Size Effect in the Damage Behaviour of Short Fibre Reinforced Composites

I. Scheider[1], T. Xiao[1], N. Huber[1], and J. Mosler[1,2]

[1] Helmholtz-Zentrum Geesthacht, Institute of Materials Research, Materials Mechanics, Max-Planck-Str. 1, D-21502 Geesthacht, Germany. E-mail: ingo.scheider@hzg.de
[2] TU Dortmund, Institute of Mechanics, Leonhard-Euler-Str. 5, D-44227 Dortmund, Germany

ABSTRACT. *The present paper is concerned with the analysis of size effects of short fibre reinforced composites. The microstructure of such composites often represents the first hierarchy level of a bio-inspired material, and thus a linear elastic organic matrix material with strong but brittle ceramic fibrous inclusions has been investigated. For such materials, previous researchers have been defined a critical size, below which such an inclusion is flaw tolerant, that is, a precracked microstructural element can sustain loads up to its residual strength. However, if this inclusion, a short fibre, is embedded in a softer matrix, the underlying physical process is significantly more complex. A size effect can be observed here, too, but the failure of the microstructure consists of a superposition of the fracture related to the isolated fibres (i.e. fibre breaking) as well as of that induced by debonding of the fibres from the matrix material. It turned out that the behaviour of the complete microstructure is also qualitatively different from that of a single fibre, namely the fracture energy does not decrease with the size of the characteristic length, but increases in case of a debonding fibre. The decision which path the crack will take, that is, whether fibre breaking or debonding occurs, depends mainly on the aspect ratio of the fibre, but only to a minor degree on its width.*

INTRODUCTION

The present paper is concerned with the analysis of size effects of short fibre reinforced composites. The microstructure of such composites often represents the first hierarchy level of a bio-inspired material, and thus a linear elastic organic matrix material with strong but brittle ceramic fibres has been investigated.

For modelling the various failure mechanisms occurring in heterogeneous materials, i.e. fibre cracking, debonding between fibre and matrix material, and matrix cracking, the overall microstructure has been represented by a three-dimensional finite element model containing cohesive interfaces for all kinds of material separation, i.e. damage and fracture, along prescribed regions in the model. However, which regions fail, is not prescribed but an outcome of the simulation.

Our previous studies [1, 2] have demonstrated that the RVE model indeed captures the size effect associated with material failure of fibre reinforced materials. In line with an analytical model proposed by Gao [3], it is shown that a surface precracked fibre reaches its theoretical strength, if the diameter is smaller than a certain threshold; however, the one-dimensional model presented by Gao usually leads to an overestimation of the flaw tolerance due to their simplifying assumptions of a 2D structure. Based on these findings, a representative volume element (RVE) containing short ceramic fibres with an aspect ratio between 1 and 6 embedded in a polymer matrix is considered. One of these fibres is surface-precracked. The failure behaviour of this RVE also shows a pronounced size effect, but the underlying physical process is significantly more complex. More explicitly, the size effect of the RVE is a superposition of that related to the isolated fibres (i.e. fibre breaking) as well as of that induced by debonding of the fibres from the matrix material. It turned out that the behaviour of the complete microstructure is also qualitatively different from that of a single fibre, namely the fracture energy does not decrease with the size of the characteristic length, but increases in case of a debonding fibre. The decision which path the growing crack will take, that is, whether fibre breaking or debonding occurs, depends mainly on the aspect ratio of the fibre, but only to a minor degree on its width.

