Numerical Simulation of Fatigue Crack Growth in Heterogeneous Material

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ABSTRACT. *Fully automatic fatigue crack growth simulation system is developed using S-version FEM (S-FEM). This system is extended to fracture in heterogeneous material. In the heterogeneous material, crack tip stress field becomes mixed mode condition, and crack growth path is affected by inhomogeneous materials and mixed mode conditions. Stress Intensity Factors (SIF) in mixed mode condition are evaluated using Virtual Crack Closure Method (VCCM). Criteria for crack growth amount and crack growth path are used based on these SIFs, and growing crack configurations are obtained. At first, basic problem is solved, and results are compared with previously reported ones. It is shown that this system gives adequately accurate estimation of SIFs. Then 2-dimentional fatigue crack growth problems are simulated using this system. crack growth problems are simulated using this system. The first example is a plate with interface between hard and soft materials. Crack tends to grow into soft material through interface. Second example is a plte with distributed hard inclusions. Crack takes zig-zag path by keeping away from hard inclusions. In each cases, crack growth path changes in complicated manner. Changes of SIFs are also shown and discussed. Finally it is shown that this system is useful for the prediction of residual fatigue life in heterogeneous material.*

INTRODUCTION

Fatigue crack growth is important problem for the integrity of structures. To avoid catastrophic accident, predictions of crack growth path and fatigue life are key technologies. As fatigue crack growth occurs in complicated structures, these predictions have met serious difficulties. FEM is usually used for these predictions, but re-meshing process is needed for modeling of growing crack configurations, it has been a bottleneck for the application of FEM to fatigue crack growth problems, especially in three-dimensional field.

Recently, several new techniques have been developed to overcome these difficulties. Element Free Galerkin Method [1], X-FEM [2] and Superposion-FEM(S-FEM[3]) have been developed to make re-meshing processes easy, and predict complicated crack paths. Authors have developed fully automatic fatigue crack growth simulation system[4], and applied it to three-dimensional surface crack problem, interaction evaluation of multiple surface cracks[5] and evaluation of crack closure effect of surface crack[6]. This system is developed for residual stress field problem, and Stress Corrosion Cracking process is simulated [7]. Residual stress field is generated by welding, and evaluation of crack growth in Heat Affected Zone (HAZ) is another important problem. In HAZ, grain size is different from other area, and mechanical properties are different from those of base metals. For the evaluation of SCC in such areas, changes of material properties should be considered. In S-FEM, local mesh is re-meshed for each step of crack growth, and local area changes its' shape in each step. It seems difficult to change material properties of local mesh following the change of local mesh shape.

In this paper, this problem is solved, and crack growth simulation system in heterogeneous material is developed. In the following, this new method is explained briefly, and example problem is simulated and compared with previous works to verify this method. Several practical problems are simulated and effect of existance of interface and changes of material properties are studied and discussed.

APPLICATION OF S-FEM TO HETEROGENEOUS MATERIAL.

S-FEM is originally proposed by J. Fish [3]. As shown in Fig.1, a structure with a crack is modeled by global mesh and local mesh. Global area, Ω^G, does not include a crack, and coarse mesh is used for the modeling of global area. A crack is modeled in local area, Ω^L, using fine mesh around crack tip. Local area is superimposed on global area and full model is made. In each area, displacement function is defined independently. In overlapped area, displacement is expressed by the summation of displacement of each

$$u_i = \begin{cases} u_i^G & i \in \Omega^G - \Omega^L \\ u_i^G + u_i^L & i \in \Omega^L \\ u_i^L = 0 & i \in \Gamma^{GL} \end{cases} \qquad (1)$$

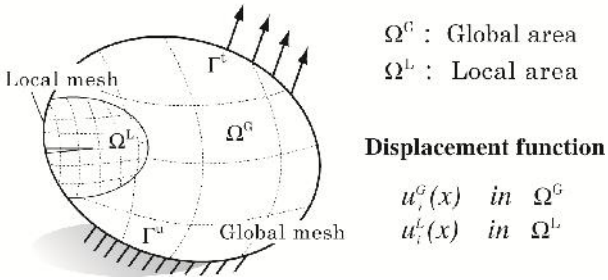


Fig.1. Concept of S-FEM.

area. To keep the continuity at the boundary between global and local area, Γ^{GL}, displacement of local area is assumed to be zero as shown in the following equation. The derivatives of displacements can be written in the same way. These displacement functions are applied to virtual work principle, as shown in Eq. 2, and the final matrix form of S-FEM is obtained as shown in Eq. 3.

