CREEP LIFETIME PREDICTION OF PRESSURIZED TUBES WITH CONTINUUM DAMAGE MECHANICS

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The Continuum Damage Mechanics (CDM) approach is applied to a thick walled cylinder pressurized at high temperatures to predict the lifetime under creep conditions. Two creep-damage models are considered, viz. the model developed by Kachanov and Rabotnov and the model developed by Hayhurst. The results obtained using the CDM approach are compared with some estimations made with simple analytical equations. These estimations are based on the elastic stress distribution or the stress distribution for the stationary creep.

INTRODUCTION

Structural components operating at high temperatures suffer creep deformation and creep damage as a result of the formation and the growth of micro cracks and micro cavities. The occurrence of this damage might finally lead to the failure of the component. For the design and the safe operation of such components it is necessary to be able to calculate the (residual) lifetime and the influence of the loading conditions on the lifetime.

In the "classical" continuum mechanics the creep problem is solved using phenomenologically derived constitutive equations which have proven to be accurate to describe creep deformations. As these constitutive equations do not consider the occurrence of creep damage the decrease of strength can not be quantified directly and failure prediction has to be done using the results calculated for undamaged material, which can lead to unreliable lifetime predictions. To avoid unsafe predictions a conservative approach will be used generally, which might be unfavourable economically. In the field of the CDM damage constitutive equations

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are derived providing a tool to calculate simultaneously the distribution of stress, strain and damage as a function of time. In this way it is possible to calculate the lifetime of components subjected to a multiaxial stress field. Because analytical solutions can be obtained for a few cases only the Finite Element (FE)-code FAME-D was developed by Braam and Haverkate (1). This code provides a tool by which damage in connection with creep behaviour can be calculated for 2-D and axisymmetric geometries.

The CDM based FE-code FAME-D was applied to a pressurized thick-walled cylinder for three different axial loading conditions, viz. a plane strain ring, a cylinder with an axial force due to the sealed ends and a cylinder with zero axial force (Braam and Haverkate (2)). The material parameters in the creep and damage equations were determined by fitting the 1-D formulation of the constitutive equations to uniaxial creep curves of a modified 9 Cr steel T91/P91 at $\approx$ 600 °C. Besides the FE-analyses analytical equations are presented for the lifetime estimation of a pressurized thick-walled cylinder. The results of these estimations are compared with the FAME-D results.

DAMAGE CONSTITUTIVE EQUATIONS

General formulation

The creep behaviour of several metals using the equations developed by Hayhurst for uniaxial and multiaxial creep behaviour was considered by Dunne et al (3). The damage evolution function and the creep law take the form

$$\dot{D} = K t^{-m} \frac{(\sigma_D)^q}{(1-D)^r} \tag{1}$$

$$\dot{\varepsilon}^c_{eq} = B t^{-m} \left(\frac{\sigma_{eq}}{1-D} \right)^n \tag{2}$$

where K, m, q, r, B and n are the material parameters and σ_{eq} is the equivalent Von Mises stress. The equivalent damage stress σ_D is equal to the tensile stress in case of uniaxial creep experiments. For multiaxial stress conditions several definitions for σ_D are given in literature. A definition proposed by Hall and Hayhurst (4) is:

$$\sigma_D = \alpha_s \sigma_{eq} + (1-\alpha_s)\sigma_1 \tag{3}$$

where σ_1 is the maximum principal stress and α_s is a material constant. This definition is preferred because the number of parameters to be determined is only one. For the uniaxial situation with a constant stress σ_o, the damage is obtained by integrating Eq. (1) with the initial condition $D = 0$ at $t = 0$

$$D(t) = 1 - \left(1 - \left(\frac{t}{t_f}\right)^{1-m}\right)^{\frac{1}{1+r}} \tag{4}$$

where t_f is the time to rupture given by

$$t_f = \left(\frac{1-m}{(r+1)K\sigma_o^q}\right)^{\frac{1}{1-m}} \tag{5}$$

Substituting Eq. (4) into Eq. (2) and integrating with the initial condition $\varepsilon^c = 0$ at $t = 0$ the axial creep strain is obtained.

