A NOVEL MODE-II FRACTURE CRITERION

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The initiation process of a shear or mixed mode loaded crack is investigated. Not only the stress concentration at the tip of the initiated kinked crack but also the stress concentration field that builds up at the additionally formed notch is found to influence the instability event. The strain energy release rate of the initiated crack is predicted on the basis of the true energy available for crack extension that results from the energy transfer between the stress concentration fields controlling the instability process. The predictions are verified by experimental data measured with shadow optical techniques in instability experiments.

INTRODUCTION

The fracture behaviour of shear or mixed mode loaded cracks has extensively been investigated over the last years. A practical fracture mechanics methodology, however, with test procedures and application rules - as it has been developed for mode-I conditions of loading - has not been established yet. The reason is that the instability behaviour of shear or mixed mode loaded cracks is much more complex than of tensile mode-I loaded cracks.

A tensile mode-I loaded crack, when it becomes unstable extends in its original direction, the geometry of the crack before instability is the same as the geometry after instability, i.e., during the process of instability the geometry of the crack remains self similar. A shear or mixed mode loaded crack, however, when it becomes unstable extends in a direction different from that of the original mother crack, the crack

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geometry before and after instability thus undergoes changes. Due to the geometrical self similarity that applies for the mode-I instability process the conditions that control the end stage of the mother crack just prior to instability are the same as the conditions that control the initial stage of the initiated crack just after instability. Since geometric similarity, however, does not apply for the instability process of shear or mixed mode loaded cracks, predictions on parameters that control the initial phase of an initiated crack just after instability from the conditions that control the end stage of the mother crack just prior to instability can only be made on the basis of additional assumptions or postulated criteria.

This paper considers the fracture behaviour of cracks initiated from shear or mixed mode loaded cracks. The stress concentration fields that control the initiation process are investigated and the energies transferred during this process are quantified. A criterion for predicting the strain energy release rate of the initiated kinked crack is established on the basis of the true energy available for crack extension. Experimental data on the actual fracture behaviour measured in instability experiments by means of the shadow optical method of caustics in combination with high speed photography verify the established criterion.

STRESS CONCENTRATION FIELDS INFLUENCING INSTABILITY

Model experiments are performed for getting an illustrative view on the development of the tensile mode-I stress field at the tip of an initiated kinked crack out of the original stress field around a purely shear mode-II loaded mother crack.

Crack tip stress fields can be made visible in the form of isochromatic fringe patterns in photoelasticity (1) or in the form of caustics by applying shadow optical techniques (2). Typical results for pure tensile mode-I and shear mode-II conditions of loading are schematically shown in Fig. 1. The isochromatic fringe pattern characterizes the whole stress field around the crack tip, the caustic curve gives only information on the stress distribution along a circular line (initial curve) of given radius around the crack tip. With caustics that apply for different radii of the initial curve, however, the stress field at various distances from the crack tip can be monitored.

The results of photoelastic studies on a shear mode-II loaded mother crack with an additional kinked crack (at 70° angle) of length a_z = 0/2/5 mm are shown in Fig. 2A. The stress field around the mother crack in the absence of an additional kinked crack (a_z = 0 mm) is of mode-II nature. The kinked crack model (a_z = 5 mm) shows a typical mode-I stress field at the tip of the kinked crack, as it is expected, but, furthermore, the photoelastic pattern shows a stress concentration that builds up at the notch which is formed between the kinked crack and the original mother crack. The pattern with a_z = 2 mm shows an intermediate stage of development. In the schematic representation of Fig. 2B the regions of dominance of these stress concentrations are graphically illustrated. With increasing length of the kinked crack both the mode-I stress field at the tip of the kinked crack and the stress concentration field at the additionally formed notch grow on the expense of the original mode-II stress field around the mother crack; only for large distances from the crack tip this mode-II stress field remains still valid.

Similar results are obtained with shadow optical investigations. For the model of a mother crack with a kinked crack of 2 mm length Fig. 3 shows caustic curves that were obtained for the same loading condition with radii of the initial curve chosen 0.6/1.5/5.0 mm. For a radius of the initial curve of 5.0 mm a mode-II caustic results around the tip of the former mother crack indicating the dominance of a mode-II stress field around this crack tip at this distance. For a radius of the initial curve of 0.6 mm a mode-I caustic results at the tip of the kinked crack indicating the dominance of a mode-I stress field around this crack tip at this distance; furthermore, a light concentration at the notch formed between the kinked crack and the mother crack is observed indicating a compressive stress concentration field at this notch (for details on the interpretation of caustic patterns for tensile and compressive notch tip stress concentration fields see (2)). For a radius of the initial curve of 1.5 mm a non analyzable caustic of a mixture of three stress concentration fields at the tip of the mother crack, at the tip of the kinked crack, and at the additionally formed notch is obtained.

