

# Effect of Initial Geometric Misalignments of the Circular Notched Bar Specimen on the Fatigue Crack Growth Behavior

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**ABSTRACT.** *In this study, the effect of various misalignments of the circular notched bar(CNB) specimens on the fatigue crack propagation behavior of pipe grade polyethylene is investigated by three dimensional numerical analyses. The effect of the asymmetric crack growth of the misaligned CNB specimens on the lifetime to failure is also addressed. Combined misalignments (concentric misalignments + angular misalignments) of the CNB specimen are considered using finite element analysis. In general, as the misalignments increase, the asymmetric crack growth is accelerated so that the time to reach the critical SIF decreases. Therefore, it can be understood that the lifetime to failure of CNB specimens can vary noticeably once the CNB specimen is misaligned initially. Considering results from this study, the fatigue crack growth behavior including the estimation of the lifetime of CNB specimens should be addressed by considering the misalignment effects.*

## INTRODUCTION

Plastic pipes have become popular in pipe industry due to their distinct characteristics such as low cost, lightweight, good impact resistance, flexibility, chemical resistance and so on. However, it is known that plastic gas pipes such as polyethylene pipe do not have enough the crack growth resistance for both of ductile and brittle fracture compared to metal gas pipe in the past. To solve those critical problems, plastics industry have dedicated a lot of efforts to improve the crack growth resistance of pipe grade polyethylene for both of ductile and brittle fracture continuously. As a result, the PE100/PE125 grade polyethylene gas pipe resin has been developed by several polyethylene resin manufacturers. However, current test standards to characterize the crack growth resistance of polyethylene are not adequate due to unique crack growth

characteristics of polyethylen. American Society for Testing and Materials(ASTM) and The International Organization for Standardization(ISO) have proposed a few standard test methods to quantify the resistance to slow crack growth in commercial pipe-grade polymeric materials such as polyethylene[1].

But, because current test methods suggest to record only the time to failure, so it is impossible to observe the response of the deformation and/or the crack growth behavior of the sample during tests. So, most current test standards can not distinguish the characteristics of crack initiation and those of crack growth, a new experiment method is required [2-5]. To solve these problems, some research institutes have recently developed a new test to use circular notched bar (CNB) specimen (Figure 1). The front of the circular notch of CNB specimen is under tri-axial stress conditions, so the highest effective stress can be formed. So, accelerated tests can be achieved for any loading conditions. However, it is also known that CNB specimens are not good for characterizing the crack growth behavior of brittle materials because it is often observed that the crack path is not axisymmetric [6]. In Figure 2, experimentally observed cases of symmetric and asymmetric crack growth of PE in CNB specimens are shown [7].

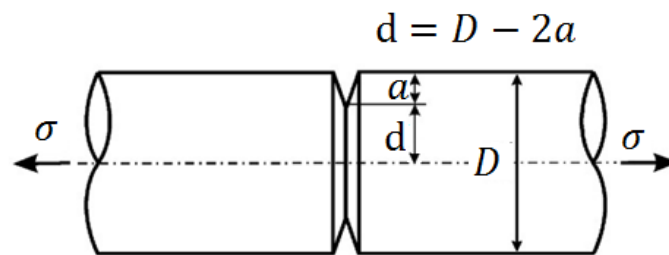


Figure 1. Geometry of a circular notched bar (CNB) specimen.

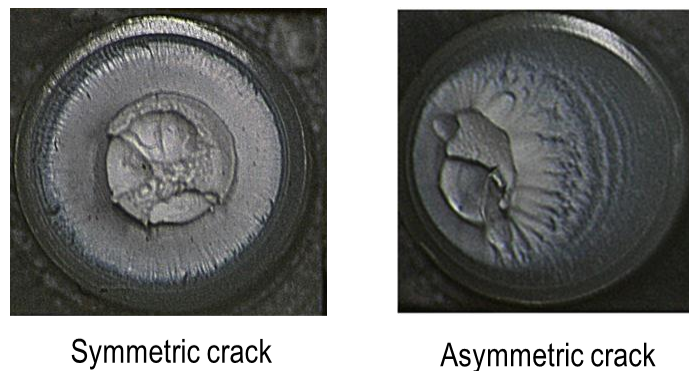


Figure 2. Two types of failure of polyethylene when using CNB specimen.

Many technical issues such as the notch sensitivity (brittleness), the anisotropy of the specimen, the initial notch geometry, the geometric alignments between the centerline of the specimen and the notch, etc., should be considered to interpret experimental data with asymmetric crack formation [6].