For $m = 0$ the Hayhurst model develops into the Kachanov model. Advantage of the Hayhurst model is that the primary creep can be taken into account, which is not possible with the Kachanov model.

Material properties

The material considered is a modified 9 Cr steel T91/P91 at ≈ 600 °C. The elastic material properties used are:

Young's modulus : E = 210000 MPa
Poison's ratio : $\upsilon = 0.3$

To fit the equation for the creep strain based on CDM to uniaxial creep curves a stable and consistent methodology was developed by Dunne et al (3). To calculate the material parameters using this optimization technique the computer program FIT was developed at ECN. This computer program is applied to the creep curves for $\sigma = 90$, 100, 110 and 120 MPa. The calculated material parameters are given in Table 1. It is clear that the material parameter α_s in the equivalent damage stress can not be determined from uniaxial experiments.

Table 1 *Material parameters*

Model	K [$h^{m-1}MPa^{-q}$]	m [-]	q [-]	r [-]	B [$h^{m-1}MPa^{-q}$]	n [-]
Kachanov	0.312e-14	-	5.337	5.337	0.657e-16	5.92
Hayhurst	0.110e-12	0.1241	4.668	7.120	0.438e-26	11.04

LIFETIME ESTIMATION

FAME-D analyses

With the FE-code FAME-D calculations are made for a thick-walled cylinder with inner radius a = 15 mm, outer radius b = 20 mm and internal pressure p = 33.3 MPa. The internal pressure is chosen such that the mean circumferential stress is 100 MPa. Because the parameter α_s could not be determined from the uniaxial experiments calculations are made for α_s = 0, 0.25, 0.5, 0.75 and 1.0. Three different axial loading conditions are considered:

- a plane strain ring, which is representative for a cylinder with the axial displacement suppressed;
- a closed cylinder with an axial force due to the sealed ends of the cylinder;
- an open cylinder with zero mean axial stress.

For the plane strain ring the circumferential stress and the damage as a function of time are shown in Figure 1 and 2 for α_s = 0. For α_s = 0 the maximum damage is at the outside of the cylinder. In the region with the highest damage (at the outside for α_s = 0 and at the inside for α_s = 1) the circumferential stress and the axial stress do decrease with time. As a result of the damage process the material will show a weaker elastic behaviour. Because of compatibility the strains in these regions can not increase freely and as a consequence the stress has to decrease. To maintain equilibrium the stresses in the region with the lowest damage have to increase. For α_s = 0 the maximum in the circumferential stress will move from the outside to the inside (Figure 1). The calculated lifetimes are enumerated in Table 2. The results for the plane strain ring and the closed cylinder do agree very well, which is consistent with the results obtained for a Norton creep law without damage. In case of a Norton creep law the creep behaviour of a plane strain ring and a closed cylinder is identical.

Table 2 *Predicted lifetime in hours*

Geometry	Creep model	α_s				
		0.0	0.25	0.5	0.75	1.0
Plane strain ring	Kachanov	1006	1051	1071	1069	1046
	Hayhurst	889	998	1104	1173	1157
Closed cylinder	Kachanov	1004	1049	1068	1066	1043
	Hayhurst	888	995	1105	1168	1155
Open cylinder	Kachanov	1053	951	853	762	676
	Hayhurst	1056	1012	942	845	750

Analytical solutions

Two methods for lifetime estimation of a pressurized thick walled cylinder are presented in this paper. Both methods are based on Eq. (5) which gives the time to rupture for a constant uniaxial stress according to the Hayhurst model. For the first method the uniaxial stress in Eq. (5) is substituted by the equivalent Tresca stress. The Tresca stress is calculated with the mean value of the circumferential stress and the mean value of the radial stress. These stress components can be calculated directly from the equilibrium equations. For the second method the uniaxial stress in Eq. (5) is substituted by the mean value through the thickness of the equivalent damage stress, σ_D, where the equivalent damage stress is calculated for the stationary stress distribution as derived for a Norton creep law with $n = 5.92$. A comparison between the estimated lifetimes and the FAME-D results obtained with the Hayhurst model is made for the closed cylinder in Figure 3. It appears that the estimated lifetimes calculated with the Tresca stress are conservative as compared to the FAME-D results. The estimated lifetimes based on the mean damage stress do overestimate the FAME-D results for small values of α_s and do underestimate the FAME-D results for $\alpha_s \geq 0.5$. The maximum difference occurs for $\alpha_s = 0$ and is about 21 %.