$$\begin{aligned} &\int_{\Omega^G} \delta u^G_{i,j} D_{ijkl} u^G_{k,l} d\Omega + \int_{\Omega^L} \delta u^G_{i,j} D_{ijkl} u^L_{k,l} d\Omega \\ &+ \int_{\Omega^L} \delta u^L_{i,j} D_{ijkl} u^G_{k,l} d\Omega + \int_{\Omega^L} \delta u^L_{i,j} D_{ijkl} u^L_{k,l} d\Omega = \int_{\Gamma^t} \delta u^G_i t_i d\Gamma^{tG} \end{aligned} \tag{2}$$

$$\begin{bmatrix} [K^{GG}] & [K^{GL}] \\ [K^{LG}] & [K^{LL}] \end{bmatrix} \begin{Bmatrix} \{u^G\} \\ \{u^L\} \end{Bmatrix} = \begin{Bmatrix} \{t^G\} \\ 0 \end{Bmatrix} \tag{3}$$

where

$$[K^{GG}] = \int_{\Omega^G} [B^G]^T [D^{GG}][B^G] d\Omega \qquad [K^{GL}] = \int_{\Omega^L} [B^G]^T [D^{GL}][B^L] d\Omega$$

$$[K^{LL}] = \int_{\Omega^L} [B^L]^T [D^{LL}][B^L] d\Omega \tag{4}$$

In Eq. (3), $[K^{LG}]^T = [K^{GL}]$, and the stiffness matrix is symmetric. $[K^{GL}]$ expresses the relationship between local and global areas. They are calculated by following integrations. By calculating this term with high accuracy, accurate FEM results are obtained. By solving Eq. (3), both displacement fields of local and global areas are obtained simultaneously. The detail of the theory was presented in the literature of one of the author [8].

This method is applied to crack growth in heterogeneous material. As shown in Fig.2, material properties are different from each other in material 1 and 2. The phase boundary is easily modeled by global mesh. Local mesh is overlapped on global mesh. $[K^{GL}]$ and $[K^{LL}]$ are calculated by eq.(4), and in $[D^{GL}]$ and $[D^{LL}]$, material properties in each material, are needed for these calculations. As shown in Fig.2, integrations are

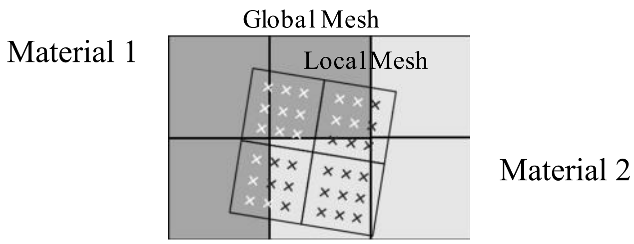


Fig.2 Global mesh and local mesh in heterogeneous material.

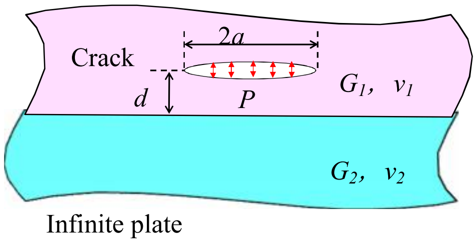
conducted using Gaussian integration method in each local element, and material properties at these Gaussian points are needed for integration. In S-FEM analysis, all Gaussian points in local elements belong to some global element. Then, material properties of each Gaussian point is same as those of global element in which Gaussian point belongs. For this meaning, local mesh needs not to have material properties, and it becomes easy to calculate eqs.(4) using material properties of global element.

VERIFICATION OF S-FEM FOR HETEROGENEOUS MATERIAL.

Figure 3 shows an example problem. A stratght crack exists parallel to interface of two materials. Inner pressure is applied to crack surfaces. Crack length is 2a and distance between crack and interface is expresses as d. In the following simulation, Shear Modulus G is changed in upper and lower materials, and Poisson's ratio is assusmed to be same as 0.3. By changing distance d, several cases are simulated and normalized Stress Intensity Factors (SIF) are evaluated. As this is mixed mode problem, mode I and mode II components are calculated.

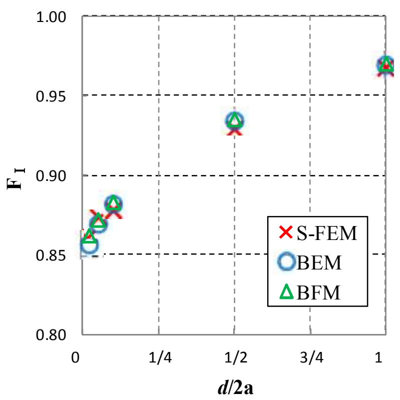
SIF is calculated using VCCM. As inner pressure is applied to crack surface, following equation is used for evaluation of energy release rate by VCCM.

$$G = \frac{1}{2h}\left\{\left(u_\alpha^{\ up} - u_\alpha^{\ down}\right)\left(f_\alpha + f_\alpha^{\ P}\right) + \left(u_\beta^{\ up} - u_\beta^{\ down}\right)\left(f_\beta + f_\beta^{\ P}\right)\right\} \tag{5}$$

