NEW MODEL FOR DETERMINATION OF RUPTURE TIME OF MATERIALS WITH DIFFERENT BEHAVIOUR IN TENSION AND COMPRESSION

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This paper deals with a new energy-based variant of a theory to estimate the time of fracture under creep and damage conditions. The theory is based on the hypothesis of coupled creep and fracture processes for anisotropic materials with different behaviour in tension and compression (e.g. some light alloys, composites, monocrystals etc.). The damage process is characterized by the specific dissipation energy. The nonclassical creep behaviour including second order effects is described by constitutive equations derived from a creep potential. In the case of anisotropic materials the potential depends on the linear, the quadratic and the cubic invariants of the stress tensor and some anisotropy tensors. In the case of isotropic materials the potential depends only on the three invariants. Finally, for a titanium alloy a comparison of theoretical predicted and experimental results is given for a multiaxial stress state.

INTRODUCTION

Most of traditional creep behaviour isotropic models postulate the existence of a creep potential including only the second stress deviator invariant. On the other side some light alloys, composites, monocrystals etc. show a creep behaviour in tests different from traditional one: for example different behaviour in tension and compression [Pintschovius et al. (1)], nonunique normalized tension and torsion curves [Kowalewski (2)], influence of hydrostatic pressure [Nishitani, (3)]. In this case the potential formulation should be extended. The simplest extension is connected with a potential formulation using in addition the first or the third stress deviatoror invariant. In general, the potential depends on an equivalent stress as a function of all three stress tensor invariants. In the case of anisotropic materials the potential depends on stress tensor invariants and some anisotropy tensors.

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Creep processes (especially the tertionary creep) are coupled with increasing damage evolution. For modelling coupled creep and damage it is necessery to introduce a damage variable. There are many possibilities of definition of damage variables. One of them is connected with the dissipation power during the deformation process. Thus the coupled creep and damage can be described by creep constitutive equation and damage evolution equation.

THE MODEL

If a creep potential

$$F = \sigma_e^2 \qquad (\sigma_e \geq 0 - \text{equivalent stress}) \tag{1}$$

exists, the small strain rate tensor $\dot{\boldsymbol{\varepsilon}}$ can be calculated by an associated creep law

$$\dot{\boldsymbol{\varepsilon}} = \lambda \frac{\partial F}{\partial \boldsymbol{\sigma}} \tag{2}$$

$\boldsymbol{\sigma}$ is the stress tensor, λ is a scalar factor. The equivalent stress suggesting the equivalent behaviour in the uniaxial and multiaxial states can be defined in the following way. Considering mixed invariants of the stress tensor and some material constant tensors characterizing the anisotropy

$$\sigma_1 = \mathbf{b} \cdot\cdot \boldsymbol{\sigma}, \sigma_0^2 = \boldsymbol{\sigma} \cdot\cdot {}^{(4)}\mathbf{a} \cdot\cdot \boldsymbol{\sigma}, \sigma_2^3 = \boldsymbol{\sigma} \cdot\cdot \left(\boldsymbol{\sigma} \cdot\cdot {}^{(6)}\mathbf{c} \cdot\cdot \boldsymbol{\sigma}\right), \tag{3}$$

the equivalent stress depends in general on all three stress tensor invariants

$$\sigma_e = \sigma_0 + \alpha\sigma_1 + \gamma\sigma_2 \tag{4}$$

There are $\mathbf{b}, {}^{(4)}\mathbf{a}, {}^{(6)}\mathbf{c}$ second, fourth and sixth order anisotropy tensors, α, γ are numerical coefficients characterizing the "specific weight" of the linear and the cubic invariants. For $\alpha = \gamma = 0$ we get the traditional anisotropic creep models (von Mises-Hill-type theory). The eq. (4) is a possible generalization of the equivalent stress formula proposed by Leckie and Hayhurst (4).

