ON THE STRESS STATE OF TWO-COMPONENT CURVILINEAR RING WITH CRACKS

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The paper deals with plane elasticity problem for two-component curvilinear ring with internal and edge cracks considered as a finite plate with cracks containing the inclusion of arbitrary shape with a hole and cracks (internal and edge). The derived system of singular integral equations was solved numerically in the case of two-component solid curvilinear ring with square hole and two-component circular ring with edge crack.

INTRODUCTION

Modern engineering structures are usually composed of the parts in the form of piece-wise homogeneous finite plates (wide elastic patches in different plates, aircraft panels, etc. with access holes). There are often used also cylindrical bodies in the form of compound ring regions (pressure chambers, wire and rod drawing, compound pipes joined by press fit). Their serviceability depends to a great extent on a level of stress concentration along the contour of the hole - the most stressed region, where fatigue cracks may initiate.

Therefore, the elaboration of general methods of solving plane elasticity problems for piece-wise homogeneous bodies with cracks has become important, especially in predicting crack paths and estimating life of structures.

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The present study lies on the consideration of a finite plate of arbitrary shape containing the elastic inclusion with inside hole. The cracks are situated in the plate as well as in the inclusion. Basing on the solution of the derived system of singular integral equations, the proposed approach allows to affect stress concentration along the hole contour by choosing the geometry and material of inclusion and to define stress intensity factors in the tips of internal and edge cracks.

FORMULATION AND GOVERNING EQUATIONS

Consider an elastic piece-wise homogeneous body occupying the finite region S_0, bounded by closed contours L_0 (external boundary) and L_2 (internal one) and consisting of two ring regions: S_1 (elastic inclusion with a hole) and S (external holder) with their common boundary L_1 being the interface (Fig. 1).

Elastic parameters of inclusion are G_1, $\varkappa_1$, of external ring - G, $\varkappa$. Assume N_0-2 curvilinear cracks L_K ($K=\overline{3,N_0}$) exist inside the inclusion with $M-2$ first of them ($K=\overline{3,M}$) terminating at the hole contour; region S contains isolated defects L_K ($K=\overline{N_0+1,N}$).

Let xOy be a Cartesian coordinate system with the vertex situated in arbitrary point of the region S. Local coordinate systems $x_nO_ny_n$ ($n=\overline{0,N}$) are associated with contours L_n. Axis O_nx_n makes an angle α_n with the axis Ox, origins O_n are defined by complex coordinates z_n^0 ($z_0^0=0$, $\alpha_0=0$) in xOy.

Consider the boundary value problem of the plane elasticity when the stresses are prescribed on the boundaries of the region S_0 and crack edges:

$$N_n + iT_n = p_n(t_n) \,, \quad t_n \in L_n \,, \; n = 0,2; \tag{1}$$

$$\left[N_n + iT_n \right]^{\pm} = p_n(t_n) \pm q_n(t_n), \quad t_n \in L_n \,, \; n = \overline{3,N} \tag{2}$$

satisfying the equilibrium conditions. The conjugation conditions hold on the interface :

$$\left[N_1 + iT_1 \right]^{+} = \left[N_1 + iT_1 \right]^{-},$$
$$\frac{d}{dt_1}\left[(u_1+iv_1)^{+} - (u_1+iv_1)^{-} \right] = g_1^{*\prime}(t_1), \quad t_1 \in L_1 \,. \tag{3}$$

Here N_n and T_n denote the normal and tangential stress components; u_n, v_n are the displacement vector projections on the axes Ox_n, Oy_n respectively; index "+" ("-") stands for the limiting value of the function when tending to the contour on the left (right), inclusion contour tracing is performed anticlockwise.

Using the known representations of the stress complex potentials for homogeneous multiconnected regions with holes and cracks and for finite regions with cracks obtained by Savruk (1), complex potentials for the problem under consideration were constructed by Savruk and Kuznyak (2). Having satisfied the conditions (1) - (3) we arrive at the system of $N+1$ singular integral equations to determine $N+1$ unknown functions $g_K'(t_K)$ ($K=\overline{0,N}$)

$$A_n g_n'(t_n') + \frac{1}{\pi}\sum_{K=0}^{N} \int_{L_K} \left[K_{nK}(t_K,t_n')g_K'(t_K)dt_K + L_{nK}(t_K,t_n')\overline{g_K'(t_K)}\,\overline{dt_K}\right] +$$

$$+ R_n(t_n') = F_n(t_n') + \frac{1}{\pi}\sum_{K=3}^{N} \int_{L_K} \left[M_{nK}(t_K,t_n')q_K(t_K)dt_K + \right. \tag{4}$$

$$\left. + L_{nK}(t_K,t_n')\overline{q_K(t_K)}\,\overline{dt_K}\right], \quad t_n' \in L_n, \; n = \overline{0,N},$$

