DISSIPATION RATE AND CTOA: THE PROBLEM OF TRANSFER OF R-CURVES TO CONTAINED YIELD

O. Kolednik* and C.E.Turner**.

The equations for energy dissipation rate during stable growth in rep material are set out. The crack tip opening angle (CTOA) model for fully plastic ductile crack growth is shown to be consistent with them. A suitably defined J-type R-curve can then be expressed in terms of CTOA. Models for growth under quasi-lefm conditons of well contained yield are examined in relation to selected experimental and computed J-R-curve data for a low and high strength steel. A model is proposed for the carry over of certain fully plastic CTOA test data to contained yield. It is concluded that this is quite plausible although no experimental data are known to substantiate it.

INTRODUCTION

J-R-curves are used to describe toughness as a function of crack growth. For amounts of growth large in relation to the ligament several different patterns of variation with width and size have been summarised by Turner, [1]. A physical meaning for different amounts of yielding was described by Kolednik, [2]. The CTOA is also widely used as a plausible criterion for ductile crack advance. Recently, the energy dissipation rate, D, and the corresponding driving force, C, have been introduced as the energy per unit area of crack advance associated with combined events of plasticity and fracture. The advantage is that, neglecting residual stress effects, these terms describe the behaviour of real elastic - plastic (rep) material. The relation C = D gives a necessary condition for equilibrium crack growth. The relation of D to CTOA provides a criterion for growth. The object of this paper is to apply

* Eric-Schmid-Institut, Österreichische Akademie der Wissenschaften, Leoben.
** Department of Mechanical Engineering, Imperial College, London.

these arguments to use with large amounts of growth in contained yield.

THE ENERGY DISSIPATION RATE MODEL

The basic equations have been developedby the Authors, [3][4], from both a macro- and micro-viewpoint. In brief, the crack driving force, C, comes from both external work, U and recoverable internal energy, w_{el} (where $w_{el} = Qq_{el}/2$; Q is load, q is displacement and sub el implies the elastic component). The dissipation is associated with both irrecoverable plastic energy, w_{pl}, and the energy for fracture, Γ. Thus, for crack of length, a, in a two dimensional body of thickness, B, advancing in a stable or equilibrium manner,

$$C = d(U-w_{el})/Bda = D = d(w_{pl} + \Gamma)/Bda \quad (1)$$

For strict lefm, $G = 2\gamma_o Bda$, where $2\gamma_o$ is the classical surface energy. For engineering lefm, $\Gamma = 2\gamma_{eff}Bda$, the Irwin-Orowan concept of effective surface energy. For epfm as often applied in terms of either J or COD to uncontained yield, several possible meanings for Γ were discussed in [3]. As Irwin and Orowan pointed out, for engineering metals the true surface energy is second order compared to the effective energy, implying the value of $2\gamma_o$ can be neglected in any study of structural cases made in terms of engineering lefm or epfm.

In lefm, it has been shown that C is identical in value to G. In rigid-plastic material C is the external work rate, as required by Rice et al, [5]. In rep material, C may be smaller or larger that dU/Bda, according to whether dw_{el} is increasing or decreasing with growth, broadly identifiable for growth with rising or reducing load respectively. Data were illustrated for all these cases in [3] and in [4] applications were made to R-curves and instability. Examples of D for fully plastic behaviour are given Fig.1 for a low and a high stregth steel.

Components of energy dissipation rate. In [4], most attention was focused on applications of D to the fully plastic state where many test data are gathered. For that case, a steady state of growth exists after a small transient regime immediately following initiation, extending for perhaps 0.5 to 1mm growth or perhaps for 5% to 10% of the ligament. Thereafter, up to the limits tested of some 60% growth, D is the sum of an areal dissipation, γ, and (mean) volumetric terms, τ and ρ, called the specific intensities of rate of energy dissipation, SIRED.

$$D = \gamma + \tau s^2/B + \rho\pi b_c \tag{2}$$

The first term, γ, clearly relates to an effective surface energy, identifiable at least in engineering lefm with the Irwin-Orowan term, γ_{eff}. The second, τ, is an energy per unit volume associated with the formation of shear-lips of size, s, represented adequately by $s = xb_o$ where b_o is the original ligament width and x a fraction, such as 0.1 for a certain titanium alloy, Turner and Braga, [6], and 0.2 for HY130 steel, Dagbasi and Turner, [7]. For side grooved pieces the τ term is of course absent. The third term, ρ, not foreseen in [1], is an energy per unit volume associated with general plasticity, i.e. hinge formation for deep notch bend (DNB) tests. It was originally written, [6], in schematic form as just ρb_c where b_c is the current ligament size. It is now stated as $\rho\pi b_c$, implying that for each increment of growth, da, a plastic hinge re-forms, of width da and roughly πb_c in circumference (for DNB).

