A DAMAGE CRITERION BASED ON THE STRESS CONDITION

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Damage in a material is starting if the stress passes over a limit. This limit is quantified as the endurance limit and a relation is proposed between the damage and the variabel endurance limit. A damage function is introduced to descripe the change of this stress limit. Plastic deformation is supposed as the consequence of the originaly caused damage if the material is able to annihilate opposite dislocations. It is calculated by the inverse form of Pragers rule. To explain the method, the theory is applied to some simple cases.

INTRODUCTION

Before a crack arises, the material in its vicinity has been damaged caused by the applied load. This damage may be completly developed, if the straining is made by a static load. It may also be accumulated step by step if an alternating load has occurred. If a crack exists, its further propagation can be described with the methods of fracture mechanics.

Thus it is very important to investigate what amount of damage has occurred and by which parameters it may be quantified. In metals and their alloys the dislocations disturbing the regularity of the crystal structure are the most important elements of damage. The quantity of dislocations and their distribution give some information about the damage. If the stresses are still very small, it is no variation in the mesh of dislocations, but increasing the load, new dislocations are created and they are not uniformly arranged as on the start of loading.

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On the other hand the dislocations form the base of the theory of plasticity and many investigations are made in this field. A summary is given by Lemaitre (1). But till now, there are no satisfactory relationships between the theory of dislocations and the continuous theory of plasticity. If we can explain the role of dislocations in the theory of damage, we can also hope to make a step forward in the continuous theory of plasticity.

STRESS LIMIT AND BACK STRESSES

We know that no increase of damage occurs, if the stresses are sufficiently small. We can define a limit for the stress tensor given by the equation

$$\mathbf{A}_{ijkl}\,(\sigma_{ij}-a_{ij})\,(\sigma_{kl}-a_{kl}) - \sigma_D^2 \le 0 \;. \tag{1}$$

An anisotropic behavior may be expressed by the symmetric tensor $\mathbf{A}_{ijkl}$. In the isotropic case it is $\mathbf{A}_{iiii} = 1$, $\mathbf{A}_{iijj} = -0{,}5$ and $\mathbf{A}_{ijij} = 1{,}5$. All the other elements are identical zero. If the expression on the left side of eq. (1) is less then zero, we do not have a growth in dislocations. If the stresses σ_{ij} increase, the sign of equality is always valid causing an alteration of a_{ij} and σ_D. Thus the value of σ_D gives us information about the density of dislocations whereas the tensor of back stresses a_{ij} indicates their nonuniform distribution. Following the increasing load, the a_{ij} change at first and if the load continue to increase, new dislocations are created and therefore σ_D will change, too. As a simplification we suppose that σ_D and the a_{ij} start to change together at the point when the sign of equality is valid. The increase of a_{ij} may then be expressed by

$$da_{ij} = \mathbf{A}_{ijkl}\,(\sigma_{kl}-a_{kl})\,d\lambda \tag{2}$$

with the unknown factor $d\lambda$. Defined in this way the incremental vector is perpendicular to the surface (1) . Simultaneously, σ_D decreases by

$$d\sigma_D = -\frac{3}{2}\,g\,\sigma_D\,d\lambda \;, \tag{3}$$

because an increasing density of dislocations means a diminution of the damage limit. The function g is a damage function, which is still to be determined. It depends from the actual state, that means from the invariants formed by σ_D, σ_{ij} and a_{ij}.

Differentiation of eq. (1) with respect to $d\lambda$ gives

$$\mathbf{A}_{ijkl}\,(\sigma_{ij}-a_{ij})\,(d\sigma_{kl}-da_{kl}) = \sigma_D\, d\sigma_D \tag{4}$$

and with regard to the symmetry of $\mathbf{A}_{ijkl}$, we get

$$d\lambda = \frac{2}{3}\,\frac{\mathbf{A}_{ijkl}\,(\sigma_{ij}-a_{ij})\,d\sigma_{kl}}{\sigma_D^2\,(1-g)} \ . \tag{5}$$

When the increasing load is applied, there are not only new dislocations created, but it may also be the case that dislocations contrarily orientated annihilate each other. From this we obtain the plastic deformation $d\varepsilon^p_{ij}$ which can be expressed by

$$d\varepsilon^p_{ij} = \frac{1}{c}\, da_{ij} \ . \tag{6}$$

This is Prager's rule, but in an inverse form, and if we replace da_{ij} by eq. (2), we see that $d\varepsilon^p_{ij}$ is normal to the surface given by eq. (1). Here, c is the function of plasticity depending like g from the invariants formed by σ_D, σ_{ij} and a_{ij}. Both c and g are specific material functions, but they are still unknown. If we make some assumption about c and g, we can obtain information about the stress-strain-diagram and the behavior of the material by alternate loading. We can see by this indirect method, whether the choice of the invariant parameters and their functions c and g was right. If σ_D becomes smaller than a lowest limit σ_{DG} a crack is initiated at this point, any further propagation of this crack is not examined in the presented contribution. First investigations for service strenght are made by Koczyk (2).

PLASTIC MATERIAL WITHOUT STRAIN-HARDENING

The well-known case of a material without strain-hardening is a special case of the theory presented here. We know that σ_D will be always constant and the back stresses a_{ij} remain equal to zero. From $da_{ij} \Rightarrow 0$ it can be concluded that $d\lambda \Rightarrow 0$ and also that $d\sigma_D \Rightarrow 0$ and the damage function is then equal zero as well. If we calculate $d\varepsilon^{p}_{ij}$ from eq. (6) we see that the function c must also be equal to zero. At least we have the limit

$$lim \frac{d\lambda}{c} => d\lambda^* \tag{7}$$

and $d\lambda^*$ is a real infinite parameter, which indicates the growth of plastic deformation. Instead of eq. (6) we get with regard to eq. (2)

$$d\varepsilon_{pij} = \mathbf{A}_{ijkl}\, \sigma_{kl}\, d\lambda^* \quad . \tag{8}$$

Defined in this way the vector of plastic deformation $d\varepsilon^{p}_{ij}$ is directed perpendicular to the surface given by eq. (1). That means, the number of dislocations created is equal to those annihilated, and the state of the material remains unchanged, though we have plastic deformation.

