Crack Growth Evolution from a Notch

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ABSTRACT. *This paper analyzes the influence of crack shape and the procedure to calculate the stress distribution on the estimated fatigue life in a notched specimen. In order to estimate the crack shape a series of interrupted tests at different number of cycles has been performed. The specimens are broken afterwards to analyze crack shape. The stress intensity factor is calculated through weight functions from the stress distribution in the specimen without crack. The stresses are calculated numerically and three possibilities are analyzed: plane stress, plane strain and 3D. For each case (2D or 3D geometries) an appropriate weight function is used. Each of these combinations gives a different evolution of the stress intensity factor and therefore different values of the propagation life. As a final result, the lives in the numerical analysis are compared to the experimental lives.*

INTRODUCTION

Fatigue is one of the predominant failure modes in mechanical systems. Therefore, in the design process it is fundamental to comprehend how materials fail under the loads applied. The disponibility of reliable fatigue prediction methods will improve the design, reducing costs. In recent years, advances in predicting methods have been made, including aspects as variable amplitude, multiaxial fatigue criteria or the behaviour in special conditions as high temperature [1].

The fatigue process is a combination of nucleation and crack propagation. Usually, the crack initiates at a stress raiser. Initially, cracks have a dimension similar to the microstructure and do not allow the direct application of linear elastic fracture mechanics (LEFM) until they reach a certain length. It is this initiation the most complicated part. Different methods have been proposed to estimate life in notched components. Some consider only initiation (using ε-N or S-N), neglecting the propagation phase [2]. This is the case of small parts with a mild notch and a high fatigue life. An important issue in these models is where to evaluate the stresses. But, it is difficult to decide, based on the geometry, loads and material, when this approximation is good enough. In any case, this procedure usually gives conservative results.

Other methods consider only the propagation phase, considering negligible the initiation of the crack due to, for example, a high stress raiser and high stresses or to already existing defects or cracks [3,4]. If the initial crack is long enough, LEFM can be used, otherwise there are several methods that take into account the growth of short cracks [4-7].

Finally, other methods analyse the fatigue process considering it as the combination of the initiation and propagation phase. The method proposed in this paper is of this kind and has already been applied to very different situations: notches and fretting fatigue with spherical and cylindrical contact, obtaining good results [8]. In this paper it will be applied to a plate with a central hole subjected to tensile stress.

LIFE ESTIMATION MODEL

The model proposed combines the initiation and propagation phases without defining a priori the crack length where initiation is assumed to finish and propagation begins. This method is extensively explained in previous papers [8,9]. Each phase is first analysed separately and then combined together. In the propagation phase, the number of cycles to propagate a crack from different lengths, a_t, to final fracture is calculated. In the initiation phase, the number of cycles to generate a crack of different lengths, a_t, is calculated based on the plain fatigue curve ε-N and the stress and strain at a certain depth. These two values are added to obtain the total life as a function of a_t. It has been shown in other publications [8,9] that close to the surface the initiation process prevails, while far from the surface it is the opposite. The value of a_t chosen, called initiation length, a_i, is the one that balances the two approaches and coincides with the one giving the most conservative fatigue life value.

Propagation phase

The crack growth law used models the behaviour of short cracks since the initiation length could be in the order of microns. This is explained in detail in [10,11]. The stress intensity factor (SIF) has been calculated in 2D and 3D. In the case of a crack in a 2D geometry, either in plane stress or strain, the SIF has been calculated integrating the weight function given by Wu [12] and shown in [13]. The stress distribution used in this calculation is obtained from an elastic-plastic finite element model of the specimens used in the tests. It is necessary to take into account the plasticity because in some of the tests the yield stress is reached in the first cycles. The commercial software ANSYS 13 has been used. In the case of a 3D problem, the SIF is obtained using the technique shown by Zhao et al. [13] for semielliptical surface cracks emanating from a circular notch in the center of a plate with a finite thickness, Fig 1. Again, an elastic-plastic finite element model is used.

