# ON THE TESTING OF THE ELASTO-PLASTIC CRACKING RESISTANCE OF THIN PLATES

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The method of Cotterell and Reddel has been applied to a study of the influence of overaging treatments on the essential work of fracture of thin plates of an aluminium alloy. The various experimental parameters accessible by this method have been revisited. It is shown that two different self-consistent estimations of the essential work of rupture can be derived by a proper combination of these parameters. Moreover, this approach allows the partitionning of the total essential work of fracture into the work for homogeneous straining of the process zone and the work for necking and decohesion along the crack path.

## INTRODUCTION

In the framework of linear elastic fracture mechanics, very large samples are needed for the measurement of the fracture toughness of ductile materials presenting a large plastic zone at the crack tip. For example, ASTM standard E - 399 [1] requires that the dimensions of the specimen be higher than 2,5 $(K_{Ic}/\sigma_y)^2$ for valid determination of $K_{Ic}$. In order to circumvent this condition, Cotterell and Reddel [2] proposed to derive the fracture toughness of ductile materials by measuring the "essential work of fracture" using a series of DENT specimens with variable ligament length l. Following Broberg's idea [3], these authors showed that the total work of fracture may be written as the sum of an essential work we spent in a process zone ahead of the crack tip and a non-essential plastic work $w_p$ dissipated in an outer plastic screening region. Cotterell and Reddel also suggested that the essential work is proportional to the area of the fracture surface while the non-essential work is proportional to the

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plastified volume. So, the total work of fracture can be written as :

$$W_f = l.t.w_e + \beta.l^2.t.w_p \quad (1)$$

where t is the specimen thickness and $\beta$ is a geometrical shape factor. The specific work of fracture, or the work of fracture per unit ligament area, $w_f = W_f / lt$ is thus equal to

$$w_f = w_e + \beta.l.w_p \quad (2)$$

It has been shown that the method provides reliable values of the plane stress fracture toughness on the condition that the ligament length lie between the limits

$$3t...5t < l < \min(L/3 , 2r_p) \quad (3)$$

where L is the width of the plate and $r_p$ is the radius of the plastic zone [4]. When the total specific work is plotted against the ligament length, the specific essential work of fracture can be obtained by a linear extrapolation of the data points to zero ligament size.

The specific essential work of fracture can also be written as [4]:

$$w_e = w_{e1} + w_{e2} = \Delta \int_0^{\varepsilon_s} \sigma d\varepsilon + \int_{\varepsilon_{ns}.\Delta}^{\delta_c} \sigma_n(\delta) d\delta \quad (4)$$

where $\Delta$ is the width (measured before straining) of the part of the plate that forms the process zone, $\varepsilon_s$ and $\varepsilon_{ns}$ are the true strain and the engineering strain at the onset of necking, $\sigma_n(\delta)$ and $\delta$ are the engineering stress and the displacement in the plane of the crack along the centre of the neck, and $\delta_c$ is the critical crack tip opening displacement. The first term of (4), $w_{e1}$, is the uniform plastic work spent up to necking in order to form the process zone and the second term of this equation, $w_{e2}$, represents the work dissipated in the localized, non-uniform deformation during cracking along the neck.

In this work, the method of Cotterell and Reddel has been applied to a study of the influence of overaging treatments on the essential work of fracture of an aluminium alloy. The various experimental parameters accessible by this method have been revisited. It is shown that two different self-consistent estimations of the components $w_{e1}$ and $w_{e2}$ of the essential work of rupture can be derived by a proper combination of these parameters.

## EXPERIMENTAL

The method of Cotterell and Reddel was applied to the measurement of the cracking resistance of 1 mm thick plates of aluminium alloy A6082, either in T6 condition (HB=102) or after an overaging treatment of 4h at 300°C (HB=44). (The overaged specimens will be denoted A6082 PH). The DENT specimen dimensions were 110x60x1 mm, with ligament sizes l ranging from 3 to 20 mm (fig. 1). A notch tip radius of about 30 µm was created by use of a razor blade. The displacement was measured by means of an extensometer with an initial gage length of 45 mm. For each specimen, the load-displacement curve ( F [N] versus d [m]) was recorded. The specific work of fracture, $w_f$ [J/m$^2$] is the area under this curve divided by the ligament area l * t. We denote

- ($F_{max}$,$d_{max}$) the values of (F,d) at the maximum of the curve
- $w_s$ [J/m$^2$] the area under the load-displacement curve from (0,0) to ($F_{max}$,$d_{max}$) divided by the ligament area
- $d_{final}$ [m], the elongation after completion of the cracking of the ligament.

