Determination of crack propagation direction using approach based on generalized linear elastic fracture

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ABSTRACT. *The objective of the paper is to investigate the direction of a further crack propagation from the interface between two elastic materials. The angle of crack propagation changes when the crack passes the interface, due to different material properties of layers. The suggested procedure makes it possible to estimate an angle of crack propagation under which the crack will propagate into the second material. The assumptions of (generalized) linear elastic fracture mechanics, which takes into account stress singularity exponent different from 1/2 are considered. The finite element method was used for numerical calculations. As an example application of the suggested approach on crack propagation through laminar ceramics (residual stresses in individual layers are taken into account) is introduced and results obtained compared with experimental data. The results obtained might contribute to a better understanding of the failure of materials with interfaces (e.g. layered composites, materials with protective coatings) and to a more reliable estimation of the service life of such structures.*

INTRODUCTION

The paper presented is focused on study of behaviour of cracks especially in ceramic layered composites. By layering of different materials can be achieved often contradictory requirements on the properties of structural materials, and therefore are these materials increasingly used in practice. For example, layering of different types of ceramics is one of the ways to increase the fracture toughness and reliability of structural ceramics, functional without a decrease in other mechanical properties (e.g. hardness or wear resistance). For this reason, many research teams trying to develop ceramics/ceramics or ceramics/metal composites convenient for industrial production in large quantities.

The increase in fracture toughness of this type of composite material is well documented in many publications. The paper is especially devoted to the crack

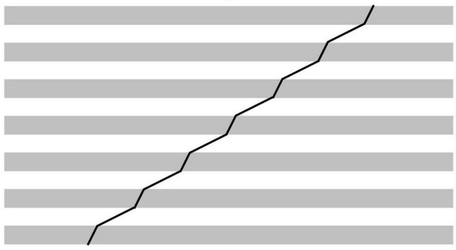

Figure 1. Scheme of stepwise crack propagation through ceramic laminate

propagation and toughening mechanism in Al_2O_3 – ZrO_2 ceramic laminates. Results from experimental measurements on mentioned kind of material can be find e.g. in [1-17]. These works point out important increase of fracture toughness of ceramic laminates in comparison with homogenous ceramics. One from the reasons of this effect is so-called stepwise mechanism of crack propagation through layers of the laminate (see Fig. 1). Behaviour of the composite and its fracture is not so brittle like in the case of homogeneous ceramics due to stepwise crack advance through individual layers. The stepwise crack propagation is connected with change of crack propagation direction at (or close to) material interfaces. The knowledge of the change of crack propagation direction at each interface is necessary for estimation of fracture properties of the layered ceramic composite. The reason for change of crack propagation at interfaces are strong residual stresses developed during manufacturing of the composite (by cooling from sintering temperature due to different coefficients of thermal expansion) and different elastic properties of applied materials.

A crack propagation in brittle materials can be generally described by linear elastic fracture mechanics (LEFM). However, in studied case the classical LEFM cannot be used due to change of stress singularity exponent of crack touching the interface between two materials (it is important configuration for description of crack propagation).

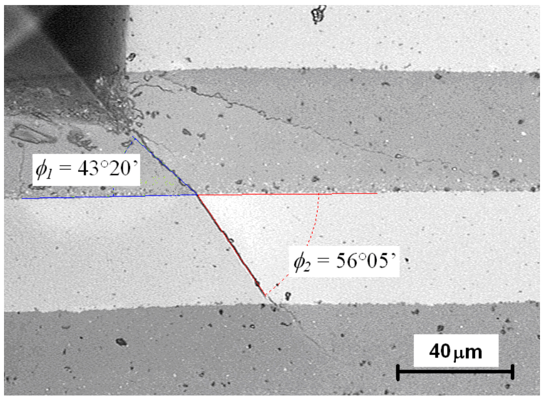

Figure 2. Change of crack propagation direction at interface between two materials. The crack was induced by indentation – by courtesy of H. Hadraba

In homogeneous material the stress singularity is of type $\sigma \approx r^{-0.5}$ [18]. In the case of crack touching the interface is of type $\sigma \approx r^{-p}$, where p is stress singularity exponent. The stress singularity exponent depends on elastic mismatch and takes values $0 < p < 1$. Due to that classical approaches of LEFM for estimation of crack propagation direction cannot be used and special procedures are needed.