A Relationship With CTOA

Experimental data, on a titanium alloy in [6] and HY130 steel in [7], shows the areal term γ in Eqn.2 to be comparable to the elastic energy term G. Here, it is identifed with it and also with Γ in Eqn.1. For simplicity, the relationship is stated for the fully yielded case with shear-lips supposed negligible, so that

$$\gamma = G; \qquad \rho\pi b_c = dw_{pl}/Bda = Qdq_{pl}/Bda = L\sigma_o b_c^2 dq_{pl}/Sda \tag{3}$$

where L is the classic plastic constraint factor on load, about 1.36 for DNB tests in plane strain, neglecting work hardening. For that case, the CTOA, α, was defined in [6] in a global sense which may not be the same as from a local measurement. Using the rotational factor, r,

$$dq/da = S\alpha/4rb_c\ ; \qquad \alpha = \delta_t/da \tag{4}$$

where S is span and rb_c defines an apparent centre of rotation. The CTOA is formed step by step from the stretch, δ_t, of the micro-ligament between the tip and the next micro-void. It was shown, [6], that applying Eqn.4 to q or to just q_{pl} affected the value α/r by only about 5 per cent. However, since an actual crack tip opening angle exists for that increment only after the micro-ligament has been severed, Eqn.4 is not formed by differentiating an expression for CTOD. This concept of CTOA is here seen as inherently plastic; an elastic CTOA at the crack tip (as opposed to some arbitrary angle subtended there) is not compatible with Eqn.4. It was later realised by the same authors, [8], that since dq/da is an incremental term, the distance to the instantaneous centre of rotation should be used at each step. These centres were found by rigid-plastic analysis of q_{pl} and mouth opening, V_{pl}. After a small transient regime they occurred at a distance r^*b_c ahead of the tip and -c into the 'other half' of the piece. This implies that δ_t is formed not only by plastic hinge rotation but also from a direct opening effect. In the two sets of experimental data analysed, from [6] and [7], $r^* \approx 0.9$, constant with growth with c small, of the order of the COD but proportional to Δa. An example of data for CTOA is given Fig.2 where q_{pl} is plotted against $\ln b_c$ to give a line of slope, now taken as $S\alpha_{pl}/4r^*$.

In confirmation of this analysis Fig.3 shows some computed 2D finite element data for DNB pieces of HY130 steel, with up to 60% growth, in which the local CTOA is formed by imposing a constant δ_t at one element from the tip; a collapsed node structure was not used.The estimate of global CTOA from Eqn.4 agrees with the local imposed CTOA, provided the elastic component of opening associated with the nodal distance behind the tip is recognised and the r* value is used rather than r. Using that meaning of Eqn.4 in Eqn.3 then gives for the steady state regime of large growth

$$\rho \approx L\sigma_o\alpha_{pl}/4r^*\pi \quad (5)$$

Thus the CTOA model for ductile crack extension is seen not just as a plausible micro- mechanism for growth but one which satisfies the conservation of energy in rep material.

A related R-curve. Despite the origin of J as a contour integral, all the forms used in experimental work are based on the area, A, under the load-displacement curve, which is the work done

$$J = \eta A/Bb \quad (6)$$

where η is a geometric factor, closely 2 for DNB cases. Eqn.6 is often split into elastic and plastic parts but for the present DNB examples, that need not be done. dJ may be formed by differentiation of Eqn.6, giving rise to a term in db/da, or in the increment, in which case that term does not occur. In the following, all such are referred to collectively as J^A curves despite the differences between the several forms. An alternative is to write without the η factor in dJ, as Turner did in [9],

$$J_{red} = J_i + \Sigma dJ_{red} = J_i + \Sigma D da/b \quad (7)$$

since after initiation the factor is not required in forming an increment in energy dissipated. For no shear-lips, Eqn.5 can then be used to give