Among them, the geometric misalignments of CNB specimen during installing it can be a very important issue practically. There are two types of possible geometric misalignment, i.e. concentric and angular misalignments, and those two misalignments can be combined. The individual effect of concentric misalignment and angular misalignment on the crack growth behavior are already studied previously by authors [6]. The effect of the geometric misalignments of polyethylene on the crack growth behavior at the early stage of crack growth is not that significant, but the asymmetric crack is developed gradually as the crack grows.

In this study, the effect of various geometric misalignments of the CNB specimen on the fatigue crack growth behavior of pipe grade polyethylene is investigated by three-dimensional numerical analyses. Combined misalignments (concentric misalignments and angular misalignments) and the effect of directions of angular misalignments ( $0, \pi/2, \pi, 3\pi/2$ ) of the CNB specimen are considered based on practical difficulties of test conditions. The variation of the stress intensity factors with the progress of a two-dimensional (2D) crack under fatigue loading conditions based on conventional Paris' equation is studied using three-dimensional (3D) finite element analysis (FEA). In addition, experimental observations of the asymmetric fatigue crack growth is compared with 3D FEA results, and the effect of the asymmetric crack growth due to combined initial geometric misalignment on the lifetime to failure of the CNB specimen is also discussed.

## FINITE ELEMENT ANALYSIS

Combined misalignments of the CNB specimen are studied by 3D finite element analysis. Two types of possible geometric misalignment, i.e. concentric and angular misalignments, are considered, and combined misalignments of them are also addressed. The normalized concentric misalignment ( $e/R$ ) is varied as 0, 0.004, 0.012 and 0.020 with the radius of the CNB specimens ( $R$ ), 5mm. At the same time, the angular misalignment ( $e_\theta$ ) is varied as 0, 0.1, 0.2 and 0.4. The directions of the angular misalignment are  $0, \pi/2, \pi$  and  $3\pi/2$ . Combined misalignments are the combination of concentric misalignments and angular misalignments. The conditions that are studied are shown in Table 1.

A three-dimensional (3-D) half model for the CNB specimens is used for FEA, and a commercial FEA program, ABAQUS, is used for this study. All crack tips are remeshed for each calculation of stress intensity factors (SIFs) by considering the crack tip singularity. The type of element is C3D20 (a 20-node quadratic brick), and the numbers of elements and nodes for each specimen are about 60000 and 250000, respectively. Physical properties of the material for FEA are shown in Table 2.

Stress intensity factors(SIFs) are calculated from sixteen(16) node points of the circular (notch) crack contour at the degrees ( $\theta$ ) of  $0, \pi/8, \pi/4, 3\pi/8, \pi/2, 5\pi/8, 3\pi/4, 7\pi/8, 3\pi/4, 11\pi/8, 3\pi/2, 13\pi/8, 7\pi/4, 15\pi/8$  and  $2\pi$ . Based on the calculated SIF from each node, the amount of the crack growth for each node is defined using the

conventional Paris equation with the constants,  $C=1 \times 10^{-11.6}$  and  $m=4$  [8], as shown in Table 2, assuming a fatigue interval of  $10^5$  cycles. Critical SIF ( $K_c$ ) is  $75.7 \text{ MPa}\cdot\text{mm}^{1/2}$  which is obtained from fractured CNB specimens with symmetric crack growth from the experimental results [6]. The lifetime to failure (termination of the crack-growth simulation) is determined when the SIF of any node point of the circular (notch) crack contour reaches the critical SIF. The direction of the crack growth is determined as the normal to the direction of the tangent to the circular crack (notch) contour based on the maximum tangential stress (MTS) criterion[9], as shown in Figure 3.

Table 1. Misalignment conditions for FEA

	e/R	$e_\theta$
Concentric misalignment	0.004, 0.012, 0.020	0
Angular misalignment	0	0.1, 0.2, 0.4
Combined misalignment	0.004, 0.012, 0.020	0.1, 0.2, 0.4

Table 2. Physical properties of the material for FEA

Young's modulus $E$ (MPa)	Poisson's ratio $\nu$	Constant for Paris' equation $C$	Constant for Paris' equation $m$	Remote stress range $\sigma$ (MPa)
1250	0.4	$10^{-11.6}$	4	10.8