CONCLUSIONS

It can be concluded that the CDM based FE-code FAME-D has the capability for lifetime prediction of structural components operating at high temperatures suffering creep deformation and creep damage. Before the CDM approach can be used for practical applications it has to be demonstrated that experimentally obtained lifetimes and patterns of damage evolution can be predicted accurately. Because the CDM based FE-code FAME-D requires a powerful computer it is not suitable for all situations the lifetime has to be calculated. Therefore a number of simple methods for lifetime prediction were presented.

REFERENCES

(1) Braam, H and Haverkate, B.R.W., "FAME-D, a finite element code based on continuum damage mechanics", in "Topics in Applied Mechanics, Integration of Theory and Applications in Applied Mechanics". Edited by Dijksman, J.F. and Nieuwstadt, F.T.M., Kluwer, 1993.

(2) Braam, H and Haverkate, B.R.W., "Creep analyses of a pressurized thick-walled cylinder using continuum damage mechanics", ECN-I--93-020, 1993.

(3) F.P.E. Dunne, A.M. Othman, F.R. Hall and D.R. Hayhurst, , *Int. J. Mech. Sci.*, 32 (1990), pp. 945-957.

(4) F.R. Hall and D.R. Hayhurst, *Proc. R. Soc. Lond.* A (1991) 433, 383-403

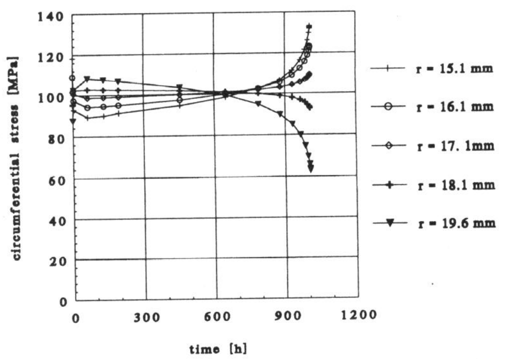

Figure 1 *Circumferential stress in plane strain ring for* $\alpha_s = 0$

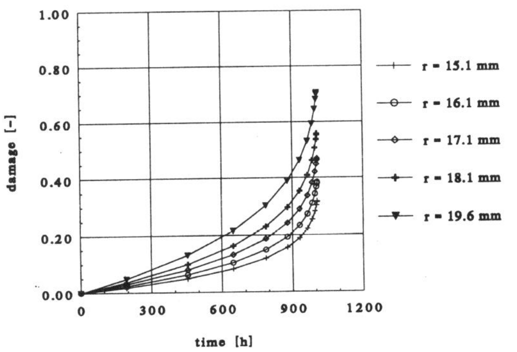

Figure 2 *Damage in plane strain ring for* $\alpha_s = 0$

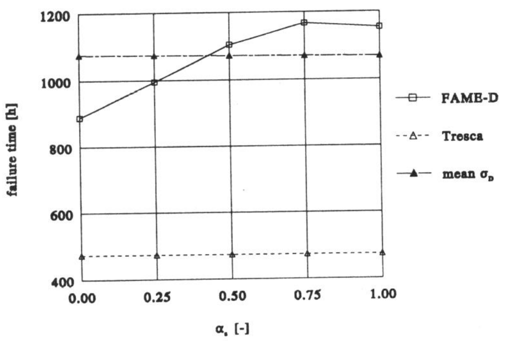

Figure 3 *Lifetime based on Hayhurst model for closed cylinder*