When classical LEFM fails in studied case the procedures of so-called generalized LEFM, e.g. [19-21] can be used. For example generalized form of Sih's strain energy density factor (SED) criterion [22] is described and applied for estimation of crack propagation direction in mentioned works. The crack propagation direction is given by expression:

$$\left(\frac{\partial A_{11}}{\partial\theta}+2\frac{H_2}{H_1}\frac{\partial A_{12}}{\partial\theta}+\left(\frac{H_2}{H_1}\right)^2\frac{\partial A_{22}}{\partial\theta}\right)_{\theta_m}=0\,, \tag{1}$$

where H_1, H_2 are generalized stress intensity factors [MPa.m^p] and A_{11}, A_{12}, A_{22} are known functions (see e.g. [20] for details). In comparison to classical LEFM quantities H_1, H_2 don't belong explicitly to crack loading modes, but contain contributions of both (normal and shear) modes. Functions A_{11}, A_{12}, A_{22} depend on radial distance r from the stress concentrator (crack) tip. Distance for determination of crack propagation direction depends on material properties of material where the crack propagates to what is disadvantage of this criterion.

Table 1. Observed angles of initial crack orientation relative to the interface and direction of futher crack propagation (average experimental values)

Al_2O_3		ZrO_2	
ϕ_1 [deg]	ϕ_2 [deg]	ϕ_1 [deg]	ϕ_2 [deg]
52.5	60.4	58.0	43.9
73.0	77.5	64.5	53.4
82.5	87.0	78.5	72.0

Table 2. Material characteristics of individual components of the laminate [6,17]

material characteristics	component	
	Al_2O_3	ZrO_2
Young modulus [MPa]	$3.8.10^5$	$2,1.10^5$
Poisson's ratio [-]	0.26	0.31
CTE [K^{-1}]	$8.5x10^{-6}$	$10.3x10^{-6}$

EXPERIMENTAL DATA

As a base for determination of crack propagation in layered ceramics the experimental results published in [8,17] were used. The experimental samples were of size 2 x 2.5 x 25 mm (width x height x length) prepared by electrophoretic deposition. Samples contained 59 layers of alumina (Al_2O_3 a mean particle size ~ 400nm) and zirconia (ZrO_2 a mean particle size ~ 150nm) with thickness of 42 μm. The change of direction at interfaces of propagating crack induced by Vickers indentation was observed (Fig. 2). Selected data

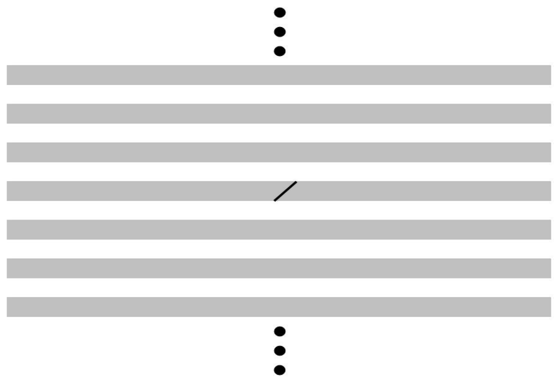

Figure 3. Scheme of numerical model: layered ceramics with initial internal crack with tips at material interfaces

from experimental measurements are shown in table 1. The angle ϕ_1 represents initial crack orientation relative to the material interface. The angle ϕ_2 represents direction of next crack propagation when the crack has passed the bimaterial interface, see Fig. 2.

NUMERICAL MODEL

Crack propagation in the ceramic laminate was modelled by means of finite element method (system Ansys was used). The scheme of the numerical model is shown in the Fig. 3. The 2D (conditions of plane stress approximation were considered) model contained initial internal crack with tips touching the material interfaces.

The presence of initial crack was considered in both alumina and zirconia layers. Numerical calculations were performed for both initial conditions. The crack initial orientations were taken from table 1. Material characteristics used in calculations were found in the experimental works [6,17] and summarized in the table 2.

A solution of fracture mechanics problems needs special mesh with high density of elements around the crack tip. It was reason for reduction of material layers in numerical model. The reduction did not influence the results obtained.