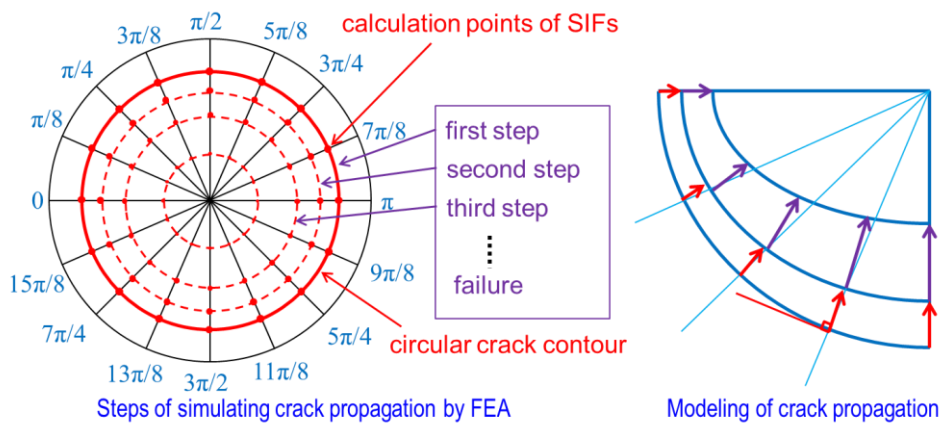


Figure 3. Modeling of crack propagation

## RESULTS AND DISCUSSION

The accuracy of FEA is confirmed by comparing the SIFs calculated for a symmetric crack from FEA and the analytical SIFs for CNB as expressed as [10],

$$K = \frac{1}{2} \sigma_{net} \sqrt{\frac{\pi a d}{D}} \left( 1 + \frac{1}{2} \lambda + \frac{3}{8} \lambda^2 - 0.363 \lambda^3 + 0.731 \lambda^4 \right) \times \left( 1 + 0.1 \sqrt{\frac{2a}{D}} \left( 1 - \frac{2a}{D} \right) \right) \quad (1)$$

where  $\sigma_{net}$  is the net section stress,  $a$  and  $D$  is the crack length and diameter,  $d=D-2a$ ,  $\lambda=d/D$ . It can be confirmed that the SIFs obtained from FEA and from the analytical solution are almost identical.

As shown in Figure 4, it can be confirmed that the SIFs obtained from FEA and from the analytical solution are almost identical.

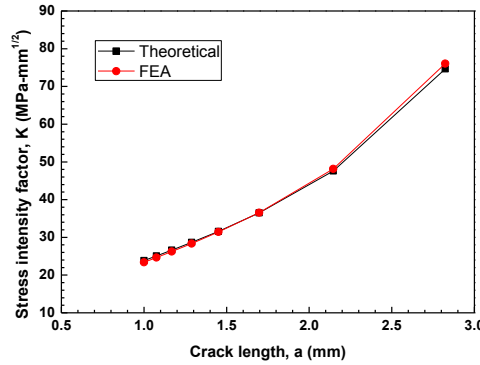


Figure 4. Comparison of SIFs calculated from FEA and the analytical solution.

### *Concentric misalignments and Angular misalignments*

In Figures 5, the FEA results of the variation of SIFs around the crack contour for concentric misalignments ( $e/R=0.020$ ) (Figure 5(a)) and angular misalignments ( $e_0=0.4a$ ) (Figure 5(b)) are shown. It can be observed from Figure 5(a) that the initial SIF varies slightly, but the difference between the maximum and minimum SIF values increases as the crack grows. From Figure 5(a), it can be noticed that the variation of SIFs becomes pronounced as the crack grows. Because the maximum SIF is observed at the degree of  $\pi$  ( $\theta = \pi$ ) and the minimum SIF is observed at the degree, zero ( $\theta=0$ ), it is interesting to observe the increase in  $(K_{max}-K_{min})$ , i.e., the calculated SIF, rather increases as the crack grows. The large difference of  $(K_{max}-K_{min})$  causes even the crack-closure behavior of a part of the circular crack [8], and the crack contour becomes more asymmetric as the crack grows. It is also observed that as the concentric misalignments increases, asymmetric crack propagation is accelerated so that the time to reach the critical SIF decreases at  $\theta = \pi$ . Hence, the normalized lifetime to failure decreases as the concentric misalignment increases, and, for example, the normalized time to failure with 2% (0.02) of the concentric misalignment to the radius of CNB specimen decreases by almost 19% by comparing with that without any misalignment as shown in Figure 6(a).