The model investigations indicate: 1) The shear mode-II stress intensity field around a mother crack is transferred into a tensile mode-I stress intensity field at the tip of the initiated kinked crack, but, parts of the original shear mode-II field are also transferred into a compressive stress concentration field that builds up at the notch formed between the kinked crack and the mother crack. The validity regions of the stress concentration fields at the kinked crack and at the notch grow continuously with increasing length of the kinked crack at the expense of the validity region of the stress field around the original mother crack.

ENERGY CONSIDERATIONS ON STRESS CONCENTRATION FIELDS

Three stress concentrations are involved in the instability process of shear or mixed mode loaded cracks, i.e. the stress concentration at the tip of the original mother crack before instability and the stress concentrations at the tip of the initiated kinked crack and at the additionally formed notch after instability. The elastic energies stored in these three types of stress concentration fields have been quantified by analytical considerations. In order to allow for an easy comparison of the data all energy quantities are given in normalized form relative to the energy stored at the tip of a tensile mode-I loaded crack. The different singularities of the crack tip stress field ($r^{-0,5}$) and of the notch tip field ($r^{-0,415}$ for a notch formed by 70° crack extension) have been taken care of in the calculations; but, whereas exact expressions were derived for the crack tip energies only approximate relationships are given for the notch tip energy.

For an arbitrary mixed mode loaded crack $K_{MMI} \neq 0$, $K_{MMII} \neq 0$, including the special cases of pure tensile mode-mode-I loading, $K_{MMI} \equiv K_{PMI}$, $K_{MMII} \equiv 0$, and pure shear mode-II loading, $K_{MMI} \equiv 0$, $K_{MMII} \equiv K_{PMII}$, the following expressions hold for the crack tip energy U_C and the notch tip energy U_N (see reference (3))

$$\frac{U_{C,MM}}{U_{C,PMI}} = \left(K_{MMI}^2 + \frac{2\kappa+3}{2\kappa-1} K_{MMII}^2\right) \frac{1}{K_{PMI}^2} \quad \text{and} \quad \frac{U_{N,MM}}{U_{C,MM}} = \frac{1}{4}\left(\frac{K_{MMII}}{K_{MMI}+K_{MMII}}\right)^{5/2} \tag{1}$$

with $\kappa=(3-\nu)/(1+\nu)$ for plane stress, or $\kappa = 3-4\nu$ for plane strain, where

U_C = elastic energy stored around a crack
U_N = elastic energy stored around a notch that is formed by crack initiation under mixed mode loading conditions
K = stress intensity factor
$_{PMI\,,\,PMII}$ = subscripts characterizing a pure mode-I or a pure mode-II loading condition

$_{MM}$ = index characterizing mixed mode conditions of loading

$_{MMI}$, $_{MMII}$ = indizes characterizing the mode-I or the mode-II portion of a mixed mode loading condition

Figure 4 shows the dependence of the crack tip energy U_C and of the notch tip energy U_N on the ratio of mode mixity under the special assumption of equivalent conditions of loading expressed in terms of stress intensity factors, i.e.

$$\sqrt{K_{MMI}^2+K_{MMII}^2} = K_{PMI} = K_{PMII} \tag{2}$$

The amount of mode mixity in Fig. 4 is characterized by the ratio $K_{MMII}/(K_{MMI} + K_{MMII})$. The value 1 of this quantity represents pure mode-II loading, $K_{MMI} \equiv 0$, $K_{MMII} \equiv K_{PMII}$; the value 0 represents pure mode-I loading, $K_{MMII} \equiv 0$, $K_{MMI} \equiv K_{PMI}$. Data are given for the states of plane stress and plane strain and different values of Poisson's ratio.

It is recognized: 1) For a mode-II loaded crack the stored elastic energy is about a factor of 3 larger than the elastic energy stored at an equivalent mode-I loaded crack, under the assumption that $K_{PMI} = K_{PMII}$. 2) For a mode-II loaded crack the elastic energy stored at the notch that is additionally formed between the initiated kinked crack and the original mother crack is about ¼ of the elastic energy stored at the original mode-II loaded mother crack. 3) For mixed mode conditions of loading the quantities are accordingly lower, for a crack under pure mode-I loading the notch does not built up at all and the notch tip energy accordingly would of course be zero.