$$J_{red} = J_i + \Sigma(\gamma + \rho\pi b_c)da/b = J_i + \Sigma(G da/b + m^*\sigma_o\alpha_{pl} da) \quad (8)$$

$$m^* = Lb_c/4r^*b \quad (9)$$

There is an uncertainty on whether b in the denominator of Eqns.8 and 9 should be taken as b_o, as in [9], or as b_c. It is remarked without discussion that, with hardening, L rises appreciably so that $Lb_c/b_o \approx$ constant. In Eqn.8, dJ is in the form of the conventional term

$$J = m\sigma_o\delta; \quad dJ/da = m\sigma_o d\delta/da \tag{10}$$

but $m \neq m^*$ and α is formed from Eqns.4, not from $d\delta/da$.

Contained Yield

Parameters used for contained yield are denoted by sub cy. The term G written as G_{pzc}, implies the Irwin plastic zone correction (pzc) has been used to update the strict lefm value. It is generally agreed that such a correction gives a working approximation to J up to about 0.8 of the fully plastic load. For stable growth in engineering lefm, the driving force is

$$C_{cy} = (dU - dw_{el})Bda \approx G_{pzc} \tag{11}$$

It is therefore clear that the Irwin-Orowan concept, if identified with G_{pzc}, is the energy dissipation rate within the whole plastic zone rather than just a fracture process zone. In short, the energy rate balance for growth, notionally at G_{pzc} in contained yield, might be written as a dissipation over the swept volume at each increment, $2r_pBda$, to give

$$C_{cy} \approx G_{pzc} = 2\rho_{cy}r_p = 2\gamma_{eff} = D_{cy} \tag{12}$$

By their nature, ρ in Eqn.2 and ρ_{cy} in Eqn.12 are some multiple, μ, of yield stress and average strain in the plastic region, perhaps of the order of 10 to 100, so that

$$\rho \approx \mu\sigma_o\varepsilon_o \tag{13}$$

The value of μ may differ as the physical picture changes from a plastic volume of an 'eye' to an 'eye' plus slip lines that, as full yield is approached, spread across the net section.

The writers are not aware of experimental studies of crack tip details for appreciable growth in contained yield giving flat fracture, so turn to the contrast between [2], where growth at constant G was discussed, and recent finite element modelling by Shan et al, [10], where growth is at constant CTOA. Studies mainly in full yield were extended to contained and small scale yield (ssy) by increasing the size of CT pieces from B = W = 50mm for full yield to 200 and 800mm using a low strength steel ST37, σ_o = 298MN/m^2, tensile stregth = 426MN/m^2. For each size, three values of b_o/W were used, 0.3, 0.46 and 0.6. An initiation COD of 0.073mm was followed by 3mm growth at a step size of 0.1mm using a constant CTOA of 0.3rad for all cases. The R-curves for a 2D plane strain model, defined by J^A, showed dJ/da at W = 800mm was practically constant with growth, reducing very slightly from about 95 to $90MN/m^2$ as b_o/W was increased from 0.3 to 0.6. These values are 10-15% lower than in full yield where a 10% 'wider-lower' effect was seen. Clearly, the toughness defined by constant CTOA is giving a near constant increase in toughness, dJ^A/da, rather than a constant J ($\approx G_{pzc}$) for this assumed model of growth in small scale yielding. This behaviour for growth in ssy at constant CTOA is very different from that discussed in [2], where growth at constant G is equivalent to $dJ = \Gamma/Bb_o$, giving an inverse proportionality with initial width.

Here, using A for the area as in Eqn.6, and with stable growth at $G = G_R$,

$$G = \eta w_{el}/Bb = G_R; \qquad dA = dU = dw_{el} + BG_R da \tag{14}$$

$$dG = \eta dA/Bb - G(\eta - 1)da/b \tag{15}$$

The values in the computed data of [10] shows that J ($\approx$ G) after 3mm growth exceeds the initial J ($\approx$ G) by some six fold. The near independence of dJ^A/da on b in [10] must therefore follow from the dominance of the dA term in Eqn.15 over the G term, with the increment of work, dA, proportional to the volume, consistent with the very large CTOA of the ST37 steel. In that case, for dimensional reasons dJ^A/da will not be dependent on size and the fracture process has a trivial effect on the work done, the reverse of the behaviour discussed in [2].