The aspect ratio of the crack has to be introduced for the calculation of the SIF. Two different cases have been studied: a fixed aspect ratio and a variable aspect ratio. The latter is obtained by simulating a semielliptical crack growing from the surface of an unnotched specimen. The SIF at the deepest point of the crack, $\varphi = 0$, and at the surface, $\varphi = \pi/2$, are obtained from [14]. Assuming that the crack at every point grows according to the value of the SIF, the evolution of the aspect ratio can be obtained by integrating the equation

$$\frac{db}{da} = \left(\frac{K_{\varphi=\pi/2}(a,b)}{K_{\varphi=0}(a,b)} \right)^n \tag{1}$$

where n is the exponent of Paris crack growth law. Figure 2 shows that the initial value of the aspect ratio has no influence after the crack has grown 10 microns.

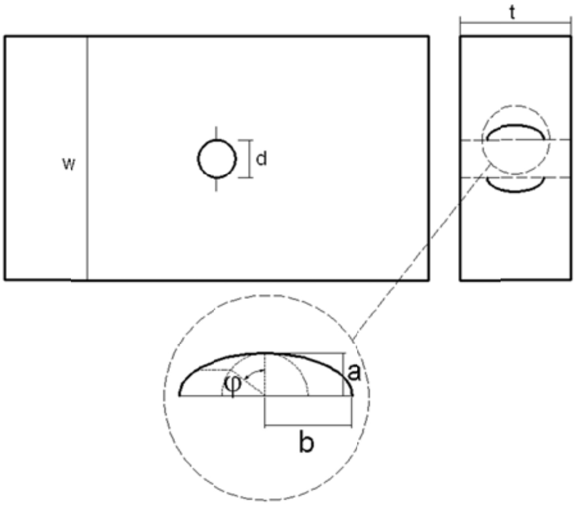


Figure 1. Semielliptical cracks emanating from the center of the notch.

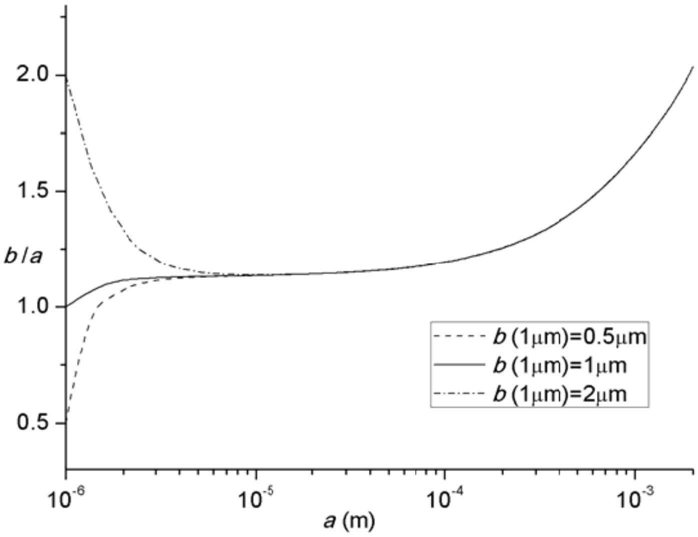


Figure 2. Crack aspect ratio evolution in a crack growing from the surface in an unnotched specimen. Initial crack lengths were b = 2, 1 and 0.5 μm.

Initiation phase

The life estimating model analyses the initiation phase based partially on the work of McClung et al. [15] in notches. The first step is to obtain the fatigue curve, $\varepsilon - N|_{a_t}$, in plain specimens, which gives the number of cycles necessary to generate a crack of length a_t depending on the strain applied. For each strain level, ε_j, the number of cycles in this curve, $N_{\varepsilon_j a_t}$, is obtained through the equation

$$N_{\varepsilon_j a_t} = N_{\varepsilon_j} - N_p(a_t) = N_{\varepsilon_j} - \int_{a_t}^{a_f} \frac{da}{f(a)} \tag{2}$$

where N_{ε_j} is the number of cycles until failure obtained in a plain fatigue test subjected to the strain ε_j, $N_p(a_t)$ is the number of cycles to propagate a crack from a_t to final failure, a_f is the critical crack length at which fracture is produced and *f(a)* is the crack growth law. Solving Eq. 2 for different crack lengths, a_t, to be initiated will produce a family of curves $\varepsilon - N|_{a_t}$. The combination of the stresses and strains produced in the notched specimen with this family of curves would allow obtaining a curve that gives the number of cycles necessary to generate a crack of a certain length, a_t, in the specimen of interest. Actually, due to the stress state, the problem is solved in terms of the value of amultiaxial fatigue parameter versus the number of cycles. The parameter chosen is Fatemi-Socie, which is widely used [16]. The procedure is further explained in [17].