where

$$K_{nK}(t_K,t_n') = e^{i\alpha_K}\left(C_n Z_{nK} - B_n e^{-2i\alpha_n}\overline{Z}_{nK} \frac{\overline{dt_n'}}{dt_n'} \right)/2 ;$$

$$L_{nK}(t_K,t_n') = -B_n e^{-i\alpha_K}\overline{Z}_{nK}\left(1 - e^{-2i\alpha_n}\frac{\overline{Z}_{nK}}{Z_{nK}}\frac{\overline{dt_n'}}{dt_n'} \right)/2 ;$$

$$M_{nK}(t_K,t_n') = i\left[2(\varkappa_* + 1)^{-1}K_{nK}(t_K,t_n') + B_n e^{i(\alpha_K-2\alpha_n)}\overline{Z}_{nK}\frac{\overline{dt_n'}}{dt_n'}\right] ;$$

$$N_{nK}(t_K,t_n') = -2i(\varkappa_* + 1)^{-1}L_{nK}(t_K,t_n') ;$$

$$Z_{nK} = (T_K - T_n')^{-1}; \quad T_K = t_K e^{i\alpha_K} + z_K^o ; \; T_n' = t_n' e^{i\alpha_n} + z_n^o ;$$

$$F_n(t_n') = X_n(t_n') - C_n\Phi(t_n') + B_n\left[\!\left[\Phi(t_n') + e^{-2i\alpha_n}\frac{\overline{dt_n'}}{dt_n'}\left[T_n'\overline{\Phi'(t_n')}e^{-i\alpha_n} + \overline{\Psi(t_n')}\right]\right]\!\right] ;$$

$$X_n(t_n') = 2G_1\delta_n g_1^{*\prime}(t_1') + (1-\delta_n)p_n(t_n') \;;$$

$$\Phi_n(t_n') = -(X+iY)/W_n \;;\; W_n = 2\pi(1+\varkappa_1)(T_n'-\overset{o}{z}_2) \;;$$

$$\Psi_n(t_n') = \left[\varkappa_1(X-iY) - \overline{\overset{o}{z}}_2(X+iY)/(T_n'-\overset{o}{z}_2)\right]/W_n \;;$$

$$R_n(t_n') = \frac{B_n e^{-2i\alpha_n} M_2}{2\pi i(\overline{T}_n'-\overset{o}{z}_2)^2}\frac{d\overline{t}_n'}{dt_n'} + \frac{\Delta_n}{\pi}\left[a_n\frac{ds_n'}{dt_n'} - \frac{M_o e^{-2i\alpha_n}}{2i\overline{T}_n^{2\prime}}\frac{d\overline{t}_n'}{dt_n'}\right] \;;$$

$$M_o = i\sum_{K=0}^{M}\int_{L_K}\left[\frac{g_K'(t_K)e^{i\alpha_K}dt_K}{T_K} - \frac{\overline{g_K'(t_K)e^{-i\alpha_K}d\overline{t}_K}}{\overline{T}_K}\right] \;;$$

$$M_2 = i\sum_{K=2}^{M}\int_{L_K}\left[\overline{T}_K g_K'(t_K)e^{i\alpha_K}dt_K - T_K\overline{g_K'(t_K)e^{-i\alpha_K}d\overline{t}_K}\right] \;;$$

$$a_o = \int_{L_o} g_o'(t_o)dt_o \;;\quad a_j = \sum_{K=2}^{M}\int_{L_K} g_K'(t_K)dt_K \;,\; j = \overline{2,M} \;;$$

$$A_1 = i(1+\varkappa_1+\Gamma_1(1+\varkappa))/2;\; B_1 = 1-\Gamma_1;\; C_1 = \varkappa_1-\Gamma_1\varkappa;\; \Gamma_1 = \frac{G_1}{G} \;;$$

$$A_j = 0,\; B_j = -1,\; C_j = 1,\; j = 0,\; j = \overline{2,N};$$
$$\Delta_j = 1,\; j = 0,\; j = \overline{2,M} \;;\quad \Delta_j = 0,\; j = 1,\; j = \overline{M+1,N} \;;$$

X , Y are the projections on axes Ox and Oy of the main vector of stresses applied to the contour L_2 ; $\varkappa_* = \varkappa_1$, $K = \overline{3,N_o}$; $\varkappa_* = \varkappa$, $K = \overline{N_o+1,N}$.

When tracing the contours of isolated cracks, the following conditions should be held:

$$\int_{L_n} g_n'(t_n)dt_n = 0, \qquad n = \overline{M+1,N} \tag{5}$$

thus providing the single-valued displacements.

NUMERICAL RESULTS

The system of singular integral equation (4) was solved numerically by means of developed in the reference (1) mechanical quadrature method for particular cases of practical interest.