In Figures 5(b), the FEA results of the variation of SIFs around the crack contour for angular misalignments are shown. The distribution of SIFs depends on the direction of angular misalignment. The maximum SIF is observed at the degree of zero ( $\theta=0$ ), and the minimum SIF is observed at the degree,  $\pi$  ( $\theta=\pi$ ). The angular misalignments increases, asymmetric crack growth is also accelerated so that the time to reach the critical SIF decreases, and, for example, the normalized time to failure with  $0.4^\circ$  of the angular misalignment decreases by almost 31% by comparing with that without any misalignment as shown in Figure 6(b). However, this level of the angular misalignment is very possible practically, so, it can cause a large scatter of test results and prevent quantitative analysis of fatigue crack growth behavior of CNB specimens. As shown in Figure 7, it can be confirmed that the crack propagation contours obtained from FEA and the experimental result are similar each other.

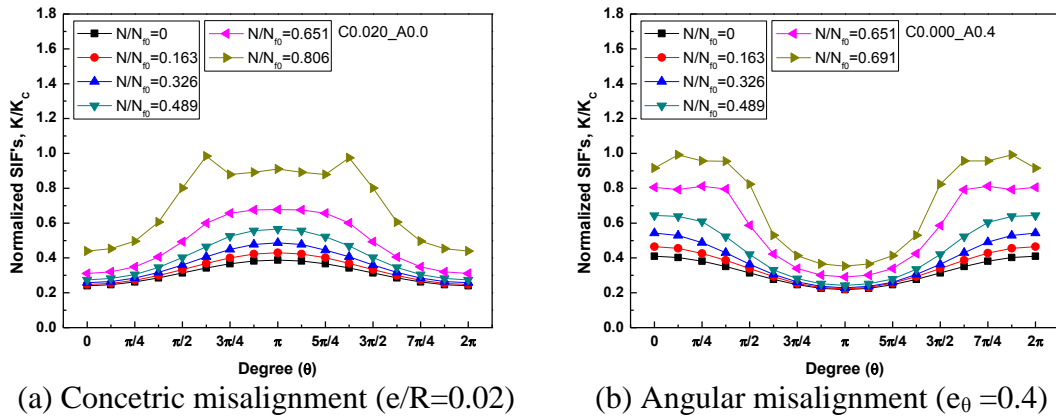


Figure 5. The example of the variation of normalized SIFs at the crack contour for concentric and angular misalignment

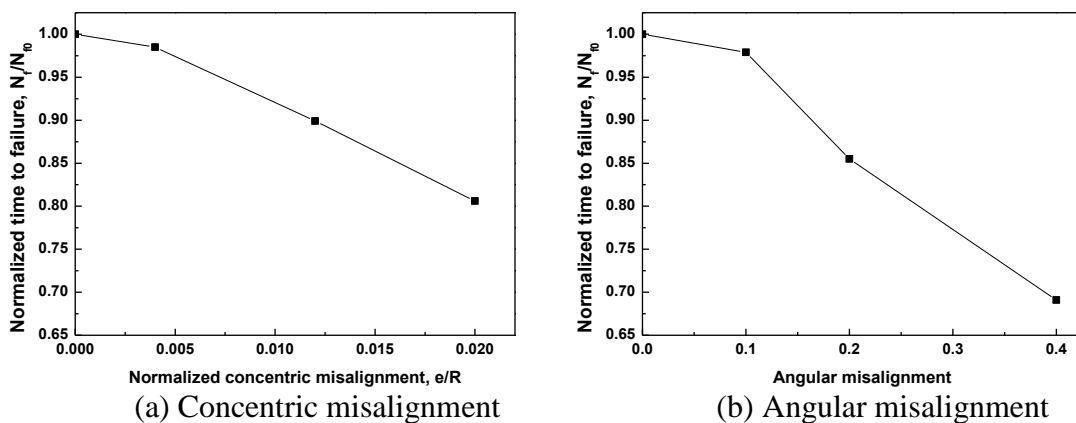


Figure 6. Variation of the normalized time to failure for various concentric and angular misalignments