Table 1. Properties of the aluminum alloy 7075 T651 used in tests [9,10].

E **(GPa)**	**ν**	**σ_u (MPa)**	**σ_y (MPa)**
71	0.33	572	503
C	**n**	**ΔK_{th} (MPa m$^{0.5}$)**	**σ_{FL} (MPa) (10^6 cycles)**
$8.83\ 10^{-11}$	3.322	2.1	169
σ'_f (MPa)	**b**	**ε'_f**	**c**
1610	-0.155	0.26	-0.806
Grain size (μm)			50

TESTS

The fatigue tests on notch specimens were performed by the authors with an alloy 7075 T651, which properties are shown in Table 1. The fatigue properties were taken from [9] for a stress ratio of $R = -1$ and crack growth properties from [10] for $R = 0$. The geometry of the specimen is a plate 8 mm thick and 50 mm wide with a center hole of 4 mm diameter, Fig. 1. The values of the maximum nominal stress based on the net section were: 260, 234, 211, 190, 172, 155, 149 and 140 MPa.

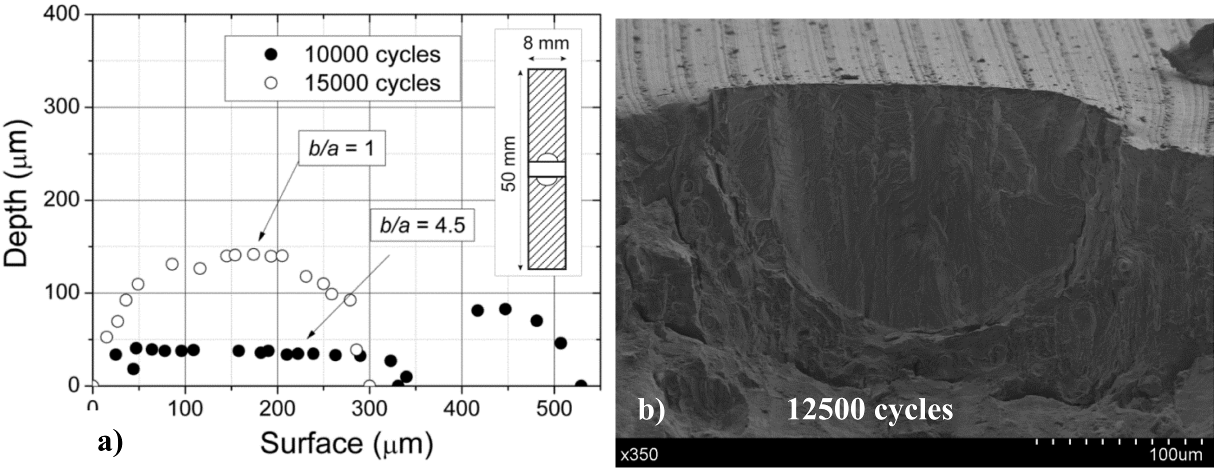
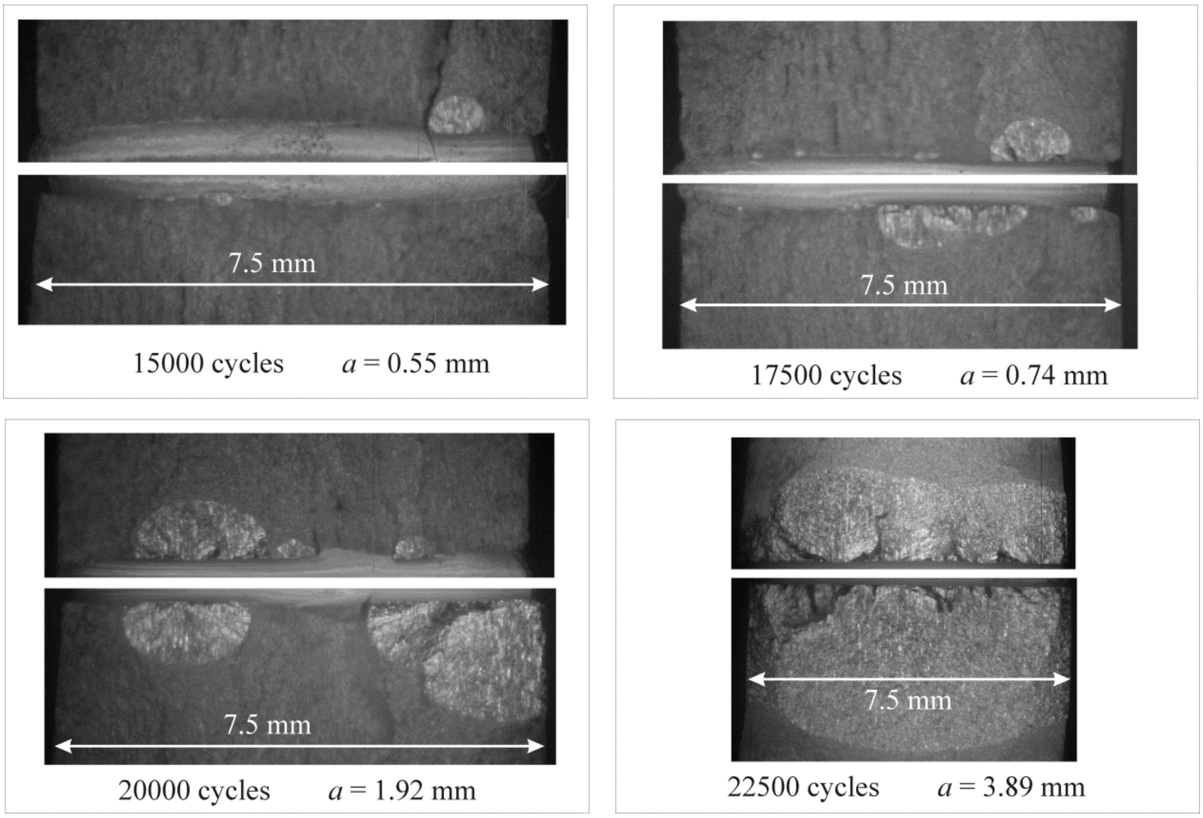


