X-FEM based modelling of complex mixed mode fatigue crack propagation

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ABSTRACT. This papers aims at describing the usage of the eXtended Finite Element Method and more widely the fracture mechanics in a general framework. First, the numerical approach with its hypotheses is described. Then, an experimental procedure is developed to validate some assumptions about crack orientation, the numerical tool has been used to estimate an a priori crack propagation behavior. Finally, conclusions are drawn around the opportunity of using such predesign tool.

INTRODUCTION

Cost reduction in combination with performance improvement implies weight and shape optimization, whithout forgetting safety aspects. Failure analysis can be carried out with fail-safe, safe-life or damage tolerant approaches. While in the first two, studied parts should undergo a given number of cycles without cracking, as they could not be changed or repaired when a crack occurs, the latter addresses the problem of parts that are regularly inspected and can be changed or repaired if a detected crack would grow to a critical size before the next inspection. More generally, damage tolerance is used to define inspection intervals by considering an undetectable crack of the biggest possible size located in the most critical area and determining the number of loading cycles until a critical configuration is reached. In aeronautics and other industrial sectors, a linear elastic behavior is observed enabling the use of the Linear Elastic Fracture Mechanics [1] assumptions where the eXtended Finite Element Method [2] has shown to be a numerical method with very powerful possibilities in the last years. This method has been thoroughly validated on multiple applications where Low Cycle and High Cycle Fatigue are considered. Still, the question remaining on the physics of a crack propagation cannot be answered by a numerical tool. For that reason, an experimental procedure has been developed. Using the XFEM as a predimensionning tool for the specimens and load cases, the experiments are built in several steps in order to be able to test a mixed-mode crack propagation under varying load conditions.

EXTENDED FINITE ELEMENT METHOD FOR LINEAR ELASTIC FRACTURE MECHANICS

Developed for about a decade, the eXtended Finite Element Method (XFEM) is a numerical method based on the standard Finite Element Method (FEM). The fundamental idea is to simplify procedures (such as meshing a propagating crack) that appear to be complex when using FEM, while keeping the robustness of the underlying method. The core of the method is the ability to generate additional degrees of freedom in areas of interest where the local physics is known *a priori* such as a discontinuity in a field or its gradient, without modifying the existing mesh or interpolation functions. The benefit is that the need in mesh refinement is much smaller to achieve a given precision. The functions associated to these additional degrees of freedom are built by simply multiplying the interpolation functions of the existing FEM problem by functions related to the considered physics. As a corollary, the main advantage is that it is possible to have an evolving local representation without modifying the FE basis. In the framework of fracture mechanics, this point is of high interest mainly in 3D. Indeed, in FEM, the surface of the crack has to be represented by the mesh, leading to the need of repeating this step at each time the crack is propagated. If a remeshing is performed, some fields have to be transferred from the previous to the updated mesh, which is a costly operation and subject to errors. On the opposite, the XFEM enables to update the representation of the crack by modifying the enrichment functions, for as many propagation steps as the numerical representation enables.

The Level Set method is often associated to the XFEM as a support for the crack representation, and thus the building of the enrichment functions. This method describes a closed or infinite surface in space by a distance field represented by its nodal values on a mesh. The distance is signed to indicate a "above" or "under" position *w.r.t.* to the norma to the surface. The surface itself can be found by interpolating the iso-zero of the field.

Enrichment Description For The LEFM

The introduction of enrichment functions is done by using the notion of partition of unity. A partition of unity is a set of continuous functions on a domain such that each point has a neighborhood on which all functions but a limited set are equal to zero and that the sum of these functions is equal to 1.

Basic enrichment procedure

In the case of LEFM, two types of enrichments are used (see Fig. 1):
- the *discontinuous* enrichment, or *Heaviside*, which corresponds to the strong discontinuity of the displacement observed between the crack lips, it is applied to to the nodes whose support is completely cut by the crack;
- the *asymptotic* enrichment, or *crack tip*, which corresponds to the singular displacements observed in the neighborhood of the crack tip, it is applied to the nodes whose support is only partially split by the crack.