STRAIN ENERGY RELEASE RATE

From the above energy considerations the following conclusions can be drawn on the true energy available for crack extension: A mode-II loaded crack can provide much more energy for crack extension than a mode-I loaded crack (about three times more). Not all of this energy, however, is available for crack extension, i.e. is available for building up the stress concentration at the tip of the initiated kinked crack, about ¼ of the energy is lost for building up the compressive stress concentration field at the notch that is formed between the kinked crack and the original mother crack. Consequently, with eqs. (1) an expression for predicting the strain energy release rate of a crack initiated from a mother crack subjected to mixed mode conditions of loading, $(G_i)_{MM}$, can be obtained of the form

$$\frac{(G_i)_{MM}}{(G_i)_{PMI}} = \left\{1 - \frac{1}{4}\left[\frac{K_{MMII,c}}{K_{MMI,c}+K_{MMII,c}}\right]^{5/2}\right\}\left(K_{MMI,c}^2 + \frac{2\kappa+3}{2\kappa-1}K_{MMII,c}^2\right)\frac{1}{K_{PMI,c}^2} \qquad (3)$$

where c = subscript characterizing critical values of stress intensity factors of the original mother crack just prior to instability. With $(G_i)_{PMI,C} = [(1-\nu^2)/E](K_{PMI,C})^2$ (where $K_{PMI,C} \equiv K_{IC}$ according to the usual nomenclature) eq. (3) becomes (see reference (3))

$$(G_i)_{MM} = \frac{1-\nu^2}{E}\left\{1 - \frac{1}{4}\left[\frac{K_{MMII,c}}{K_{MMI,c}+K_{MMII,c}}\right]^{5/2}\right\}\left(K_{MMI,c}^2 + \frac{2\kappa+3}{2\kappa-1}K_{MMII,c}^2\right) \qquad (4)$$

This equation (4) represents an expression for determining the plane strain energy release rate of a crack initiated under shear or mixed mode conditions of loading. Different from other expressions derived by Irwin (4), Nuismer (5), Hussain et al. (6) expression (4) takes into account both the large amount of energy stored at a shear mode-II loaded crack but also the energy loss at the notch that is formed in addition to the initiated kinked crack.

CRACK INSTABILITY EXPERIMENTS

The actual fracture behaviour of cracks initiated under various mixed mode conditions of loading was experimentally investigated. The stress intensity factor and the velocity of the initiated kinked crack at short times after instability were determined by means of the shadow optical method of caustics in combination with high speed photography (2). Experiments were performed with specimens made from the model material Araldite B loaded quasi-statically in an Arcan-Richard-type mixed mode loading fixture (7,8) or they were loaded dynamically in a mixed mode impact loading arrangement developed by the authors (9).

Typical results obtained for a crack initiated under almost pure mode-II conditions of loading are shown in Fig. 5b. In this experiment, on purpose, a small amount of mode-I loading was superimposed to the dominating mode-II loading in order to avoid possible disturbances in the fracture behaviour due to friction effects between the fracture surfaces. Despite the small amount of superimposed mode-I loading the data of this experiment are referred to as "mode-II" data in the following text. The critical stress intensity factor at instability in this experiment $K_{MMII,C} \approx K_{PMII,C}$ ($\equiv K_{IIC}$ in usual nomen-clature) is 0.57 MN/m$^{3/2}$, $K_{MMI,C}$ is practically zero (see data points for t = 0). The figure shows the dynamic stress intensity factors, K_i, and the velocities, v, of the propagating initiated crack as functions of time after instability. In addition, the stress concentration factor of the compressive stress field at the additionally formed notch, C_N, is shown. Only the early time range (< 50 μs) after instability is considered that is not influenced by effects due to stress wave reflexions at the finite boundaries of the specimen. The "mode-II" results are compared to data shown in Fig. 5a for a crack initiated under pure mode-I conditions of loading, with $K_{PMI,C}$ ($\equiv K_{IC}$ in usual nomenclature) of 0.67 MN/m$^{3/2}$ (see data point for t = 0).