## RESULTS

Figures 2 to 4 report the values of $F_{max}$, $d_{max}$, $d_{final}$, $w_f$ and $w_s$ as a function of ligament size, for the two series of specimens. The linearity of all the graphs is conspicuous. Various parameters can be extracted from these graphs by linear regression through the experimental points. Writing $F_{max} = \alpha$ l, we can derive the slope $\alpha$ from the graph ($F_{max}$ versus l). From the graphs ($d_{max}$ vs l), ($d_{final}$ vs l), ($w_f$ vs l) and ($w_s$ vs l), we extract the values of $\delta_m$, $\delta_{final}$, $w_e$ and $w_{es}$ by extrapolation to zero ligament size of the data points for $d_{max}$, $d_{final}$, $w_f$ and $w_s$, respectively. All these values are reported in table 1 for the two types of heat treatment of the alloy.

## DISCUSSION

The parameters listed in table 1 are not independent of one another. Indeed, $w_e$, and even the partitioning of $w_e$ into its two components $w_{e1}$ and $w_{e2}$, can be derived by use of the three parameters $\alpha$, $\delta_{max}$ and $\delta_{final}$. This may be demonstrated as follows.

First, we propose to identify $\delta_{max}$ with the elongation $\varepsilon_{ns}\Delta$ at the onset of necking. Indeed, for a vanishingly small ligament size, no work hardening can develop outside the process zone and the maximum of the load is thus observed when necking begins in the process zone. This hypothesis can be assessed if one assumes that the constitutive equation for the alloy obeys the usual power law

$$\sigma = K\,\varepsilon^n \qquad (5)$$

$\varepsilon_s$ should then be equal to the work hardening exponent n. From the stress - strain curve of an unnotched plate of alloy A6082 T6, we found n ≈ 0.11. Using this value of n with the data of table 1 brings $\Delta \approx 0.73$ mm. The fact that this value is similar to the thickness of the plate (t=1mm) is in agreement with the proposition of Mai and Cotterell [4].

Second, we follow the suggestion of Cotterell and Reddel [2] that $\delta_{final}$ must be interpreted as equal to $\delta_c$, the crack tip opening displacement. This assumption is used by various authors for the estimation of $\delta_c$ [for example, see 5-8].

A third assumption is necessary for making possible the integration of equation (4). For the first term, the power law (5) can be used. For the second term, we need the value of $\sigma_n(\delta)$. Following Mai and Cotterell [4], we can consider that $\sigma(\delta)$ is a constant : $\sigma(\delta) = \sigma_u$. $\sigma_u$ can be estimated from the relation

$$F_{max} = \frac{2}{\sqrt{3}} \sigma_u l t \qquad (6)$$

proposed by Hill [9],which yields

$$\sigma_u = \frac{\sqrt{3}}{2} \frac{\alpha}{t} \qquad (7)$$

Equation (4) can then be rewritten as

$$w_e = \Delta \left( \frac{n}{n+1} \right) \sigma_u + \sigma_{nu} \left( \delta_{final} - \delta_{max} \right) \qquad (8)$$

and we obtain finally

$$w_e = \frac{\ln\left(1 + \frac{\delta_{max}}{\Delta}\right)}{1 + \ln\left(1 + \frac{\delta_{max}}{\Delta}\right)} \Delta \frac{\sqrt{3}}{2} \frac{\alpha}{t} + \frac{\sqrt{3}}{2} \frac{\alpha}{t} \frac{\left(\delta_{final} - \delta_{max}\right)}{\left(1 + \frac{\delta_{max}}{\Delta}\right)} \qquad (9)$$

Except for $\Delta$ which can be approximated by t, this expression for $w_e$ contains only the three experimental parameters $\alpha$, $\delta_{max}$, and $\delta_{final}$. The values of $w_e$, $w_{e1}$ and $w_{e2}$ calculated using equation (9) are reported in table 1. It may be seen that these calculated values compare very well with the values of $w_e$, $w_{es}$ and $w_e$-$w_{es}$ measured experimentally by linear regression on the experimental graphs. Hence, the different assumptions bringing to equation (9) seem to be consistent.

## CONCLUSION

It is proposed that the fracture toughness of thin sheets may be characterized by three new parameters determined by tensile testing of DENT plates according to the method of Cotterell and Reddel [2]. The parameter $\alpha$ is related to the ultimate tensile stress. The parameter $\delta_{final}$ corresponds to the total straining of the process zone necessary for crack propagation. The parameter $\delta_{max}$ is directly dependent on the work-hardening rate. Moreover, the measurement of $\delta_{max}$ makes possible an intrinsic partitionning of the total essential work of fracture in two parts: the work for homogeneous straining of the process zone and the work for localised necking and decohesion along the crack path.