The constitutive equations (2) with respect to eq. (1) and (4) can be expressed in the following way

$$\dot{\boldsymbol{\varepsilon}} = 2\lambda\sigma_e \left(\frac{^{(4)}\mathbf{a} \cdot \cdot \boldsymbol{\sigma}}{\sigma_0} + \alpha \mathbf{b} + \gamma \frac{\boldsymbol{\sigma} \cdot \cdot {}^{(6)}\mathbf{c} \cdot \cdot \boldsymbol{\sigma}}{\sigma_2^2} \right) \tag{5}$$

Multiplying eq. (5) with $\boldsymbol{\sigma}$ finally we get the dissipation power

$$W = \boldsymbol{\sigma} \cdot \cdot \dot{\boldsymbol{\varepsilon}} = 2\lambda\sigma_e^2 \tag{6}$$

The intensity of the creep process can be characterized by the specific dissipation power W (6), in this case a measure of damage is the energy dissipated during the creep deformation

$$\varphi = \int W \, dt \tag{7}$$

For a given temperature and coupled creep and damage we can introduce the following constitutive assumption

$$W = f(\sigma_e, \varphi) \tag{8}$$

For example non-hardening material can be described by

$$W = \frac{\vartheta(\sigma_e)\varphi_*^q}{(\varphi_* - \varphi)^q} \tag{9}$$

φ_* is the value of the specific dissipation energy in the time of rupture, q - a material parameter. The general non-linear tensorial constitutive equation for coupled creep and damage can be expressed

$$\dot{\boldsymbol{\varepsilon}} = \frac{\chi(\sigma_e)\varphi_*^q}{(\varphi_* - \varphi)^q} \left(\frac{^{(4)}\mathbf{a} \cdot \cdot \boldsymbol{\sigma}}{\sigma_0} + \alpha \mathbf{b} + \gamma \frac{\boldsymbol{\sigma} \cdot \cdot {}^{(6)}\mathbf{c} \cdot \cdot \boldsymbol{\sigma}}{\sigma_2^2} \right) \tag{10}$$

The system of material dependend equations is complete, if a damage evolution equation is defined

$$\frac{d\varphi}{dt} = \frac{\vartheta(\sigma_e)\varphi_*^q}{(\varphi_* - \varphi)^q} \tag{11}$$

The function $\chi(\sigma_e) = \vartheta(\sigma_e)/\sigma_e$ in eq. (10) should be determined by tests. The simplest assumptions are the power law σ_e^n, the hyperbolic sinus $\sinh(\sigma_e/d)$ or the exponential function $\exp(\sigma_e/p)$ (n, d, p are constants). It can be shown, that eq. (10) is similar to the equation suggestet by Rabotnov (5).

ISOTROPIC CREEP

Starting from the general constitutive and evolution eq. (10), (11), special cases of material symmetry can be derived: orthotropic, transverse-isotropic and isotropic relations. For isotropic coupled creep and damage we get

$$\dot{\boldsymbol{\varepsilon}} = \frac{\chi(\sigma_e)\varphi_*^q}{(\varphi_* - \varphi)^q}\left(\frac{AI_1\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_0} + \alpha B\mathbf{I} + \gamma\frac{DI_1^2\mathbf{I} + E\boldsymbol{\sigma}\cdot\boldsymbol{\sigma} + \frac{K}{3}(I_2\mathbf{I} + 2I_1\boldsymbol{\sigma})}{\sigma_2^2}\right) \tag{12}$$

with $\sigma_1 = BI_1$, $\sigma_0^2 = AI_1^2 + CI_2$, $\sigma_2^3 = DI_1^3 + KI_1I_2 + EI_3$, $I_1 = \boldsymbol{\sigma}\cdot\cdot\mathbf{I}$, $I_2 = \boldsymbol{\sigma}\cdot\cdot\boldsymbol{\sigma}$, $I_3 = \boldsymbol{\sigma}\cdot\cdot(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})$. The 6 material-dependend parameters can be determined by some basic tests. Such test are