STRESS LIMIT AND ENDURANCE LIMIT

The basic equations derived before look like those in the theory of plasticity, but there is quite a difference. At first we have to attach the stress limit to a real quantity. Of course, it is not the yield stress which happens to be used in the theory of plasticity. We can determine this parameter only by an indirect method. If we have a specimen, loaded by a cyclic force with constant amplitude, we know there exists an endurance limit. If the stress amplitude is lower, no crack is initiated but if it passes over this limit the specimen will be destroyed after a finite number of oscillations. Consequently we can conclude, that damage starts if the stress exceeds the endurance limit and we see, the stress limit σ_D is exactly this quantity. At first it has an initial value σ_{D0}, which will be changing with increasing load. Generally, if we pass this point on the stress-strain diagram, we cannot observe any plastic deformation. It starts lowly when the stress is growing up. In previous investigations (3), (4) the assumption was made, that any damage is caused by plastic deformation, whereas it is supposed now, that damage starts before plastic deformation occurs.

ENDURANCE LIMIT AND MAXIMAL STRESS

Characterizing the position of the stress point on the limit surface given by (1) we introduce the invariant parameter q (fig. 1), defined as

$$q = \frac{1}{\sigma_D^2} \boldsymbol{A}_{ijkl}\, a_{ij}\, (\sigma_{kl} - a_{kl}) \quad . \tag{9}$$

At the initial state q equals zero. With an increasing load q becomes positiv on the "loading side" of the limit surface whereas it will be negativ on the "unloading side" (see also fig. 1). Defined in such way q becomes positive for an increasing stress in the case of tension (fig. 2) and (9) may be reduced to

$$q = \left(\frac{\sigma^{(o)}}{\sigma_D} - 1 \right) \tag{10}$$

and it is negative for unloading

$$q = -\left(\frac{\sigma^{(u)}}{\sigma_D} + 1 \right) \tag{11}$$

In the simplest way the damage function g may be a uniformly increasing function of q for tension, because damage is accumulated with increasing load. In the same manner the plasticity function c(q) will decrease for more plastic deformation by increasing load. In each case the tensile stress is limited by a maximum when the endurance limit has reached it's critical value (see fig. 2).

CONCLUSIONS

The theory presented above doesn't need a special parameter D to indicate the increasing damage. The yield condition for plastic deformation is replaced by a damage condition and the plastic deformation develops step by step if the stress has passed this limit. The difficulty is to find out the unknown material-functions g and c. Especially, the application to problems of service strength proposed by in (2) requires a more exact determination of the damage function g. Previously, it was assumed that damage and plastic deformation are starting simultaneously (3), (4). The experiences show that damage starts at stresses below those creating noticeable plastic deformation. This fact has been considered here. Plastic deformation has been assumed as a consequence of the damage caused by new created dislocations before.

REFERENCES

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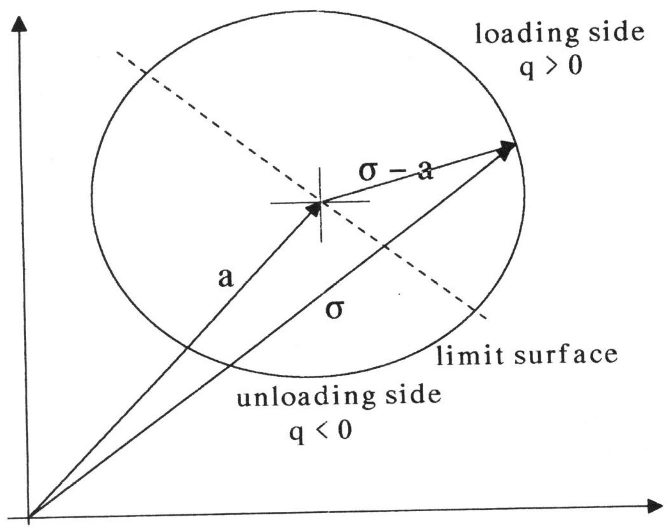

Figure 1 Limit surface corresponding to eq. (1)

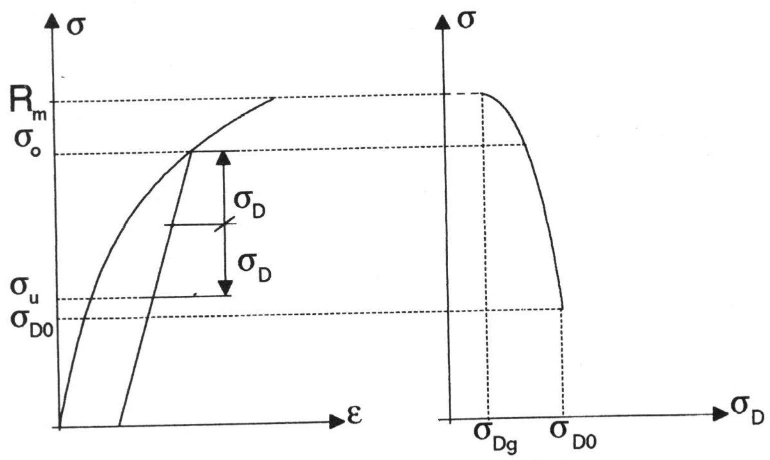

Figure 2 Stress maximum and endurance limit