear propagation of crack determine the fractal dimension of crack (6).

The mechanism of branching and curvilinear propagation of crack has been decribed by Panassyuk et al. (3) as follows. Crack grows as a result of formation of microcracks in front of its tip and joining of the microcracks to the crack. The greater the stress concentration around the crack tip , the greater amount of microcracks which are formed in front of the crack. So , the probability of formation of microcracks in points which are not lie in the plane of crack increases with increasing stress concentration around the crack tip. As a consequence the probability of curvilinear propagation of crack or its branching increases as well. Thus , one can write

$$p(N) \propto 1 / M \qquad (9)$$

The value $1/M$ can be considered as the ratio between the amount of points of possible microcrack formation which ensure rectilinear crack propagation and total amount of these points in front of crack.

It is evident that the greater the probability of microcrack formation q , the greater their amount M : $M \propto q$. As discussed earlier the relationship between q and the crack size looks as follows: $q \propto l^{m}$, where $m = 1.5 - D$, when the crack is small, and $m = 1 - D$, when the crack is relatively large. Integrating eq.(8), with eq.(9), we obtain

$$N \propto l^{m+1} \qquad (10)$$

Confronting eq.(10) with (4) we have : $D = 1.25$ when the crack is small, and $D = 1.0$ when the crack is large. It follows herefrom that when the velocity and size of crack are small , the roughness of crack surface is much more than at large velocity and size of crack.

It is of interest to compare the obtained values of fractal dimension of crack with the data of Mandelbrot et al (7) which have determined D for cracks in martensitic steels , and derived $D = 1.26$.

From eqs.(6) and (7) follows that when the crack is small the crack velocity increases with crack size. When the crack is rather large the opposite situation can be seen : the more is the crack , the smaller the crack velocity

SYMBOLS USED

a = average distance between the crack tip and a point in which the microcrack is formed

D = fractal dimension of crack

k_o = coefficient of the mechanism of microcrack initiation which is equal to 2 or to 12 when the microcrack is formed by the mechanism of accumulation of dislocations, or by the mechanism of locked shearing, correspondingly

q = frequency of unit increments of the crack by joining with microcrack (or probability of unit increments of crack)

M = amount of microcracks which are formed in front of crack tip

p (N) = probability of increase in crack size when a microcrack joins to the crack

REFERENCES

(1) Yokobori,T., "An Intersdisciplinary Approach to Fracture and Strength of Solids", Wolters-Noordhoff Ltd, Groningen, 1968

(2) Finkel, V.M., "Physics of Fracture", Metallurgiya, Moscow,USSR, 1970.

(3) Panassyuk, V.V. et al. , "The Foundations of Fracture Mechanics of Materials ", Vol.1, Naukova Dumka, Kiev, Ukraine , 1988

(4) Smirnov, B.M., "Physics of Fractal Aggregates", Nauka, Moscow, Russia, 1991

(5) Turkevich , L. and Scher,H., "Fractals in Physics", Proc.6th Int.Symp. on Fractals in Physics. Edited by L.Pietronero and E.Tosatti, North-Holland, NY,1986

(6) Heping,X. Int.J.Fracture, Vol.41, No.4, 1989, pp.267 - 274

(7) Mandelbrot , B.B. et al. Nature, Vol.308, 1984, pp.721 - 722