These enrichment functions are multiplied by the interpolation functions used for the finite element discretization in order to build a richer approximation space.
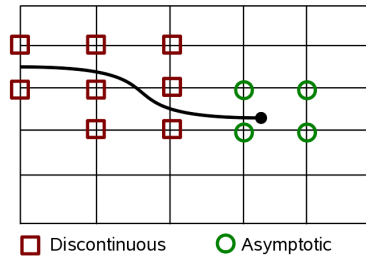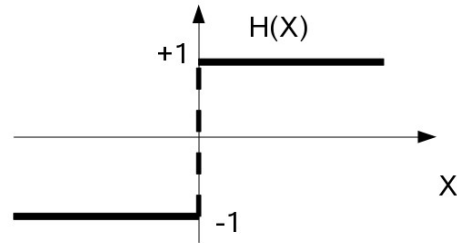

Figure 1: Nodes for enrichment

Figure 2: Discontinuous enrichment

Discontinuous enrichment (Fig. 2)

This enrichment type ensures the representation of the full splitting of an element by the crack surface. The function *H* used is a modification of the standard Heaviside function *h*, the relation is given by, for any *X*, *H(X)=2.h(X)-1*. Where *X* is a function of the position in space *x* that will be determined with the normal level set function defined in a following section of the document. As a consequence, this function is equal to +1 on one side of the crack surface and -1 on the other side of the crack surface.

Enriching nodes that do not have their support split by the crack would lead to the generation of a singular stiffness matrix as there would be linearly dependent functions on the same elements.

Asymptotic enrichment

The second enrichment type ensures the representations of the asymptotic displacement field in the neighborhood of the crack tip. It is based on the assumption that locally, the crack will behave such has a straight crack in a infinite medium with remote loading. For example, the displacement field in the such a case with remote tension orthogonally to the crack surface has been given by Westergaard [4] under the following form:

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \frac{K_I}{2\mu} \sqrt{\frac{r}{2\pi}} \begin{bmatrix} \cos\frac{\theta}{2}(\kappa - 1 + 2\sin^2\frac{\theta}{2}) \\ \sin\frac{\theta}{2}(\kappa + 1 + 2\cos^2\frac{\theta}{2}) \end{bmatrix}$$

where K_I is the stress intensity factor for mode I (opening), *r* and θ are the local polar coordinates associated to the crack. For the two other kinematic modes (in-plane shearing and out-of-plane tearing), the fields have similar aspects. Based on this form of physical functions, a basis is defined in order to span all the functions encountered. The following four functions are commonly used:

$$\{F_i\}_{i=1..4} = \{\sqrt{r}\sin(\theta/2) \ ; \ \sqrt{r}\cos(\theta/2) \ ; \ \sqrt{r}\sin(\theta/2)\sin\theta \ ; \ \sqrt{r}\cos(\theta/2)\sin\theta\}$$

The first function is the only one having a discontinuity accross the crack surface (between $-\pi$ and π). The other ones have discontinuities in their gradient and are also compulsory to achieve a good representation of the physics.

It is possible to increase the number of nodes concerned by the asymptotic enrichment in order to have a larger area where the physics is taken into account. Two approaches exist: the first considers that all nodes within a circle of a given radius around the crack should be enriched, the second uses the topology of the mesh and the choice of the enrichment is led by a number of layers around the crack tip.

Level Set Functions For Crack Representation

A crack is by definition a finite or semi-infinite surface limited by a line. This leads to the definition of two level set functions. The first one defines the surface on which the crack will be located, it is called „normal level set" function, *lsn*. The second is defined such that it limits the part of the space where the crack really exists, it is called „tangential level set" function, *lst*. Conventionnally, the crack is defined by „the iso-zero of *lsn*, where *lst* is negative". It is recommended to build these functions such that their gradients are orthogonal and normed to 1 at any point. They are generally defined as analytical surfaces when possible, or computed from a mesh for more complex initial geometries. If the orthonormality of the gradients is verified, the level set functions can be used to define the local crack coordinates with the following relationships:

$$r = \sqrt{lsn^2 + lst^2}$$
$$\tan\theta = \frac{lsn}{lst}$$

Also, the modified Heaviside function can be defined thanks to *lsn* with the relationship: „H(x)=sign(*lsn*(x))".