Figure 3. Crack shape at interrupted tests with σ = 211 MPa.



15000 cycles *a* = 0.55 mm

17500 cycles *a* = 0.74 mm

20000 cycles *a* = 1.92 mm

22500 cycles *a* = 3.89 mm

Figure 4. Cracks found in interrupted tests with σ = 211 MPa.

The tests were run until failure, but also several tests at 211 MPa were interrupted before failure. Some of these specimens were sectioned and polished to look for cracks with the microscope. The test interrupted at 5000 cycles (19% of total life) showed no cracks, although this method can not detect cracks smaller than 10 to 20 microns. The analysis of the tests interrupted at 10000 cycles (38% of total life) and 15000 cycles (57% of total life) showed cracks inside the specimens, Fig. 3a. This figure shows what could be two cracks close to one another in the 10000 cycles test. Considering it as one crack the aspect ratio would be approximately $b/a = 4.5$. In the 15000 cycles test the aspect ratio of the crack is approximately 1.

Other specimens were tested and interrupted at 12500, 15000, 17500, 20000 and 22500 cycles, but then they were tested in traction until failure with the objective of discovering the possible cracks inside. The specimens used in these tests were 7.5 mm thick but the nominal stress was still 211 MPa. Figure 3b shows one fatigue crack found in the test stopped at 12500 cycles ($a = 148$ μm). The distance of the center of this crack to the free surface is 1.76 mm. The ondulated surface at the top of this figure is the surface of the hole, which has a roughness of $R_a = 0.3$ μm approximately.

Figure 4 shows the crack surfaces in the other four tests (15000, 17500, 20000 and 22500 cycles) at both sides of the notch. The crack length, a, defined as in Fig. 1, of the biggest crack is also indicated. It is observed how the cracks are initiated inside the specimen, although, sometimes also in the corner (20000 cycles, bottom). The aspect ratio of the cracks is usually around 1, but in some cases (17500 cycles, bottom) two cracks join and form a much wider crack. This possibility will be simulated in this paper through the $b/a = 4.5$ crack. In each test multiple initiations of cracks were observed. The surface of the notch appears deformed due to the high stresses at rupture in the tensile tests performed.