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TABLE 1

| | experimental values (figures 1 to 4) | | | | | | theory (eq. 9) | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\alpha$ [N/mm] | $\delta_{max}$ [μm] | $\delta_{final}$ [μm] | $w_e$ [kJ/m$^2$] | $w_s$ [kJ/m$^2$] | $w_e$-$w_s$ [kJ/m$^2$] | $w_e$ [kJ/m$^2$] | $w_{e1}$ [kJ/m$^2$] | $w_{e2}$ [kJ/m$^2$] |
| A 6082 T6 | 314 | 85 | 160 | 40 | 20 | 20 | 39.3 | 20.5 | 18.8 |
| A 6082 PH | 141 | 85 | 470 | 57 | 14 | 43 | 52.5 | 9.2 | 43.3 |

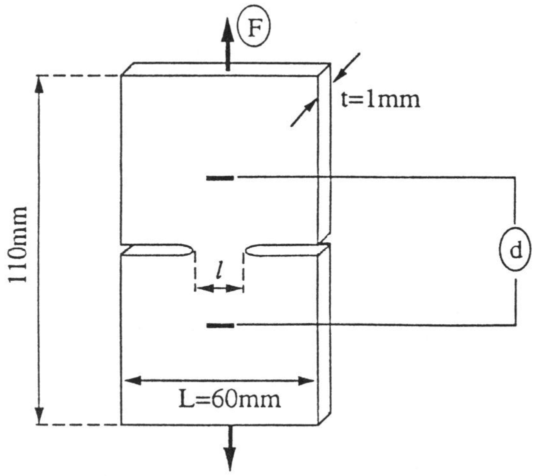


FIG 1 DENT specimen

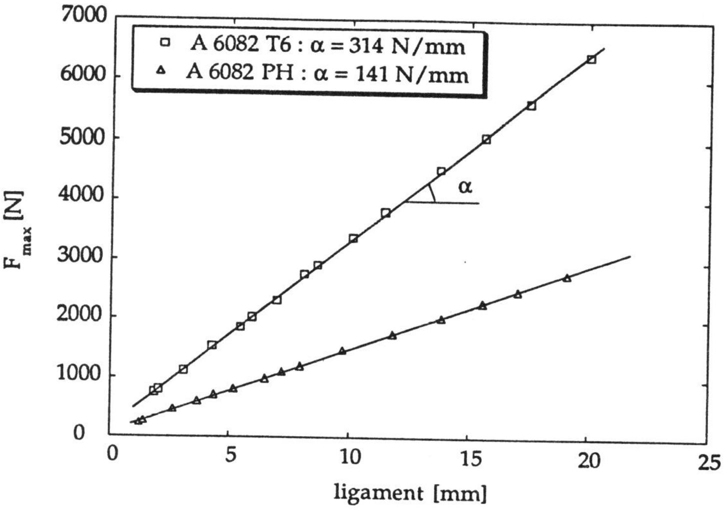


FIG 2 $F_{max}$ as a function of the ligament size; determination of the slope α

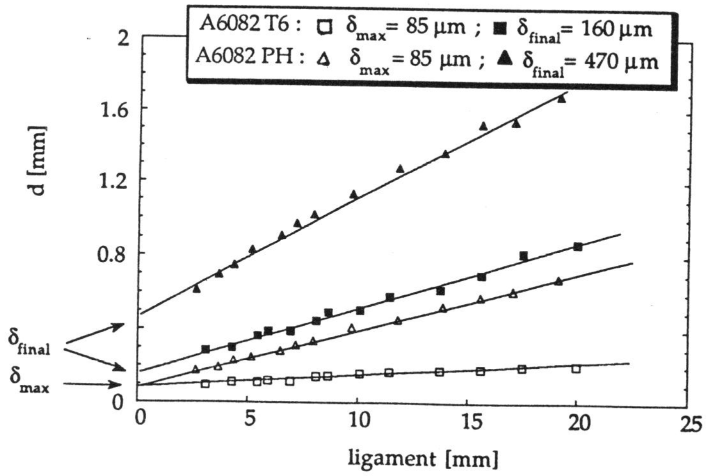


FIG 3 Elongations as functions of the ligament size: determination of $\delta_{max}$ and $\delta_{final}$

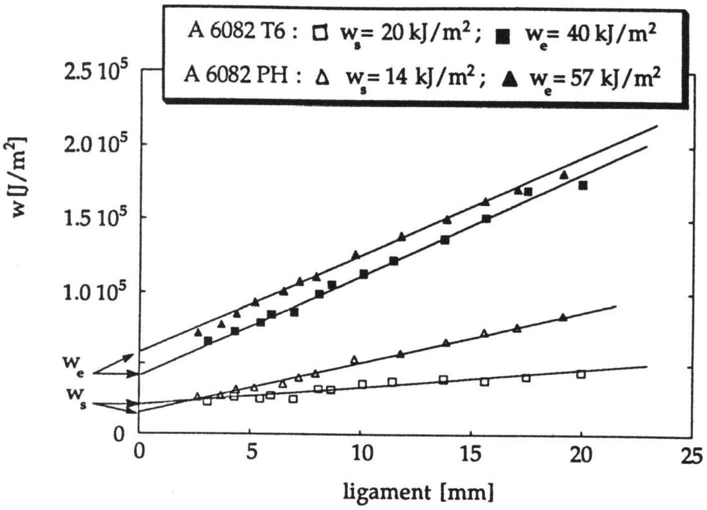


FIG 4 Specific works as functions of the ligament size; determination of $w_s$ and $w_e$