The distribution of the level set functions signs and the crack polar coordinates can be observed on Figure 3.
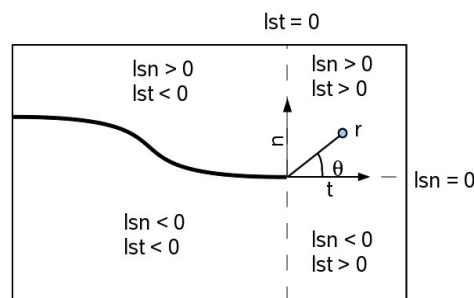

Figure 3: Relationships

Stress Intensity Factors Computation

The J-integral is computed under its domain form on a domain surrounding the crack tip, using the Eshelby tensor. The three stress intensity factors are extracted using the interaction integral method where the test fields correspond to each crack kinematical mode taken separetely.

Crack Propagation

A Paris-Erdogan propagation law is used in order to determine the crack increment related to the number of cycles. The maximum hoop stress criterion is used for the propagation angle determination. It is computed based on the average values of the stress intensity factors during a cycle:

$$\theta_p = 2 \arctan \left(\frac{1 - \sqrt{1 + 8\alpha^2}}{4\alpha} \right) \qquad \alpha = \frac{K_{II\,av}}{K_{I\,av}}$$

PRE-DESIGNING EXPERIMENTS

The ultimate goal of the experiments is to validate two points: first, ensure that the direction criterion is sufficiently representative; second, verify the validity of the propagation law in complex cases. The definition of the experiments is led by the cases that can be found in industrial applications. The simulations were done with Morfeo [3].

Test Case Definition

Let us consider a housing of an aircraft engine. It will be subjected to complex loadings including pressure and vibrations. From that point, several strong assumptions have to be made in other to make preliminary experiments with a reasonable cost. First, the targetted part is thin in some areas, with evolving thickness, the experiment will take a constant thickness metal sheet. Then, the representation of different types of loadings will be performed by applying a constant traction in one direction and a variable traction in the other direction.

Geometry definition

For that reason, a cross-shaped specimen will be chosen. A preliminary step of pre-cracking will be performed in order to have a real crack rather than a notch, which will need an initial geometry more complex than a cross. So, the experiments will be carried out in several steps:
- Pre-cracking of the notched specimen (phase 1, Fig. 4)
- Re-manufacturing to obtain a cross-shaped participants
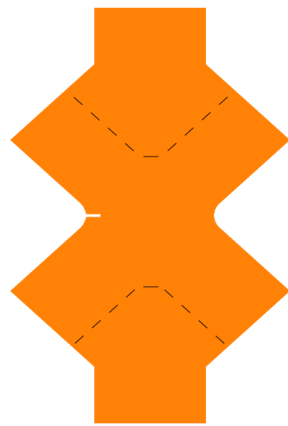- Complex loading crack propagation (phase 2, Fig. 5)
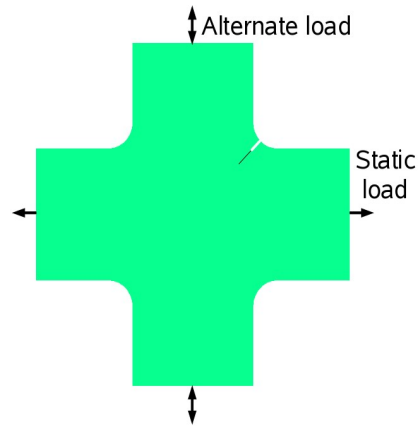
Figure 4: Phase 1   *Figure 5: Phase 2*
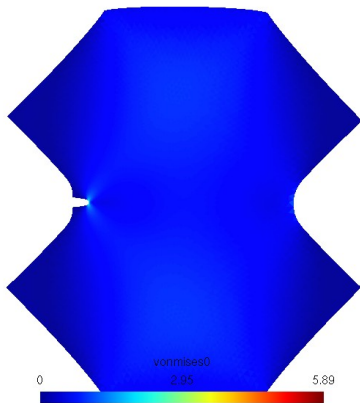
Loading definition for phase 1