APPROACH AND PARAMETERS

History

In 2006 Gao [3] investigated the flaw tolerance of biological materials such as bones, teeth or shells, which in general have a hierarchical microstructure. The main advangtage of these structures is that on the lowest scale, hard mineral particles are embedded in a soft protein matrix. This composite structure forms particles or fibres on a higher scale, where it is resembled again with a soft matrix. Nature has improved the microstructure such that even flaws do not deter the strength of the overall part. Gao investigated a microstructural element, idealized as a thin plate with a centre crack, with fracture mechanics approaches and found out that the strength of a precracked particle is equal to its theoretical strength of a particle with a cross section equal to the remaining ligament, if the width of the particle, h, is lower than a critical value, h_{ft} (the index ft stands for flaw tolerance), which he derived to

$$h \leq \frac{\mathcal{G}_f E}{\sigma_c^2} =: h_{\mathrm{ft}} \tag{1}$$

The fracture strength is here denoted as G_f, E is the Young's modulus, and σ_c is the theoretical strength of the material. A characteristic point in his study is that this critical size is independent of the crack length.

The advantage of the definition of a cricital size is that it can used on all levels of the microstructure, that is, on a higher level, the microstructural element can still be flaw tolerant, if the critical size of the that element is increased by either increasing the fracture toughness or the Young's modulus, or by decreasing σ_c. Since especially the fracture toughness may vary by orders of magnitude, flaw tolerant microstructural elements are possible at several levels of the hierarchy.

Numerical model

In order to model the damage and failure of the fibre and the various failure mechanisms in the representative volume element, a cohesive model is applied. This model describes the crack propagation in a structure by means of interface elements located between the faces of two neighboring 3D continuum elements. The constitutive behaviour is a so-called traction-separation law, which relates the opening vector of the interface (the displacement jump in a continuum mechanics sense) to the traction acting on the interface. In general the opening can be divided in a tangential and a normal part with individual sets of model parameters. In general for each opening mode two parameters are used: The cohesive strength, T_0, and the cohesive energy, Γ_0. The shape of the traction-separation law is bilinear, that is, the interface behaves linearly elastic with a high stiffness until the cohesive strength is reached, and then the traction decreases linearly until the cohesive element has failed completely at a critical opening, δ_0, which can be calculated from the parameters given above by $\delta_0 = 2\ \Gamma_0\ /\ T_0$. This model has been used by the authors for several years, see e.g. [1, 4]. The element used in this publication is the one implemented in ABAQUS finite element code [5].

Outline of investigation

In order to prove the applicability of the flaw tolerance as defined by eq. (1), a numerical study has been performed. The investigation is structured in three parts. First, the applicability of the cohesive model on a small scale was verified by simulation of a centre cracked strip similar to the one studied by Gao, see Fig. 1a. In the second step, the transferability of the flaw tolerance size was investigated by simulation of a surface cracked fibre, Fig. 1b. This study was used to find out whether the flaw tolerance size is still valid for a more complex crack configuration. These studies are already reported in [2] and do not need to be detailed here again, but the results are briefly presented in the next section.

The third part of the investigation, which is highly relevant to the transfer of the idea of a size effect to composite structure and also to the issue of crack path deviation, consists of an assembly of fibres in a soft matrix with one fibre obscured by an initial crack. This study shows the effect of a surrounding matrix material, where additional failure mechanisms appear: the fibre may not only break, but also debond from the matrix, and the matrix material must fail in order to break the whole structure as well.

While for the parts 1 and 2 only the width of the specimen has a high impact on the results, the width as well as the height of the fibres affect the behaviour of the composite, and thus both are varied in this investigation. The independence of the flaw tolerance of the crack length has not been investigated here. The size effect was investigated for a constant relative crack length.