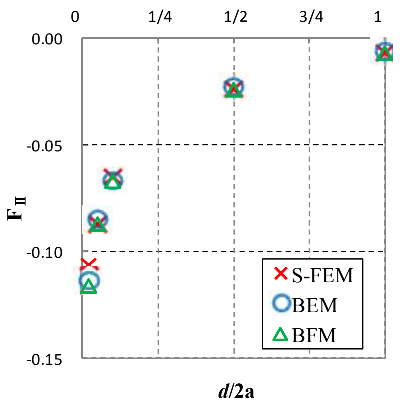


a = 1.0 [mm]
P = 100 [MPa]
Shear modulus ratio
$\Gamma = G_2/G_1$
Poison`s ratio
$\nu_1 = \nu_2 = 0.30$

Fig. 3 Interfacial parallel crack loaded inner pressure



(a) Mode I



(b) Mode II

Fig. 4 Relationship between F and d/2a (Γ= 2) where u is displacement, f is nodal force, and superscript P means nodal force due to inner pressure.

Figure 4 (a) and (b) show results when G_2=2G_1. By changing distance d, several cases are sumulated. Ordinates of these figures are Normalized stress intensity factors in mode I and II, and abscissa is d/2a. Results by previous papers by Boundary Element Method [9] and Body Force Method [10] are also shown in these figures. Results by S-FEM agree very well with other solutions within 1% differences. It is shown that this system gives enoughly accurate results.

CRACK GROWTH IN TWO-PHASE MATERIAL

Figure 5 shows a two-phase plate with slant interface. Young's modulus of material 1 and 2 are expressed by E_1 and E_2, respectively. Poisson's ratios are assumed to be same with each other. Two cases, where ratio of E_1 to E_2 is 4.0 and 0.25 are simulated. Initial crack is assumed to be in Material 1, and crack length is a, as shown in this figure.

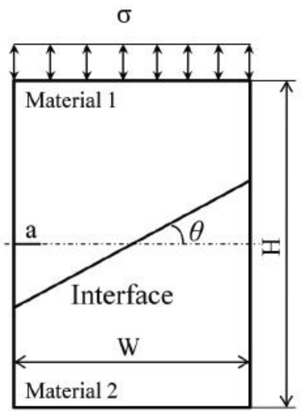
Crack growth is assumed to occur due to fatigue by cyclic stress. Crack growth rate is determined by Paris' law[11], shown in eq.(6), where C=1.67x10^{-12} and n=3.23 assuming aluminum alloy A7075-T6. Crack growth direction, φ, changes by the existance of interface, which satisfied eq.(7) [12], where K_I and K_{II} are mode I and mode II stress intensity factors, respectively.

$$\frac{da}{dN} = C\left(K_{eq}\right)^n \qquad (6)$$

$$K_{\mathrm{I}} \sin\varphi + K_{\mathrm{II}}\left(3\cos\varphi - 1\right) = 0 \qquad (7)$$

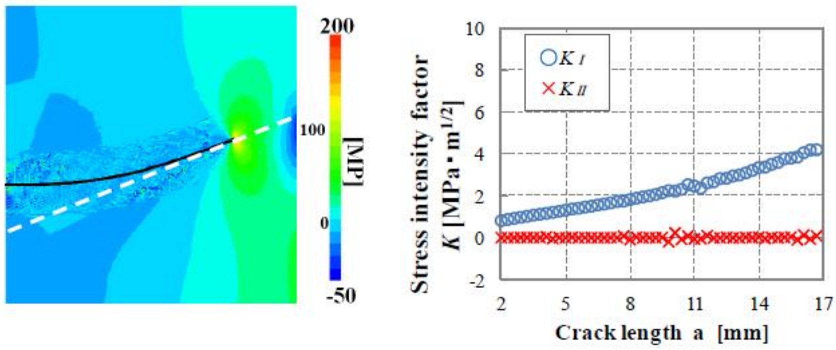
Figure 6 show result when E_1/E_2=4.0 where Young's modulus of material 1 is smaller than that of material 2. Figure 6 (a) shows crack path, and 6 (b) shows changes of K_I and K_{II} during crack growth. As crack tip becomes near to interface, crack path changes gradually, and grows along interface. It does not grow into material 2 across interface. It means that a crack in material 1 prefers to exists in the same material, and does not grow in material 2. During these crack growth process, K_{II} value keeps nearly zero, and K_I increasese monotonically. It means this crack growth is mode I dominant process.