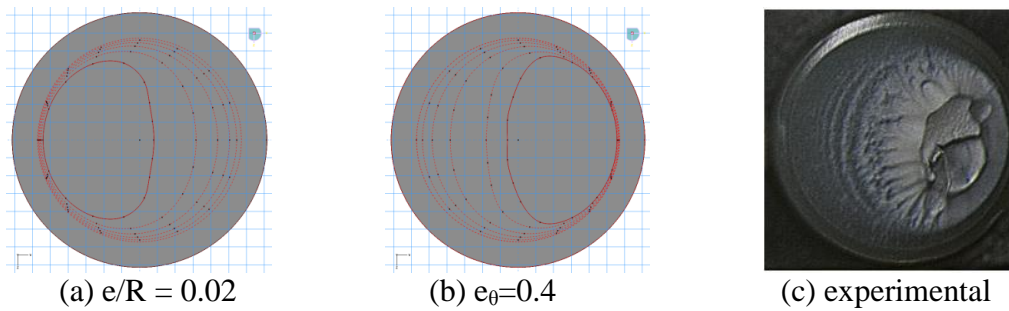


Figure 7. Crack propagation contours compare from FEA and experimental

### ***Combined misalignments***

Combined misalignments (concentric misalignment + angular misalignment) and the effect of directions of angular misalignments ( $0, \pi/2, \pi, 3\pi/2$ ) of the CNS specimen are considered based on practical difficulties of test conditions.

In this study, combined misalignments are defined by the combination of concentric misalignments ( $e/R$ ), i.e., 0.004, 0.012 and 0.02, and angular misalignments ( $e_\theta$ ), i.e., 0.1, 0.2 and 0.4. In addition, the direction of the angular misalignment are varied as  $0, \pi/2, \pi$  and  $3\pi/2$  was analyzed by FEA.

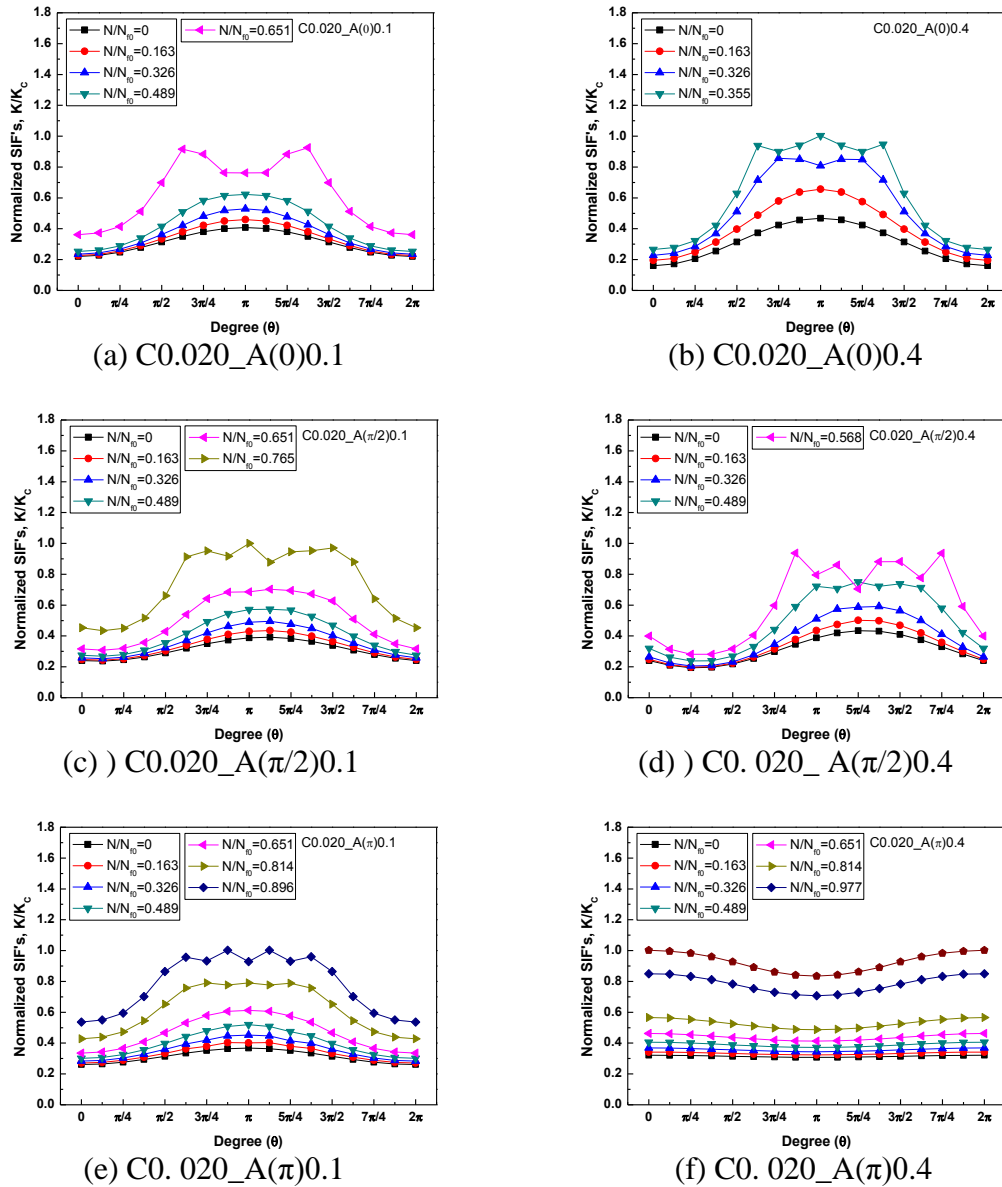
In Figure 8, the FEA results of the variation of SIFs around the crack contour with concentric misalignment ( $e/R$ ) as 0.020, angular misalignment ( $e_\theta$ ) as 0.1 and 0.4 are shown. For all cases, the direction of angular misalignments varies as  $0, \pi/2$  and  $\pi$ .