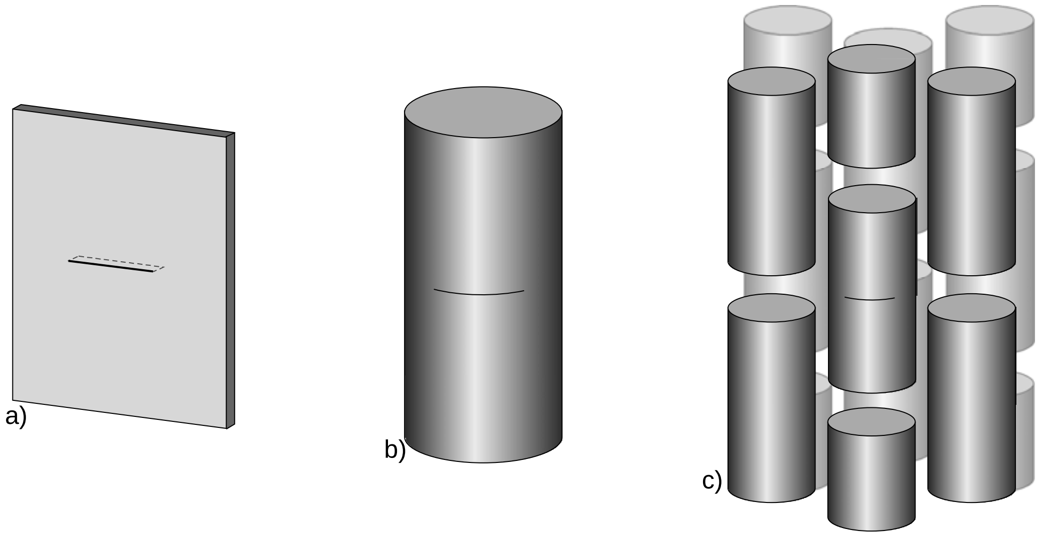

Figure 1: Structures under investigation: a) centre cracked plate; b) surface cracked fibre; c) assembly containing fibres in a matrix, one of which is surface precracked.

Material parameters

All deformation is assumed to be linearly elastic. The hard fibre and the soft matrix material are assumed to be elastic, the parameters are taken from [6]. The damage properties for fibre breaking and for polymer failure are also taken from [6]. The additional damage mechanisms, namely the delamination of the fibre from the matrix, is included with its own parameter set. The elastic properties are summarized in Table 1, the damage properties in Table 2. Please notice, that the critical separation is not an independent parameter, but is calculated from cohesive energy and strength by $\delta_0 = 2\frac{\Gamma_0}{T_0}$.

Table 1: Elastic parameters for the plate/fibre and the matrix material

	Youngs modulus [MPa]	Poisson's ratio
Plate / Fibre	250.000	0.27
Matrix material	3.000	0.33

Table 2: Cohesive zone parameters for the three failure mechanisms

	main failure mode	cohesive strength, T_0 / MPa	Fracture energy, Γ_0 / (J/m²)	Critical separation, δ_0 / µm
Crack extension in plate/fibre	normal	600	320	1.0667
Matrix cracking	normal	40	120	6.0
Fibre delamination	tangential	40	40	2.0

From the parameters in Table 1 and 2, one can identify the critical size from (1) with $\Gamma_0 = G_f$ as h_{ft} = 440 µm for the ceramic. In case of the composite material, it is actually not possible to define a single strength and energy. Therefore, general trends are more appropriate to be explored by finite element simulations.

RESULTS

Verification and Validation

As reported in [2] the thin plate has been simulated using a 2D plane stress model taking advantage of double symmetry of the structure. The initial centre crack is assumed to be 10% of the total cross section area. A single line of cohesive elements has been inserted ahead of the crack tip along the symmetry line. The element length of the cohesive elements is 0.25 mm; a study to determine size independent results has been performed. From this study it turns out that the indeed the critical size can be reproduced. In particular, when the structure is smaller than the critical value, the strength of the structure is defined by its theoretical failure strength (which is the cohesive strength), see Fig. 2a, and the failure energy is equal to the cohesive energy, Fig. 2b. However, this does not hold for the surface cracked fibre. Even though the qualitative behaviour is similar, that is, the fracture strength is constant if the size is below a critical value, the critical size is significantly smaller for the fibre than for the plate. The same effect holds for the cohesive energy, even though the critical value identified from the strength and energy are not the same here.