RESULTS

The fatigue model has been applied to four load levels: 260, 211, 172 and 140 MPa. Different results are obtained depending on how the stresses are calculated. Ten different combinations have been tested. Their results are plotted in Figure 5. In the first two, a 2D problem is assumed: plane stress and plane strain. In the following four, the stresses have been calculated through the 3D model assuming a constant aspect ratio of $b/a = 4.5$. The difference between them resides in where the stresses are calculated to analyse the initiation phase: at the center of the specimen, at the surface and two points in between, see Fig. 5. The last four combinations are similar but assuming a crack with a variable aspect ratio, Fig. 2.

Several conclusions can be withdrawn from Fig. 5. Firstly, when the stresses used to estimate the initiation phase are calculated with the 3D model at the surface, the estimations are poor and very different from the others. The results obtained using the 2D model are similar to the ones obtained from the 3D with the fixed aspect ratio $b/a = 4.5$. It seems that this aspect ratio makes the crack flat enough as to be comparable to a through crack in the 2D problem. It can also be observed that in the region of low cycle fatigue, the estimated lives obtained with the variable aspect ratio are higher than the ones obtained with the fixed aspect ratio, but for high cycle fatigue they are similar. This is because in low cycle fatigue the initiation phase is short and appears the effect of the higher SIF for the crack with *b/a* = 4.5. On the oher hand, in high cycle fatigue the propagation is very short and total life is not affected by differences in crack geometry. The variable aspect ratio in the 3D model produces very good results in estimating life.

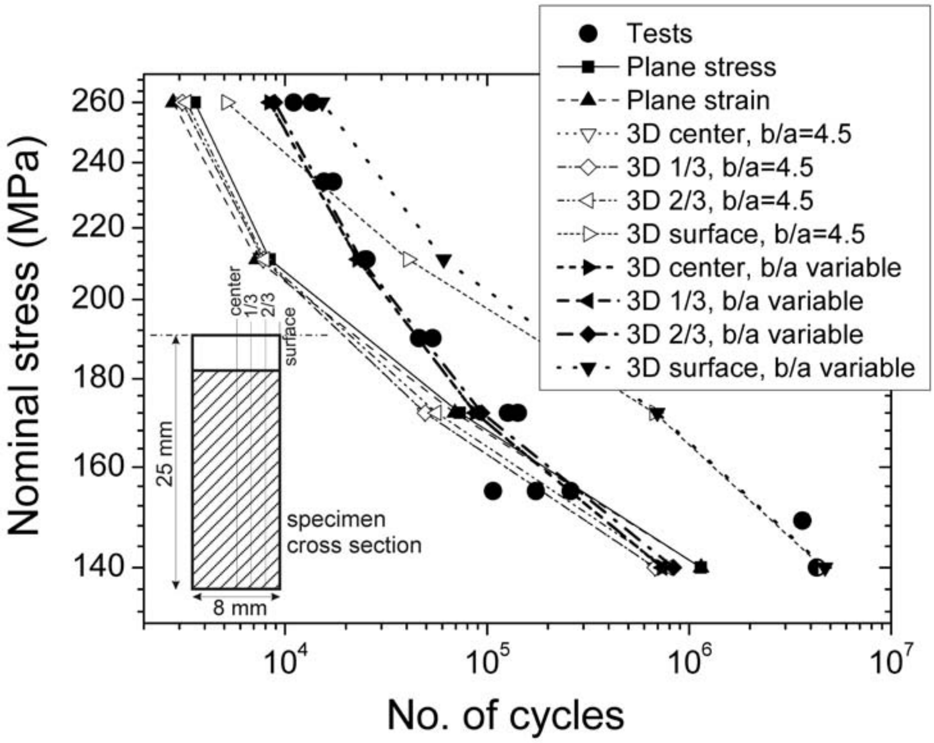


Figure 5. Fatigue curves obtained with different models.

CONCLUSIONS

This paper presents a fatigue life predicting model that combines the initiation and propagation phases without defining a priori when the first one finishes and the second one begins. This makes the model versatile and applicable to different situations with stress concentrations like fretting fatigue and notches.

Plane stress and plane strain models give very similar fatigue life predictions. These results are similar to the ones obtained with the 3D model assuming a nearly flat crack. Nevertheless, the best results have been obtained when a 3D model including the evolution of the aspect ratio of the crack is used. It is worth noting that this aspect ratio is close to one, as it is also observed in the experiments.

A further step in this model would be to take into account the existence of several cracks that join at some point.

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