Two-component solid curvilinear ring with square hole without cracks. Let inclusion (region S_1) with a square hole (contour L_2) reside in the centre of a region S_0 with external boundary L_0 (circular contour of radius R_0). Suppose that following Savin (3) contour L_2 is given in parametric representation:

$$t_2 = \omega_2(\theta) = \frac{3}{5} a(e^{i\theta} - \frac{1}{6} e^{-3i\theta}) , \quad t_2 \in L_2 , \ 0 \leq \theta < 2\pi . \qquad (6)$$

It is a matter of general experience that stress concentration is a maximum in the points of contour L_2 close to $\theta = (2m+1)\pi/4$, $m=\overline{0,3}$. The purpose of this study was to investigate the possibility of stress concentration decrease by changing the elastic constants of the rings materials, inclusion shape and its distance from the outer boundary. Particularly, the following forms of inclusion contour L_1 (being the interface) were examined: a quirked epicycloid; a set of curves, formed by successive approximations of the initial curve in the shape of epure of stresses; square contour of the form (6) with linear parameter b. The harnessing of the last one yields the best results. In this case a 30 % reduction in stress concentration at the above mentioned points was achieved at $G_1/G=0.5$; $\varkappa=\varkappa_1=2$; $(b-a)/a=0.4$; $R_0/a=10$. Similarly, optimal geometry and elastic constants of materials can be selected providing for near-constant stress concentration along the square hole.

Two-component circular ring with an edge crack. Consider a ring formed by concentric circumferences L_K ($K=\overline{0,2}$) of radius R_K with a straight edge crack L_3 of length $2l$ terminating at the internal boundary of the ring. Table 1 presents the calculated dimensionless stress intensity factors $Y_{Ip}=K_I/p\sqrt{2\pi l}$ and $Y_{I\delta}=K_I/(\delta/R_1)G\sqrt{2\pi l}$ for various values of dimensionless crack length $\lambda=2l/R_2$ in the case of pressure p acting on the hole contour L_2 and of interference fit δ of internal ring, respectively (the rest contours are free of external loading) at $G_1/G=3.049$; $\varkappa=1.88$; $\varkappa_1=2.12$; $R_0/R_2=13.33$; $R_1/R_2=5.17$.

TABLE 1 - Dimensionless stress intensity factors (SIF) Y_{Ip} and $Y_{I\delta}$ obtained for the cases of internal pressure p and interference fit δ.

Crack length	SIF		Crack length	SIF	
$\lambda = 2l/R_2$	Y_{Ip}	$-Y_{I\delta}$	$\lambda = 2l/R_2$	Y_{Ip}	$-Y_{I\delta}$
0.2	0.847	0.801	1.2	0.347	0.534
0.4	0.660	0.698	1.4	0.310	0.516
0.6	0.540	0.633	1.6	0.281	0.503
0.8	0.456	0.589	1.8	0.257	0.493
1.0	0.394	0.557	2.0	0.238	0.486

REFERENCES

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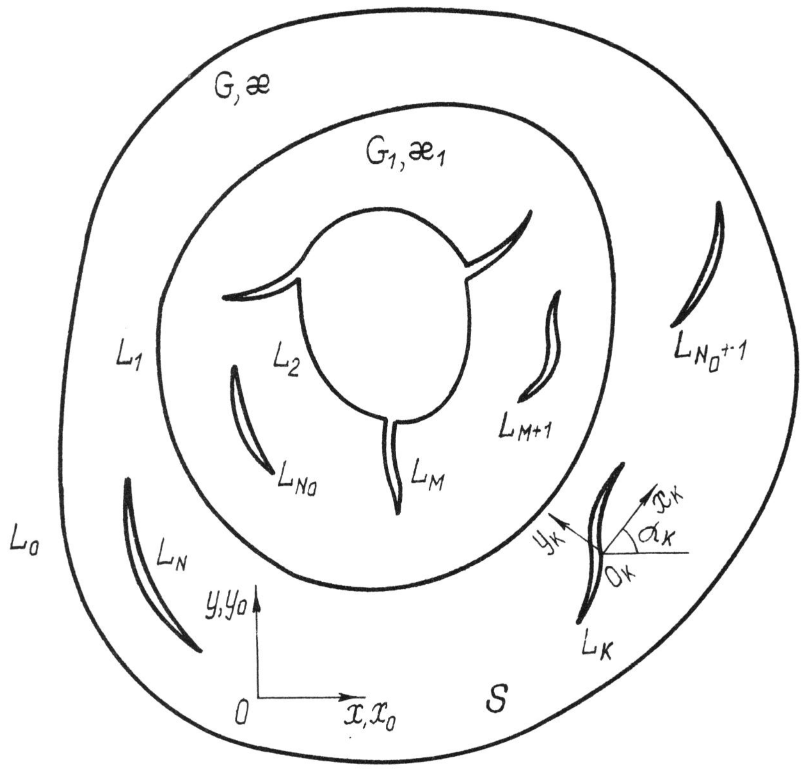

Figure 1 Two-component curvilinear ring with internal and edge cracks