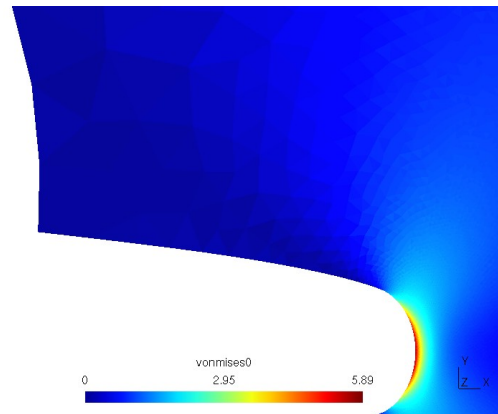*Figure 6: Von Mises stress, 1MPa load*   *Figure 7: Zoom on notch*

The loading conditions for both phases have to be defined carefully. Indeed, the stress concentration has to be taken into account. The stress level has to be verified in order to remain in the LEFM framework. The figures (6 and 7) show the von Mises stress for an applied stress of 1 MPa. The simulation of an imposed displacement (as the mean of the measured displacement) leads to identical fields. The notch (here: 5mm depth and 1mm height) has a stress concentration factor of about 5.9, and the influence area is small compared to the specimen. This information, with the knowledge of a Wöhler curve, can help estimate the number of cycles at a given load required to initiate the crack.

As the crack has to be grown up to 2-5 millimeters to be settled, simulations were performed with cracks of lengths 0.5, 1, 1.5, 2mm. The Table 1 represents the case of a crack of 0.5mm with several thicknesses for specimen (as the stress is imposed, the relations are linear and "almost independent" from the material chosen, only the row in grey was computed).

Table 1

Thick. mm	Imp. stress (MPa)	Force equ. (N)	Force equ. (kg)	Max. stress no crack, Mpa	C (exemple) (dK in MPa.sqrt(mm))	m	K1 pour 0.5mm (MPa.sqrt(mm))	K1 pour 0.5mm (MPa.sqrt(m))	dN for da = 0.1mm
1	1	56.57	5.77	5.9	3.80E-11	3	3.82	0.12	4.73E+08
2	1	113.14	11.53						
3	1	169.71	17.3						
4	1	226.27	23.07						
1	10	565.69	57.66	59	3.80E-11	3	38.18	1.21	4.73E+05
1	20	1131.37	115.33	118	3.80E-11	3	76.36	2.41	5.91E+04
1	30	1697.06	172.99	177	3.80E-11	3	114.54	3.62	1.75E+04
1	40	2262.74	230.66	236	3.80E-11	3	152.72	4.83	7.39E+03
1	50	2828.43	288.32	295	3.80E-11	3	190.9	6.04	3.78E+03
1	100	5656.85	576.64	590	3.80E-11	3	381.8	12.07	4.73E+02
2	10	1131.37	115.33	59	3.80E-11	3	38.18	1.21	4.73E+05
2	20	2262.74	230.66	118	3.80E-11	3	76.36	2.41	5.91E+04
2	30	3394.11	345.98	177	3.80E-11	3	114.54	3.62	1.75E+04
2	40	4525.48	461.31	236	3.80E-11	3	152.72	4.83	7.39E+03

This table can be used to choose a device depending on the load to be applied. The plastic area has to be monitored as it should remain "small" for the LEFM.
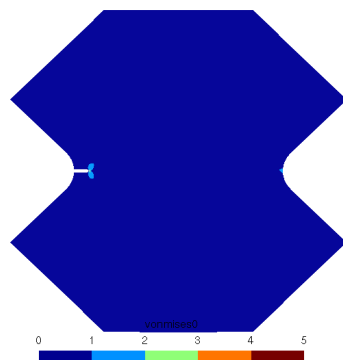
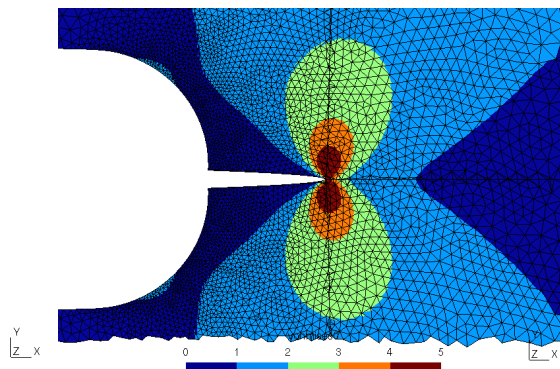