DISCUSSION AND SPECULATION

The conventional lefm picture is that the source of crack extension, by growth or by cutting a longer initial crack, does not matter. But is it to be argued that the profile will have the same shape for both a 'grown longer' and 'cut longer' crack? For the fully plastic case it is well recorded that the opening at the tip of an advancing crack, δ_t, is much less than the initial opening, δ_i, so surely in contained yield the tip opening $\delta_{t,cy}$ cannot be comparable to δ_i as a conventional interpretation $G = m\sigma_o\delta$, would imply ? It could be imagined that for a much less tough material than ST37, the dA term in Eqn.15 might be comparable to the G term, in which case dJ^A/da would decrease as size, b, increased within contained yield. In the limit, if dG were negligible compared to G, growth at constant G would be obtained, as in [2].

In [5], two extremes of behaviour seen for micro-ductile crack growth, a recognisable CTOA for very ductile materials and an ill defined quasi-elastic opening for rather inductile materials. Values of tearing modulus, $T = (E/\sigma_o^2)(dJ/da)$, of perhaps 200 are suggested to define the former and 20 for the latter. It so happens, these values are representative respectively of the ST37 and HY130 steels already instanced. But should the CTOA value, the dJ/da value or even the G value be carried over from full yield to contained yield cases?
In the lack of data for comparable tests in both contained and full yield, a model is proposed that, though expressed as CTOA, is essentially the final tip opening stretch, δ_t, as given by Wnuk using a modified Dugdale model, [11]. In full yield there is direct experimental data, [12-14], that δ_t /da is maintained constant with growth as a steady state CTOA for large distances behind the tip. It has already been remarked that in full yield there is a transient regime of high CTOA before the steady state growth described by Eqns.2 and 4 becomes established, as shown in the upper diagram of Fig.4. It is now suggested that it is this region, transient in full yield, that is carried forward in the steady state for contained yield, as shown in the lower diagram of Fig.4, where two limiting interpretations arise. If the CTOA is rather small, the same steady state CTOA as for full yield, α_{ss}, can be established across the plastic zone; if the CTOA is high, the value can only apply to the actual tip with some much lower angle extending back across the plastic zone. In both cases, the stretch, δ_t, once formed, is at each subsequent step enlarged until, with sufficient growth, it finally attains the value $\delta = G/m\sigma_o \approx \delta_i$.

A simple estimate is made for the limiting CTOA value possible within an lefm model, using $2v_{180}$, the lefm displacement 'behind' the crack tip at a distance $r = r_p$, and the (unknown) tip opening, δ_t. Allowing δ_t to be zero for low T material, e.g. HT130, and δ_i for high T material, e.g. ST37,

$$\alpha_{cy} \approx 2(v_{180} - \delta_t)/r_p \qquad (16)$$

The values are $\alpha_{cy} \approx 0.073$ and $\alpha_{cy} \approx 0.007$ radians for low and high T cases repectively. The former is about half the HY130 values in full yield but not full plane strain, from [7], Fig.2. The latter is about one-fortieth of the mid-section values for CT pieces of ST37 in [10]. This implies the second interpretation given above must apply by a very large margin for ST37 but the first interpretation is perhaps nearly possible for full plane strain behaviour of HY130.

Transfer of CTOA from full yield cases to contained yield therefore seems plausible where the constraint at the tip is similar; a high or low value of CTOA can be accepted in lefm as just shown. It cannot be demonstrated that such transfer is correct, without actual data. One argument against it is that the local strain fields differ for the two cases; it is reasoned that the two interpretations of Fig.4 accomodates that difference. An uncertainty is the different values of CTOA given by different methods, particularly the inference from global data as in Eqn.4 and the measurement from local data by infiltration in [12][13] or photogrammetry in [14].

An estimate for dJ/da in contained yield can now be attempted from the CTOA in full yield, itself best found with $B = b_o$ to avoid width effects and also in full thickness or with side grooves. The term will be dJ_{red}/da, using Eqns.8 and 9. Taking the data assumed for the computations of Fig.3, $m^* \approx 1.8/4(0.9)$ with $\alpha_{pl} = 0.052$rad giving $dJ_{red}/da \approx 25MN/m^2$. The experimental data analysed as in Fig.2 (where $\alpha_{pl} \approx 0.14$ rad. and is not in full plane strain) gives $dJ_{red}/da \approx 60MN/m^2$. For ST37 the L and r* values are not known; supposing $dJ_{red}/da \approx (dJ^A/da)/2$ to allow for the omission of η then for $\alpha = 0.3$ and $dJ^A/da \approx 110MN/m^2$ as in [10], agreement in full yield is found for an increase in L of about 15% and a similar reduction in r* over the HY130 data, plausible for the higher work hardening, but uncertain. As ssy is approached some change in the value of L/4r*, with L now just a scale factor and r* re-evaluated, may well be required.