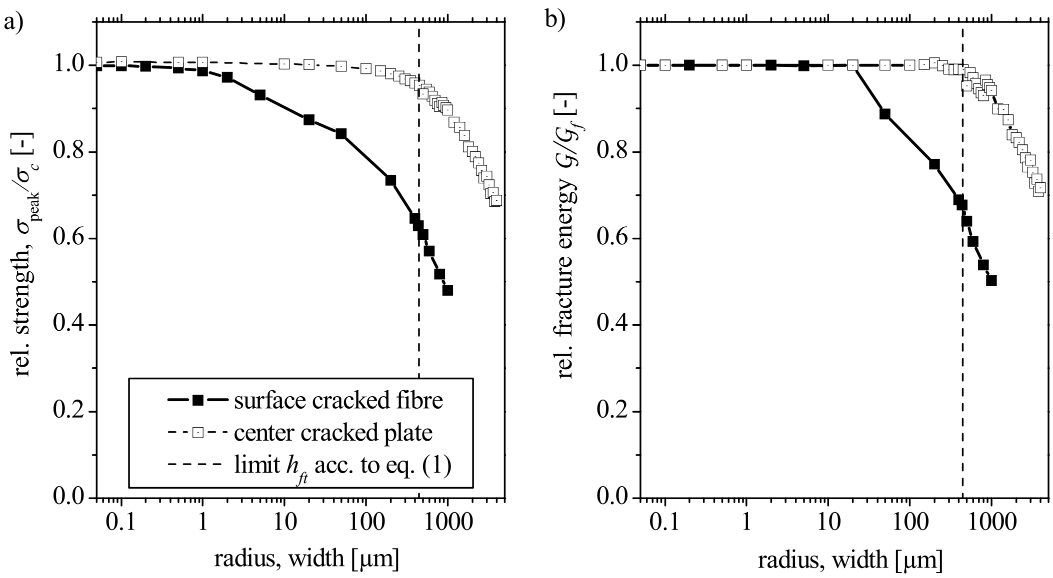

Figure 2: Size effect for a center crack plate (hollow symbols) and a surface cracked fibre (solid symbols) a) on failure strength, and b) on fracture energy.

Application

For the investigation on the composite, a microstructural element containing matrix material and several fibres, all oriented in loading direction, are modelled. One of these fibres is precracked, and the fibres are assembled in a regular manner as shown in Fig. 1c. As smallest possible part of the microstructure, being representative for the whole structure, this representative volume element (RVE) contains four fibres as shown in Fig. 3a). All side surfaces of this RVE have symmetry conditions as well as the bottom, where the crack of the obscured fibre is located. The top surface of the RVE is loaded by a prescribed displacement. The RVE cross-section is a square with width a ranging from 0.1 μm up to 100 μm; the length l of the RVE is always related to the width, but its effect is studied by varying the aspect ratio l/a in the range $1 \leq l/a \leq 6$. The fibre volume fraction is kept constant in this study to decrease the number of variables. The radius of the fibre is therefore always $r_{\text{fib}} = 0.45\ a$, and the length is $l_{\text{fib}} = 13/15\ l$.

Even though the fibre has an initial crack, the RVE does not necessarily fail by fibre breaking, as shown in Fig. 3b, where the preexisting crack does not propagate at all, and the fibre rather debonds. For the simulation of fibre debonding, cohesive elements are included between the fibre and the matrix. Of course, in order to tear the RVE apart completely, the matrix material must fail as well, which is possible in the symmetry plane only, where additional cohesive elements are located in the matrix.