DETERMINATION OF CRACK PROPAGATION DIRECTION

For determination of crack propagation direction the procedure based on combination of numerical and analytical solution was used. The change of crack propagation direction can be under conditions of LEFM determined from the expression [23]:

$$\tan\alpha = \delta_{II} / \delta_{I}, \tag{2}$$

where δ_I, δ_{II} are displacements at the crack tip related to the mode I and II of loading (see Fig. 4), α is deviation angle from initial crack direction (see Fig. 5). Expression (2) can be for homogeneous body written in the form [24]:

$$\tan\alpha = K_{II} / K_{I}, \tag{3}$$

where K_I, K_{II} are stress intensity factors corresponding to mode I and II of loading. Relation (3) can be with good approximation used for estimation of crack propagation direction in homogeneous bodies. In the case of crack touching the interface between two materials it is possible to use modified relation (3). Taking into account the change of stress singularity in this case the relation holds:

$$\tan\alpha = H_{II} / H_{I}, \tag{4}$$

where H_I, H_{II} are generalized stress intensity factors for mode I and II of loading. It should be noticed here that quantities H_I, H_{II} don't correspond to that written in the expression (1), because they don't exactly correspond to analytical solution of the problem of crack touching the interface between two materials in the sense of references [24-26]. Quantities H_I, H_{II} express magnitude of normal and shear mode of loading respectively for crack with stress singularity different from ½ and for polar coordinate $\theta = 0$.

In the case of general stress concentrators (where crack touching the bimaterial interface belongs) it is not easy to separate individual modes of loading like in the case of a crack in homogeneous body. This fact complicates estimation of crack propagation direction after the crack passes the bimaterial interface. However, components belonging to normal mode of loading and shear mode of loading can be separated at least in special case.

On the base of numerical solution of the problem the normal and shear stress components can be obtained for $\theta = 0$ in dependence on radial distance from the crack tip ($\sigma_{\theta\theta}\left(r,\theta=0\right)$, $\sigma_{r\theta}\left(r,\theta=0\right)$):

$$\sigma_{\theta\theta} = \frac{H_I}{r^{p_I}} \cdot f_I\left(\theta = 0\right) \tag{5}$$

$$\sigma_{r\theta} = \frac{H_{II}}{r^{p_{II}}} \cdot f_{II}\left(\theta = 0\right) \tag{6}$$

In the relations (5) and (6) p_I and p_{II} are stress singularity exponents of stress components $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ under condition $\theta = 0$. Mentioned approach is formally possible, for $\theta = 0$ stress component $\sigma_{\theta\theta}$ contains even terms only (cosine terms) corresponding to mode I of loading (analogy with homogeneous case) and similarly the stress component $\sigma_{r\theta}$ contains odd terms only (sine terms) corresponding to mode II of loading for $\theta = 0$.

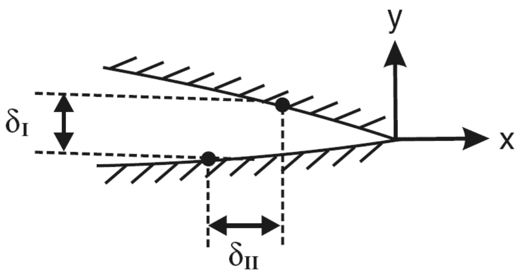

Figure 4. Displacements δ_I, δ_{II} at the crack tip

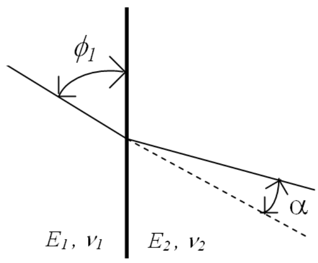

Figure 5. Scheme of crack propagation after its pass through bimaterial interface

p_I and p_{II} values can be determined from equation (5) and (6) respectively by logarithm of numerically obtained stress distribution ahead of the crack tip:

$$\log \sigma_{\theta\theta} = -p_I \cdot \log r + \log\left(H_I \cdot f_I\left(\theta = 0\right)\right) \tag{7}$$

$$\log \sigma_{r\theta} = -p_{II} \cdot \log r + \log\left(H_{II} \cdot f_{II}\left(\theta = 0\right)\right) \tag{8}$$

When p_I and p_{II} are known the functions f_i (i = I, II) can be determined as linear combination of harmonic functions. The procedure of their determination is similar that for homogeneous body. Harmonic terms of stress expansion are derived from boundary conditions (assuming free, unloaded crack faces and continuity of corresponding stress components and displacements at the material interface), see e.g. [25,26] for details.

Then the direct method (linear extrapolation) can be used for determination of H_I, H_{II} values from equations (5) and (6), see Fig. 6. The procedure is similar that known for determination of K factor in the case of homogeneous body.