CONCLUSIONS

The energy dissipation rate, D, is meaningful for growth in strict lefm (where it is constant) to engineering lefm (where it increases) to fully plastic behaviour (where it decreases).

The CTOA criterion for fully plastic ductile crack growth in flat fracture has been shown to be consistent with the energy dissipation rate model for rep material. After a small transient stage both attain a steady state condition.

No single definition of present cumulative J-type R-curves will encompass the several different meanings that have been attached to crack resistance, ranging from energy release rate when G controls growth to normalised accumulated energy when CTOA controls growth. A particular term J_{red} is formed to embrace all the meanings.

Transferance of high constraint fully plastic R-curve data to contained yield is plausible via CTOA and dJ_{red}/da but no experimental data are known to confirm or refute this model.

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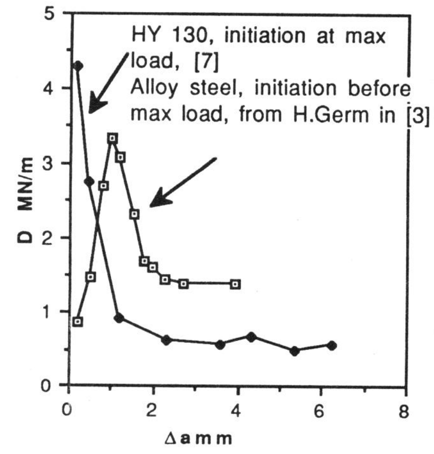

Fig.1 Energy dissipation rate - v - crack growth. Fully plastic examples.

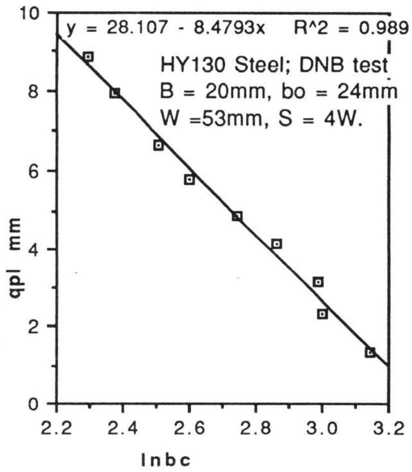

Fig.2 Plastic displacement - v - ln(current ligament); $\alpha_{pl} \approx 0.14$

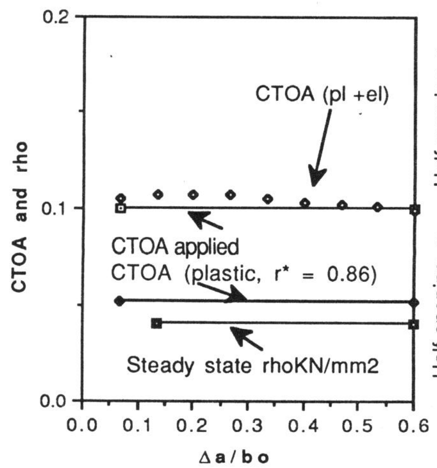

Fig.3 CTOA and ρ for fully plastic case HY130, plane strain DNB, CTOA = 0.1

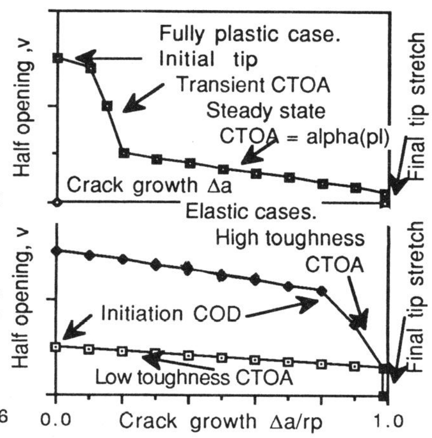

Fig.4 CTOA model for growth; upper,-v- Δa, fully plastic; lower, -v- $\Delta a/r_p$, lefm.