The following behaviour is recognized: Although the critical stress intensity factors at instability for the mode-II and for the mode-I experiment are (very roughly) similar, or, more precisely, although the critical stress intensity factor in the mode-II experiment was even somewhat lower than the critical stress intensity factor in the mode-I experiment, the stress intensity factors K_i and the crack propagation velocities v that were measured for the crack initiated under mode-II conditions of loading are considerably higher than those measured for the crack initiated under mode-I conditions of loading. This experimental finding is in qualitative agreement with the previous consideration on the true energy available for crack extension the derivation of expressions (3),(4) for predicting the strain energy release rate was based on.

For a further quantitative check on the validity of eqs. (3),(4) the results of all the experiments performed under static or impact conditions of loading were used: The dynamic stress intensity factors K_i measured with cracks initiated under mixed mode conditions of loading were transferred into strain energy release rates, $(G_i)_{MM}$, and these values were normalized by data obtained with pure mode-I experiments, $(G_i)_{PMI}$. This normalization, firstly, eliminates influences resulting from different critical stress intensity factors at instability that were observed for these different kinds of experiments, and, secondly, this normalization transfers the data into a format as used with eq. (3). These data were then compared with equivalent results theoretically predicted by means of eq. (3) from the critical stress intensity factors prior to instability determined in the experiments under consideration. Further predictions were made on the basis of the equivalent expressions by Irwin (4), Nuismer (5), and Hussain et al. (6). An overall comparison of the data is given in Fig. 6. The value 1 of the considered ratio $[(G_i)_{MM}/(G_i)_{PMI}]_{exp}/[(G_i)_{MM}/(G_i)_{PMI}]_{theor}$ represents agreement between the theoretically predicted and the experimentally measured strain energy release rates. It is seen that the criteria of Irwin, Nuismer, and Hussain are inadequate to predict the actual behaviour: The criteria of Irwin and Nuismer underestimate and the criterion of Hussain slightly overestimates the actual strain energy release rates. The derived expression (4), however, obviously correctly predicts the strain energy release rates for all ratios of load mixity.

SUMMARY AND CONCLUSIONS

For a crack initiated under shear or mixed mode conditions of loading the following characteristics of the fracture behaviour were found: 1) After instability not only the stress concentration at the initiated kinked crack has to be considered but also the compressive stress concentration field at the notch formed between the kinked crack and the original mother crack. 2) The elastic energy stored at the tip of a mode-II loaded crack is about a factor of 3 larger than of an equivalent ($K_I = K_{II}$) mode-I loaded crack. Not all of this energy is available for crack extension, however, about ¼ of this energy is lost for building up the stress concentration at the additionally formed notch. On the basis of the true energy available for crack extension a criterion and formulae for predicting strain energy release rates of cracks initiated under shear or mixed mode conditions of loading are derived. The criterion is verified by experimental data measured in instability experiments by means of the shadow optical method of caustics in combination with high speed photography.

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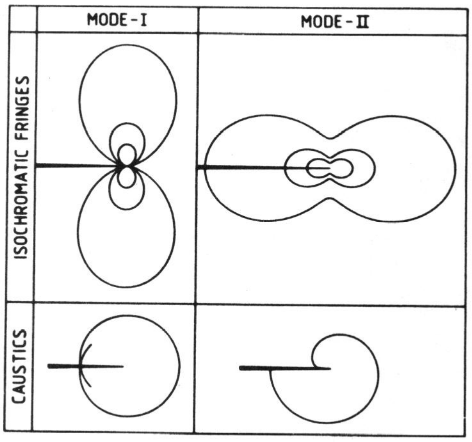

Figure 1 Visualization of crack tip stress fields

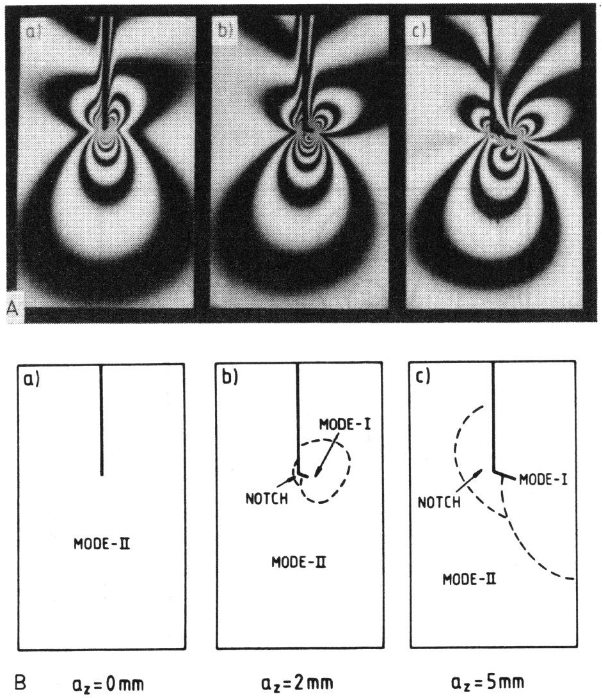

Figure 2 Photoelastic study, A) isochromatic fringe patterns, B) zones of dominance (schematically)