In Figures 8(a), and Figures 8(b) with the direction of angular misalignment  $0$ , there is a few difference in the initial SIF, but the difference between the maximum and minimum SIF values also rises as the crack grows. As expected, the maximum SIF is observed at the degree of  $\pi$  ( $\theta = \pi$ ), and the minimum SIF is observed at the degree zero ( $\theta = 0$ ). It can be noticed that when the direction of concentric misalignment and angular misalignment is same, these two effects can be cancelled out each other and the normalized time to failure decreases at high speed as the angular misalignment increases.

In Figures 8(c), and Figures 8(d) with the direction of angular misalignment  $\pi/2$ , the initial SIF varies slightly, but the difference between the maximum and minimum SIF values also increases as the crack grows. But the maximum SIF moves from around  $\pi$  to  $3\pi/2$  and the minimums SIF moves from  $0$  to  $\pi/2$  as angular misalignment increases. From this result, it can obtain that  $e_\theta$  affects crack growth rate rather than  $e/R$ . And in Figures 8(c), and Figures 8(d), as SIF values is closed to  $K_C$  when crack grows, crack is unstable comparing to the case of the direction of angular misalignment is  $0$  and  $\pi$ . Although it is different from the crack direction when the angular misalignment is  $0$ , the asymmetric crack growth is accelerated so that it can be thought that the time to reach the critical SIF decreases.

In Figures 8(e), and Figures 8(f) with the direction of angular misalignment is  $\pi$ , the initial SIF varies slightly, but the difference between the maximum and minimum SIF values increases as the crack grows. The maximum SIF is observed around at the degree of  $\pi$  ( $\theta = \pi$ ) and the minimum SIF is observed around at the degree zero ( $\theta = 0$ ) when angular misalignment is 0.1. But, when angular misalignment is 0.4, the distribution of

SIF is changed, i.e., the maximum SIF is observed around the degree of zero ( $\theta=0$ ) and the minimum SIF is observed around the degree  $\pi$  ( $\theta=\pi$ ). As a result, it can be thought that the lifetime to failure rather increases as angular misalignment increases. The distribution of SIFs and the lifetime to failure are affected by the contribution of two misalignments. In other words, the direction of misalignments can be critical to the reliability of test results using CNB specimens.



C : normalized concentric misalignment / A : angular misalignment

Figure 8. The variation of normalized SIFs at the crack contour for combined misalignments



In Figure 9, the FEA results of the variation of SIFs around the crack contour for combined misalignments are shown. When the direction of angular misalignment is 0, the lifetime to failure decreases regardless of the misalignment conditions. However, the direction of angular misalignment is  $\pi$ , the lifetime to failure rather increases up to a certain level of concentric misalignment. These results are because concentric and angular misalignments affect the distribution of SIFs of the crack contour simultaneously, so the contribution of two misalignments can be varied. For example, when the direction of the angular misalignment is  $\pi$ , the shape of the asymmetric crack growth under the concentric misalignment is opposite to that under angular misalignment. As shown in Figure 10, it can be confirmed that the crack growth contour obtained from FEA and the experimental result have similar tendency each other.