Figure 7 shows results when E_1/E_2=0.25, where Young's modulus of material 1 is larger than that of material 2. In this case, initial crack exists in material 1, and it enters into material 2 easily. When it crosses interface, K_I value decreases suddenly, and again increases gradually. K_{II} value is nearly zero, but it shows small value when crack crosses interface. In material 2, crack changes growing direction a little, and grows perpendicular to cyclic stress direction. In these simulations, strength of interface is not considered. In the real structure, strength of interface affects largely on crack behaviors in heterogeneous material. In this case, crack growth process becomes much complicated. By using this method, it is possible to simulate such complicated phenomenon.



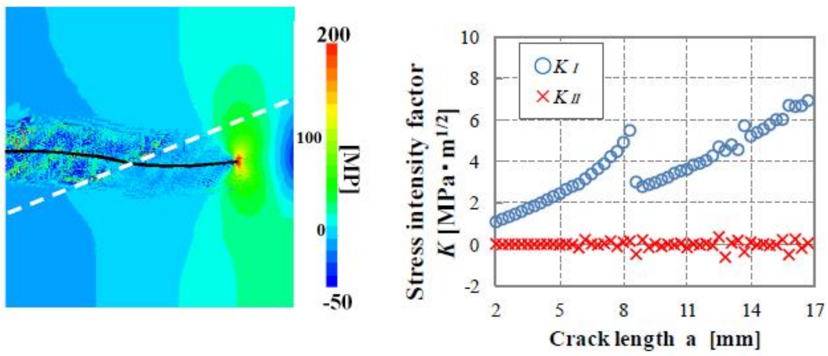
W=20mm,
H=80mm
Initial crack length
a_0=2mm
Cyclic stress $\Delta\sigma$=10Mpa
Stress ratio : R=0.1
$\theta=30^\circ$
E_1/E_2=4.0 or 0.25

Fig.5 Crack in two-phase material with slant interface.



(a) Crack path. (b) Changes of K_I and K_{II}

Fig.6 Results when E_1/E_2=4.0.



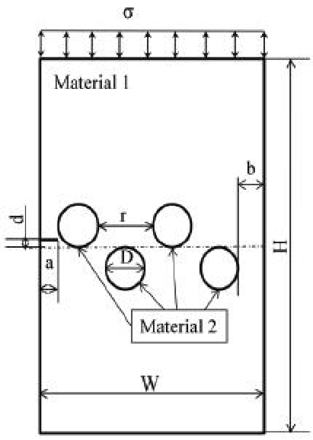
(a) Crack path. (b) Changes of K_I and K_{II}

Fig.7 Results when E_1/E_2=0.25.

CRACK GROWTH IN PARTICLE REINFORCED MATERIAL.

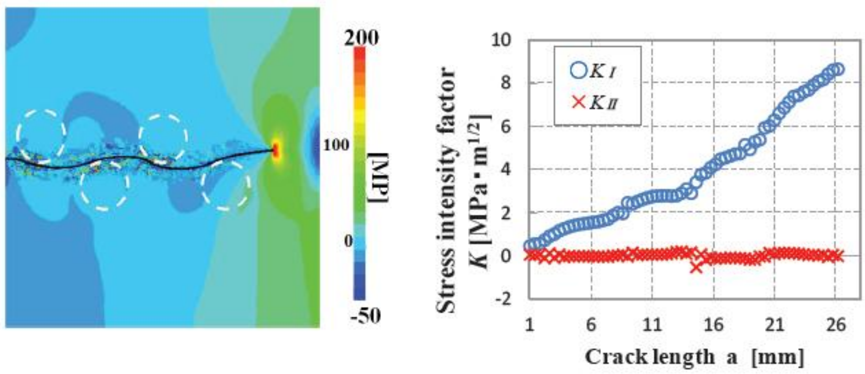
Figure 8 shows a model of particle reinforced material. Four circular particles exist in front of a crack. Young's modulus of particles is 4 times larger than tht of base material.

Crack growth simulation results are shown in Fig. 9. Crack grows winding and by-passing all particles and keeps growing in Material 1. It does not enter particles. SIF is shown in Fig.9 (b). K_I value increases due to crack growth, but increasing rate is changing. K_{II} keeps very low values, but it is not zero, which causes the change of crack path direction. Through these crack growth proceses, crack growth rate is delayed comparing with homogeneous material.



W=30mm
H=120mm
D=5mm
r=12mm
b=12.5mm
Initial crack length a_0=1mm
Cyclic stress $\Delta\sigma$=10Mpa
Stress ratio : R=0.1
E_1=51.5Gpa, E_2=206GPa

Fig.8 Model of particle reinforced plate.



(a) Crack path. (b) Changes of SIF.

Fig.9 Numerical results.

SUMMARY

It is shown that S-FEM successfully simulates complicated crack growth process in heterogeneous material. Though exampes shown in this paper are in two-dimensional fiels, it is easy to apply this method to three-dimensional problem. Using this technique, fatigue crack growth in composite material, and stress corrosion cracking in welded joint may be simulated well in near future.

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