Figure 8: Von Mises stress, with crack

Figure 9: Zoom on notch

For a 2mm-deep crack, the stress concentration is about 3 at 0.5mm from the tip and about 5 at 0.1mm from the tip (see Figures 8 and 9). These distances are quite small regarding to the specimen geometry, thus an imposed stress of about a third of the yield stress would fulfill the condition for confined plasticity.

Loading definition for phase 2
Preliminary simulations were done. Unlike the Figure 5, in the present case, the static load is vertical (1 MPa) and the alternate load is horizontal (between 0 and 0.1, 0.5, 1, 2, 10, 10000 MPa).
Starting from a crack of 2mm in a 5mm notch, Figures 10 and 11 show the pathes of the different configurations (about 20mm in total *da*). The top curved line corresponds to a maximum load of 0.1 MPa and the right curve corresponds to a maximum load of 10000 MPa. In the case where the maximum load is 2, as the mean load of that case is 1 MPa horizontally and vertically, there is a symmetry that leads to the 45°-line.
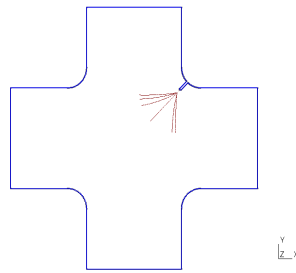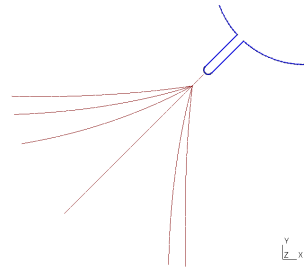
Figure 10: Crack pathes   *Figure 11: Zoom on notch*

Experiments

Some tests have been performed to validate the hypotheses for LEFM and to verify the propagation law (in mode I, Fig. 12). A first fatigue test was carried out with a load oscillating between 1.8 and 18kN (load ratio 0.1) on a specimen of 3mm-thickness in a hardened aluminium alloy. It led to a too large stress inducing large scale plasticity. The crack propagated very fast (16000 cycles, 20mm before sudden rupture) and it encountered a rotation of 45° (Fig. 13), typical of plasticity. This was expected as the theoretical stress (simulated without plasticity) exceeded 1400 MPa in the notch area.
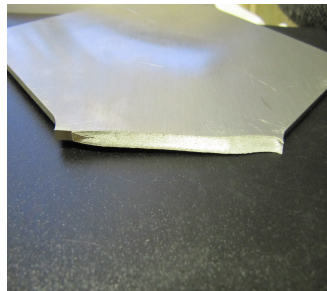

Figure 12: Experiment   *Figure 13: Crack surface*

Further experiments are being carried out at the time this document was written.

CONCLUSIONS

Though the experiments have not yet been carried out completely, the present work has shown that the simulation in particular with XFEM is a powerful tool to set up an experiment and verify assumptions. Experiments will then enable to validate more precisely the assumptions.

REFERENCES

[1] G. Irwin. Analysis of stresses and strains near the end of a crack traversing a plate. *Journal of Applied Mechanics*, 24:361–364, 1957.

[2] N. Moës, J. Dolbow, T. Belytschko. A finite element method for crack growth without remeshing, *Internat. J. Numer. Methods Engrg.*, **46**, 131–150, 1999.

[3] *Morfeo, http://www.cenaero.be*.

[4] H.M. Westergaard. Bearing pressures and cracks. *Journal of Applied Mechanics*, 49-53,1939.