- uniaxial tension ($\sigma_{11} \neq 0$)

$$\dot{\varepsilon}_{11} = K_+\sigma_{11}^n\frac{\varphi_*^q}{(\varphi_* - \varphi)^q}, \dot{\varepsilon}_{22} = -Q\sigma_{11}^n\frac{\varphi_*^q}{(\varphi_* - \varphi)^q}; \tag{13}$$

- uniaxial compression ($-\sigma_{11} \neq 0$)

$$\dot{\varepsilon}_{11} = -K_-|\sigma_{11}|^n\frac{\varphi_*^q}{(\varphi_* - \varphi)^q}; \tag{14}$$

- pure torsion ($\sigma_{12} \neq 0$)

$$2\dot{\varepsilon}_{12} = N\sigma_{12}^n\frac{\varphi_*^q}{(\varphi_* - \varphi)^q}, \dot{\varepsilon}_{11} = M\sigma_{12}^n\frac{\varphi_*^q}{(\varphi_* - \varphi)^q}; \tag{15}$$

- hydrostatic pressure ($\sigma_{11} = \sigma_{22} = \sigma_{33} = -p_0/3$)

$$\dot{\varepsilon}_{11} = \dot{\varepsilon}_{22} = \dot{\varepsilon}_{33} = -P|\sigma_{11}|^n\frac{\varphi_*^q}{(\varphi_* - \varphi)^q}. \tag{16}$$

Here the constants K_+, K_-, Q, N, M, P, n are known from tests. Suggesting Norton's creep law as function $\chi(\sigma_e) = \sigma_e^n$ we can estimate all parameters in the constitutive equations:

$$\begin{aligned}
C &= N^{2r}/2, \alpha B = M/(\sqrt{2C})^n, A = X^2 - C, \\
6\gamma^3 D &= [\sqrt{9A+3C} - 3\alpha B - (3P)^r]^3 - 3(T-\alpha B)^3 \\
&+ 18(A/\sqrt{A+C} + \alpha B + Q_+^{-nr})(T-\alpha B)^2 \\
2\gamma^3 K &= 3(T-\alpha B)^3 - [\sqrt{9A+3C} - 3\alpha B - (3P)^r]^3 \\
&- 24(A/\sqrt{A+C} + \alpha B Q K_+^{-nr})(T-\alpha B)^2, \\
\gamma^3 L &= (T-\alpha B)^3 - \gamma^3 D - \gamma^3 K
\end{aligned} \tag{17}$$

with $T = (K_+^r - K_-^r)/2, X = (K_+^r + K_-^r)/2, r = 1/(n+1)$.

From the general equations we can derive equations with less parameters. For example there are different types of 3-parameter-equations. If we get from the test

$$T = MN^{-nr}, \sqrt{9X^2 - 3N^{2r}} = 3T + (3P)^r, \tag{18}$$

then we have from (17) $\gamma = 0$ and

$$\dot{\boldsymbol{\varepsilon}} = \frac{\chi(\sigma_e)\varphi_*^q}{(\varphi_* - \varphi)^q}\left(\frac{AI_1\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_0} + \alpha B\mathbf{I}\right) \tag{19}$$

On the other side, if

$$3T^3 - [\sqrt{9X^2 - 3N^{2r}} - (3P)^r]^3 = Y = M = 0 \tag{20}$$

with $Y = X = N^{2r}/(2X) + QK_+^{-nr}$, then we get (17) $\alpha B = D = K = 0$ and

$$\dot{\boldsymbol{\varepsilon}} = \frac{\chi(\sigma_e)\varphi_*^q}{(\varphi_* - \varphi)^q}\left(\frac{AI_1\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_0} + \gamma\frac{E\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{\sigma_2^2}\right) \tag{21}$$