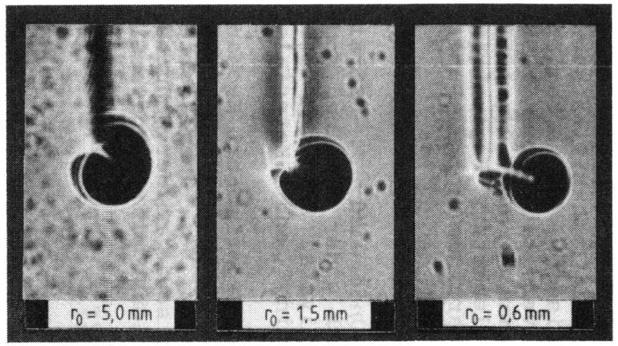

Figure 3 Caustic study

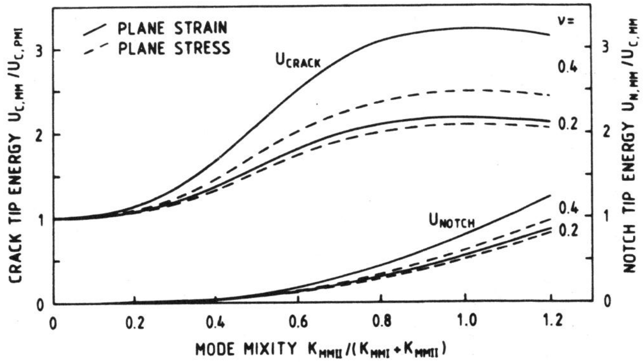

Figure 4 Energies of crack tip and notch tip stress fields

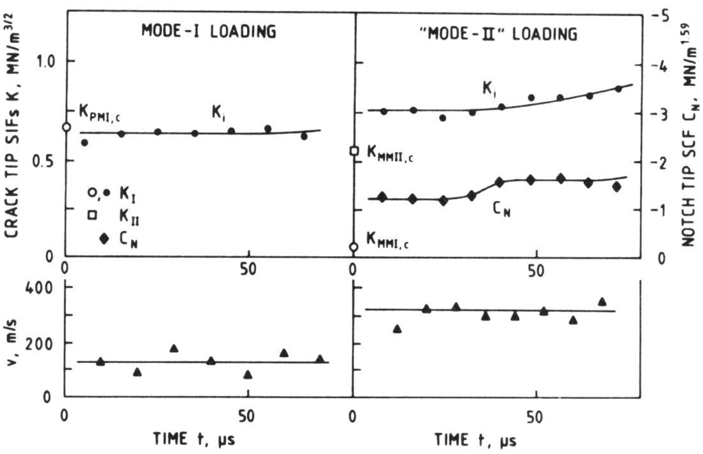

Figure 5 Fracture behaviour of initiated crack

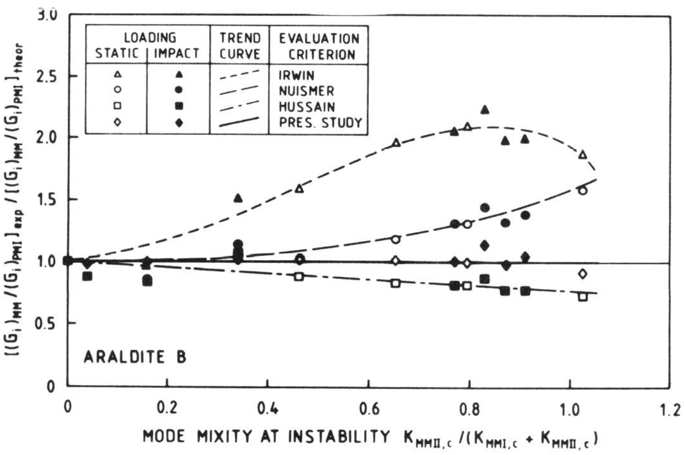

Figure 6 Comparison of theoretically predicted and experimentally measured strain energy release rates, normalized