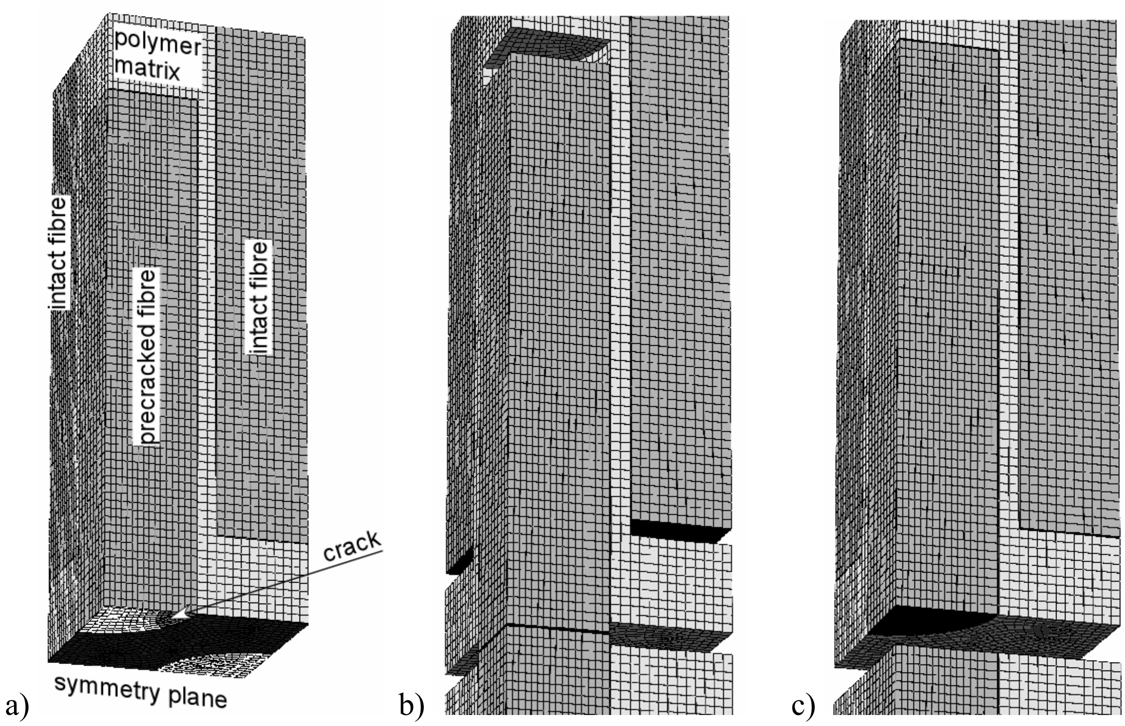

Figure 3: Represenatative volume element (RVE) of a fibre-matrix-assembly: a) undeformed RVE, b) debonding fibre, c) fibre breaking.

Simulations have been performed for various cross section widths and aspect ratios. In all cases the force and applied elongation have been evaluated. From these data, two main quantities were identified: the maximum stress sustained by the RVE (maximum force divided by cross section area), and the specific energy to failure, that is the area under the stress-displacement curve. These results are shown in Fig. 4 as contour plots. The failure mechanism is also an important result, which is indicated in the figure by the hatched region. The simulations from the upper area with higher aspect ratio fail by fibre breaking, the simulations below fail by debonding. The hatched area itself marks simulations, where only the precracked fibre breaks, but the other fibre crossing the symmetry plane fails by debonding.
The strength is shown in Fig. 4a, and one can see that the highest strength is achieved by a high aspect ratio and small width. If breaking occurs at this strength level, the change in strength is not very pronounced and depends mainly on the width. When the failure mechanism changes to fibre debonding, a different picture appears. The strength decreases significantly with decreasing aspect ratio, but the dependence on the width is negligible for small sizes, becomes slightly larger when the width is larger than approx. 3 to 5 µm, where it decreases with increasing width.
The results for the energy, Fig. 4b, are completely opposite: When the fibre breaks, the energy is very low, but when the fibres debond, higher fracture energy can be achieved, which increases with a large size and a high aspect ratio.