If we find

$$[\sqrt{9X^2 - 3N^{2r}} - (3P)^r]^3 - 9T^3 = T + 3Y = M = 0, \tag{22}$$

we get $\alpha B = D = E = 0$ and

$$\dot{\boldsymbol{\varepsilon}} = \frac{\chi(\sigma_e)\varphi_*^q}{(\varphi_* - \varphi)^q}\left(\frac{AI_1\mathbf{I} + C\boldsymbol{\sigma}}{\sigma_0} + \gamma\frac{K(I_2\mathbf{I} + 2I_1\boldsymbol{\sigma})}{3\sigma_2^2}\right) \tag{23}$$

EXAMPLE

For the titanium alloy OT-4 and the temperature 748 K some creep test data are published by Gorev et. al. (6). Using only three independent tests (tension, compression and torsion) we get the following material's parameters for this isotropic material: $n = 4, q = 2, \varphi_* = 100\ MPa, K_+ = 13.3\ 10^{-14}\ MPa^{-n}h^{-1}$, $K_- = 7.5\ 10^{-14}\ MPa^{-n}h^{-1}, N = 27.7\ 10^{-14}MPa^{-n}h^{-1}$. The function $\chi(\sigma_e)$ was taken in the form of Norton's creep law $\chi(\sigma_e) = \sigma_e^n$.

On Fig. 1 and 2 the comparison between theoretical predicted (lines) and measured (points) data for thinwalled speciments loaded by a torque and an axial load are shown. The curve 1 is calculated by the equation (19), the curves 2 and 3 - by the equations (21) und (23). Here

$$\varphi = \sigma_{11}\varepsilon_{11} + 2\sigma_{12}\varepsilon_{12}$$

We can obtain a good agreement between the theoretical and experimental results.

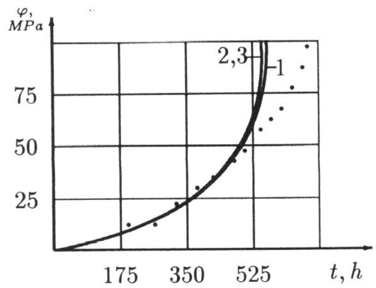

Figure 1: Specific dissipated energy versus time in the case of superposes tension ($\sigma_{11} = 194.9\ MPa$) and torsion ($\sigma_{12} = 46.6\ MPa$)

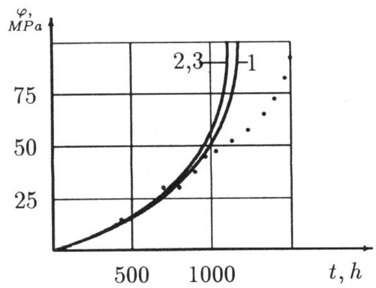

Figure 2: Specific dissipated energy versus time in the case of superposes tension ($\sigma_{11} = 156.3\ MPa$) and torsion ($\sigma_{12} = 52.1\ MPa$)

CONCLUSIONS

In the paper a theory for determination of the fracture time under creep conditions for materials with different behaviour in tension and torsion based on energy assumptions is proposed. For isotropic materials we get a good agreement between the theoretical (calculated) and experimental (measured) data in the case of multiaxial stress state.

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SYMBOLS USED

$\boldsymbol{\sigma}$	stress tensor
I_1, I_2, I_3	invariants of the stress tensor
A, B, C, D, K, E	constants in (12)
K_+, K_-, Q, N, M, P	constants in (13) - (16)
$\dot{\boldsymbol{\varepsilon}}$	rate tensor of small creep strains
$\sigma_e, \chi(\sigma_e)$	equivalent stress, function of σ_e
F	creep potential
λ	scalar factor
$\mathbf{b}, {}^{(4)}\mathbf{a}, {}^{(6)}\mathbf{c}$	second, fourth, sixth order anisotropy tensors
α, γ	numerical coefficients
$\sigma_1, \sigma_0^2, \sigma_2^3$	mixed invarients
W, φ, φ^*	dissipation power, dissipated energy, limit of φ
q, n, p	constants
p_0	hydrostatic pressure

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