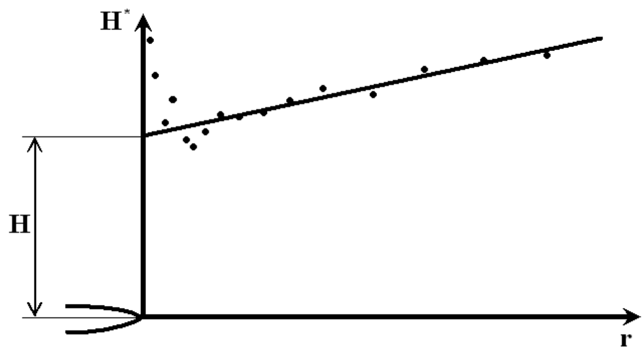

Figure 6. Procedure of H_I, H_{II} determination: linear extrapolation of $H(r)$ values for r=0

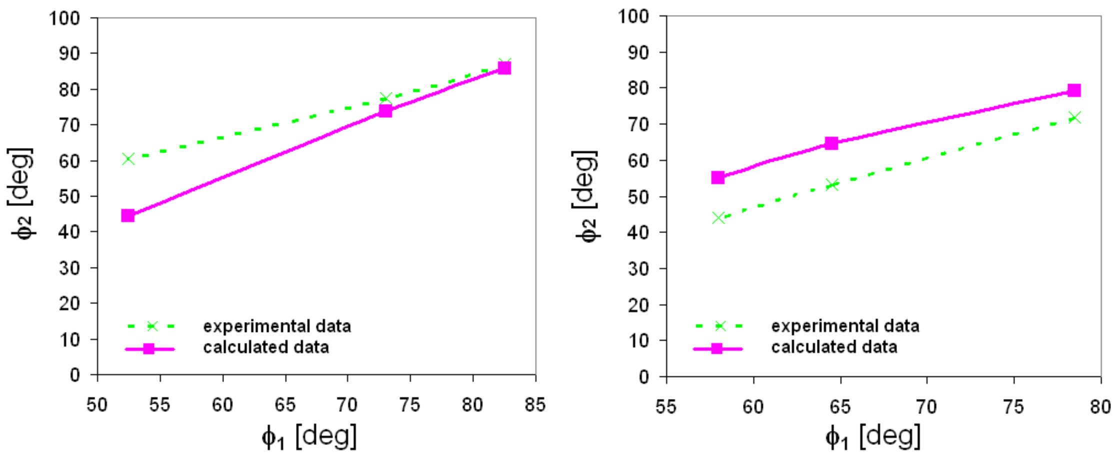

Figure 7. Further crack propagation direction behind the bimaterial interface: comparison of calculated values and experimental data (Al_2O_3/ZrO_2 interface on the left; ZrO_2/Al_2O_3 on the right)

RESULTS AND DISCUSSION

Taking into account the dimensions of the modelled experimental sample (2 x 2.5 x 25 mm) for numerical modelling 2D model under plane stress conditions was chosen. The model contained laminar structure with crack touching the interface between individual layers. The model was loaded by cooling from sintering temperature (1500°C) to the room temperature. Due to different coefficients of thermal expansion strong residual stresses (300 MPa in compression and tension) in layers in longitudinal direction developed.

The numerical solution was used for determination of stress distribution ahead of the crack tip. Then the stress singularity exponents (eqns. (7) and (8)) and values of generalized stress intensity factors H_I, H_{II} were determined. The angle of further crack propagation was then assessed from equation (3). The results obtained and the comparison with experimentally measured data are shown in Fig. 7.

A procedure combines analytical and numerical solution was chosen for determination of crack propagation direction. Ratio H_{II}/H_I was used as a quantity controlling behaviour of the crack after it passed the material interface (eqn. (4)). H_{II}/H_I ratio for $\theta = 0$ corresponds to ratio of crack loading modes. For determination of H_{II}/H_I it is necessary to know the stress components ahead of the crack tip (for $\theta = 0$) as a function of radial distance r. Results obtained are in acceptable agreement with experimental results and can be used for estimation of further crack direction behind the material interface.

CONCLUSIONS

The aim of the work was an estimation of change of crack propagation direction at interface between two materials in layered composite. The crack behaviour was modelled by means of finite element method. The analogy with crack in homogeneous material was applied. The procedure suggested assumes that controlling quantity for crack propagation is ratio of crack loading modes. The loading modes were separated for crack touching the interface between two materials for angle $\theta = 0$ and on the base of ratio of generalized stress intensity factors the direction of crack propagation behind the interface was estimated. Despite the mentioned simplification an acceptable agreement with experimental results was found. The procedure suggested can be used for estimation of toughening mechanism caused by stepwise crack propagation in layered materials.

Results obtained can be used for design of new layered materials and contribute to the better operation of ceramic structures.

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