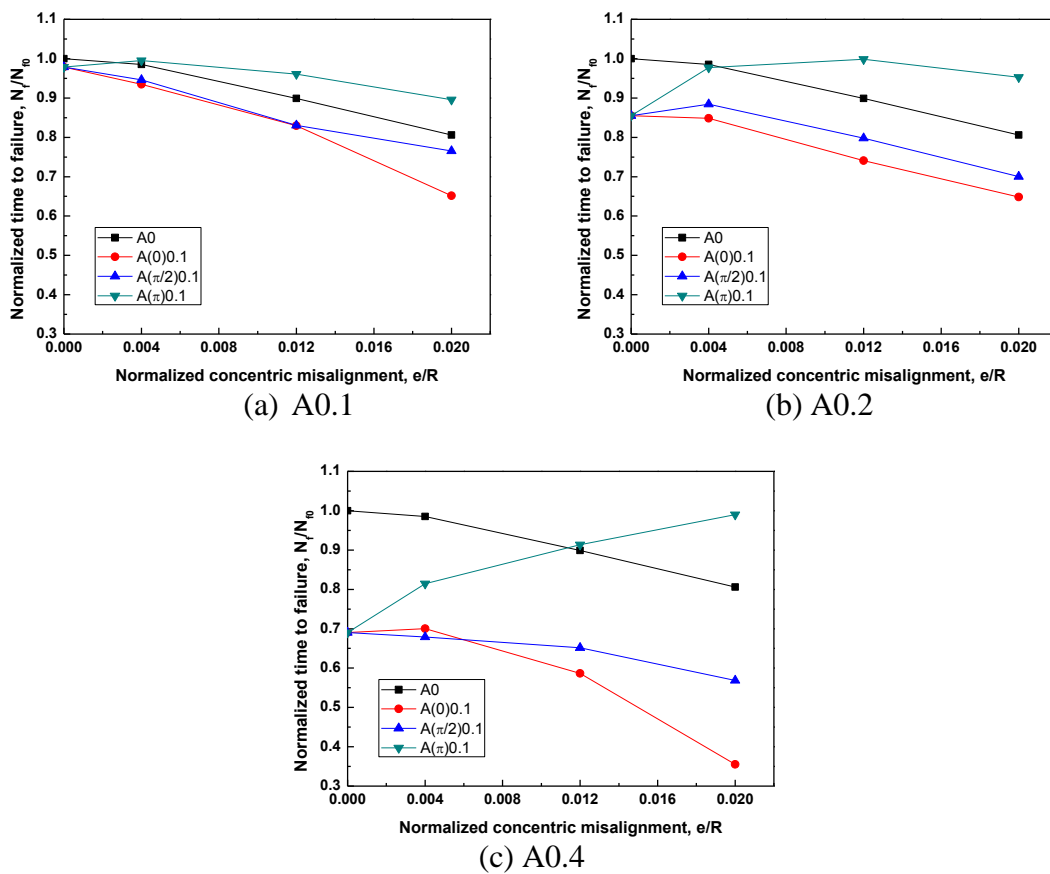


Figure 9. Variation of the normalized time to failure for combined misalignments

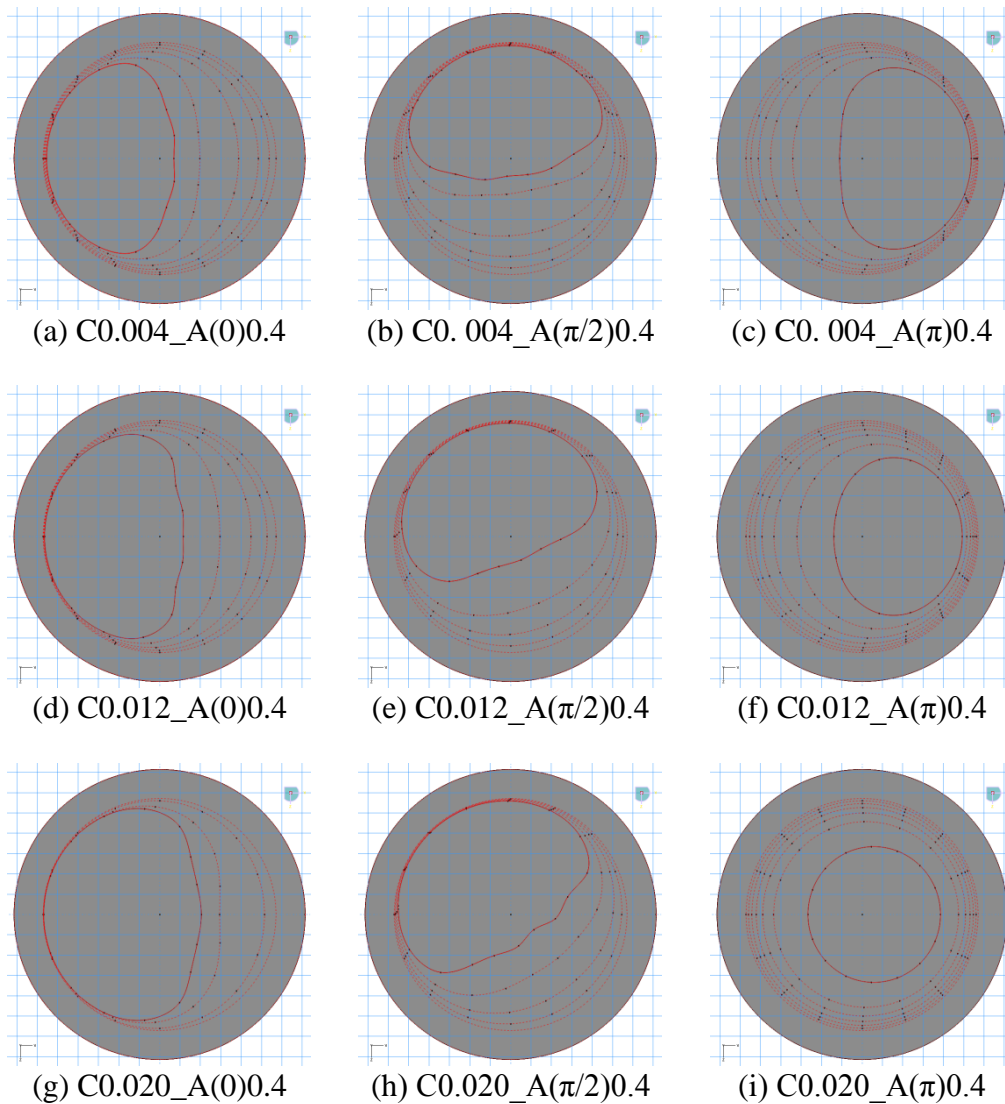


Figure 10. Crack propagation contours compare from FEA

## CONCLUSIONS

In this study, the effect of various misalignments of circular notched bar (CNB) specimens on the variation of the lifetime to failure under fatigue loads is studied by finite element analysis (FEA). Two types of misalignment of the CNB specimen, i.e., concentric misalignments and angular misalignments, which are commonly observed during installing CNB specimens for creep and fatigue tests are considered. Combined misalignments (concentric misalignments + angular misalignments) for various directions of angular misalignments ( $0$ ,  $\pi/2$ ,  $\pi$ ,  $3\pi/2$ ) of the CNB specimen are considered. Here are the summary of this study:

In case of the concentric misalignment, asymmetric crack growth is accelerated so that the time to reach the critical SIF at  $\theta = \pi$  decreases as the concentric misalignment increases. Therefore, the normalized time to failure with 2% of the concentric misalignment to the radius of CNB specimen decreases by almost 19% by comparing with that without any misalignment. In case of the angular misalignment, asymmetric crack growth is accelerated so that the time to reach the critical stress intensity factor (SIF) at  $\theta = 0$  decreases as the concentric misalignment increases. The normalized time to failure with 0.4° of the angular misalignment decreases by almost 31% by comparing with that without any misalignment. In general, as the misalignment increases, the asymmetric crack growth is accelerated so that the time to reach the critical SIF, i.e. the lifetime to failure, decreases. However, the distribution of SIF is changed once the direction of misalignments is changed, so the lifetime to failure under combined misalignments can be varied depending on the status of combined misalignments. For example, the lifetime to failure can increase as the angular misalignment for  $\pi$  direction increases with up to 2% of the concentric misalignment to the radius of CNB specimen. On the contrary, the normalized time to failure combining with 2% of the concentric misalignment to the radius of CNB specimen and 0.4° of the angular misalignment decreases by almost 65% by comparing with that without any misalignment.

Many other factors, i.e., notch sensitivity (brittleness), the anisotropy of the specimen and the notch geometry, etc., can also affect the lifetime failure and the asymmetric crack growth of CNB specimens. However, it is clear that geometric misalignments of CNB specimens can significantly affect the crack growth behavior and the lifetime to failure, so the installer should be careful to eliminate any unexpected misalignments of the CNB specimen to have reliable test results.

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