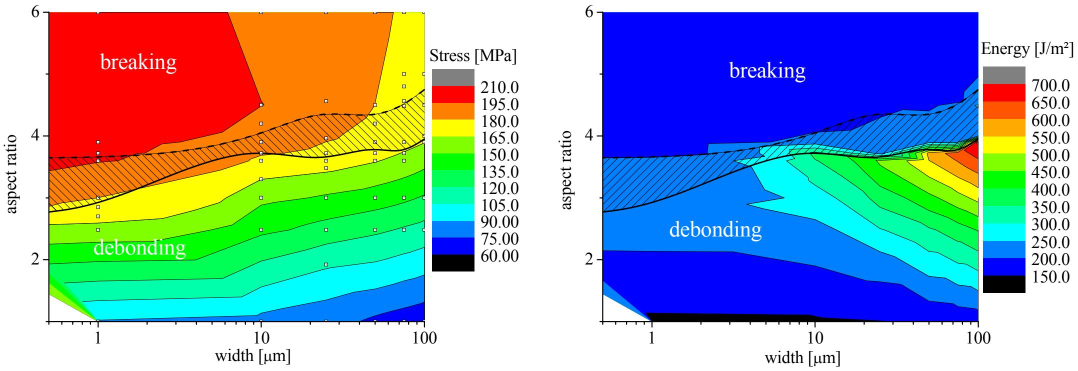

Figure 4: Results for the RVE: a) failure strength; b) failure energy.

DISCUSSION AND CONCLUSION

Regarding the size effect the appearance of a new failure mechanism leads to a completely different picture compared to the failure behaviour of a single fibre. Not only that the length of the inclusion is an additional parameter, which does not play any role in the investigation of the single fibre, but fibre debonding leads to different failure behaviour, which cannot be handled with the fracture mechanics approaches used by Gao. While the strength of the composite is simply reduced by the possibility of debonding, which was already shown in [2], the fracture energy increases significantly with size, see Fig. 4b. This behaviour, which occurs only for fibre debonding, is

opposite to the trend shown in Fig. 2b for the breaking fibre, see the maximum energy for high width and aspect ratio just below the transition region. The reason for the increased energy is the dissipated energy, which depends on the fibre surface and thus increases disproportionately with cross section.
However, the benefit of a high energy vanishes immediately as soon as the hard fibre is long enough (high aspect ratio) to transfer sufficient stress for breaking the fibre.
Beside the length of the fibre, the interface strength is therefore an important parameter for the overall behaviour of the composite. If the fibre has a high adhesive capacity, such that no debonding may occur, this increasing failure energy does not occur anymore, as was also shown in [2]. It should be noted that the maximum fracture energy is almost 700 J/m², which is higher than any of the cohesive energy parameter used, see table 2.

From this study, it can be concluded that the size effect shown by Gao [3] is valid for inclusions, where the crack extension can be described by a one-dimensional representation and the height of the inclusion does not play any role. The case of a surface cracked fibre showed that the crack front effect lead to qualitatively same results, but the critical size cannot be calculated by eq. (1). If the inclusion is embedded in a composite material where several failure modes compete and crack may deviate from its original plane or debonding comes into play, the assumptions underlying eq. (1) are not fulfilled. However, biological materials investigated in [3] always have high adhesive capacity, and thus debonding is not an issue. The investigation of the crack path is nevertheless crucial for a thorough understanding of the failure in general heterogeneous materials.

ACKNOWLEGDEMENTS

Financial support from the Hamburg Ministry of Science and Research and the Joachim Hertz Stiftung as part of the Hamburg Initiative for Excellence in Research (LEXI) is gratefully acknowledged.

REFERENCES

1. Mosler, J., Scheider, I. (2011), *J. Mech. Phys. Solids* **59**:1647-1668.
2. Scheider, I., Chen, Y., Hinz, A., Huber, N., Mosler, J. (2012). *Eng. Fract. Mech.*, accepted
3. Gao, H. (2006), *Int. J. Fract.* **138**:101-137.
4. Scheider, I., Brocks, W. (2003) *Eng. Fract. Mech.* **70**:1943-1961.
5. ABAQUS Analysis User’s Manual, V. 6.11 (2011) Dassault Systèmes Simulia Corp., Providence, RI, USA.
6. Radulovic, R., Bruhns, O.T., Mosler, J., (2010) *Eng. Fract. Mech.* **78**:2470-2485.