NURBS-based geometric fracture growth representation

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ABSTRACT. *Numerical methods for fracture propagation model fracture growth as a geometric response to deformation. In contrast to the widely used faceted representations, a smooth Non-Uniform Rational B-Spline (NURBS) surface can be used to represent the fracture domain. Its benefits include low cost, resolution-independent storage, and a parametric representation of a smooth domain. In the present work an interaction-free, deformation-informed, Gaussian-based modification algorithm of the fracture surface is presented, with localized stress intensity factor computations, and automatic resolution adjustment, which allow for geometric evolution without the need of appending or re-approximating the fracture surface. It is based on the movement of surface control points and on the systematic editing of weights and knots. It does not require trimming, and is able to shift fracture shape and capture its path evolution efficiently. Throughout growth, the number of points required for fracture representation remains fixed, and the discretization of the fracture surface is implicitly defined by the underlying parametric space. The proposed algorithm can be incorporated into any fracture propagation code that keeps track of fracture geometry and updates it as a function of deformation. The algorithm is demonstrated for a discrete finite element-based fracture propagation method.*

INTRODUCTION

Fractures in rocks are usually created by tectonically-driven events, weathering, or caused by human factors such as those triggered by explosives and hydraulic fracturing. Their creation and effects are modelled by a range of simulators. Multi-physics flow simulators usually rely on an initial, geologically-based fracture representation of the medium to reproduce reservoir conditions. These usually originate from analogue field mappings, or are stochastically generated based on site-specific criteria (see Figure 1). In this context, mechanical simulators focus on modeling the formation and growth of fractures in response to geomechanical deformation. Whereas flow codes require accurate, resolution-independent fracture representation [e.g. 1, 2], mechanical growth codes have the additional requirement of capturing geometric change as a function of fracture propagation. Although NURBS representation of fractures is already widely accepted [3], mainly due to its elegance and resolution-independent storage, the problem of geometric evolution specific to fracture growth is rarely discussed.

Approaches to capture propagation using smooth surfaces include constrained parametric-based extension, cumbersome lofting and stitching of new surface regions, and costly re-approximation of the surface.

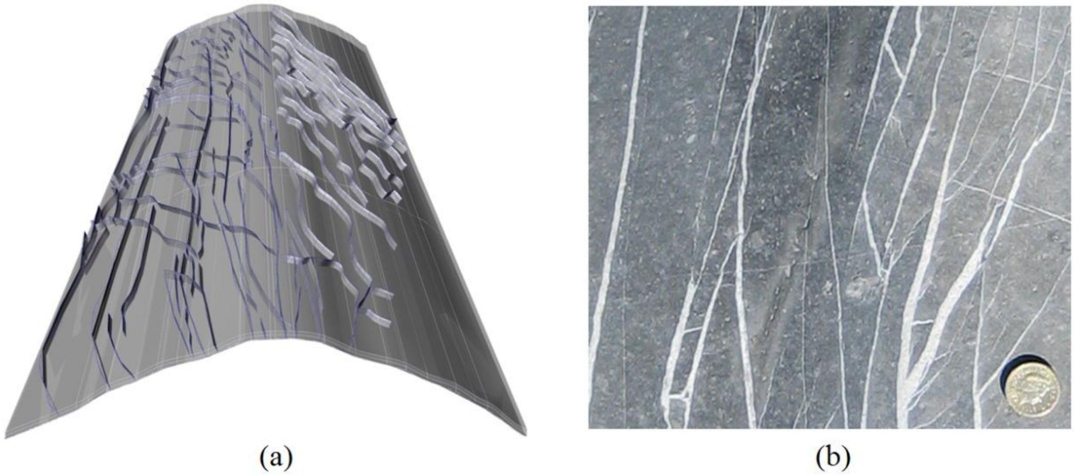


Figure 1. Fractures in the field. (a) NURBS tracing and extrusion of fractures in a limestone fold, (b) detail of centimetre-scale calcite-filled fractures in the field. Both found in Kilve Beach, UK.

Deformation of NURBS is usually assumed in the context of user-driven direct manipulation [e.g. 4, 5]. The two main approaches for direct NURBS manipulation are geometric and physically-based constraint methods. Piegl [4] describes geometric deformation of curves and surfaces in terms of movement of control points and modification of their weights. Hu et al. [6] and Pourazady & Xu [7] describe constraint-based methods for surface deformation that rely on the movement of control points to satisfy externally imposed constraints. These are costly, as they compute geometric change by solving the finite element-based deformation of the surface. Celniker & Welch [8] and Welch & Watkin [9] use linear constraints and global energy minimization functions to solve for deformation. The previous approaches are all suited for interaction-driven deformation in which constraints are typically applied to the body of the shape to sculpt or model a shape. In the specific case of fracture propagation, NURBS shape modification is based on the arbitrary –extension of a surface boundary in response to a physical event.

The focus of the present work is on non-interactive evolution of fracture geometry, expressed at the fracture boundaries, in response to geo-mechanical growth. A constraint-based geometric surface growth method suitable for the modelling of fracture propagation is presented.

FRACTURE GEOMETRY

Seeds for fracture growth are defined as sets of elliptical or circular discs, for which the two major and minor axes have a normal distribution, and whose centres obey a continuous uniform distribution. In many fracture propagation methods, fractures are defined by the mesh. In some cases, as in most FEM approaches, the fracture is defined by set of triangles in the mesh [e.g. 10]. Alternatively, XFEM defines cracks as a set of level-set functions that correspond to specific elements in the mesh, and thus, albeit somewhat independent of its form, the definition of the crack is a function of the mesh. Mesh-free methods keep track of a set of nodes which constitute the fracture. Anisotropic damage models track planes within elements in which fracturing develops. Most of these rely on faceted descriptions of the fracture during growth.

Faceted vs. smooth representation

The main advantage of faceted fracture representation is its storage simplicity. Using this approach, fractures are stored as a set of triangles which readily discretize the fracture shape (see Figure 2). The shape is usually a point-based approximation which originates from experimental or field observations, lab measurements, or numerical simulations. As in the faceted approach, the smooth surface approach also honours these points, but assumes that the geometric variation of the surface in space is smooth. This is advantageous with respect to the discrete respresentation, as it provides a resolution independent representation of the fracture, while remaining low in cost. In particular, NURBS have the additional advantage of providing an implicit and parametric representation of the domain, as has been recently exploited by the increasingly popular Isogeometric method for numerical modelling [cf. 11].

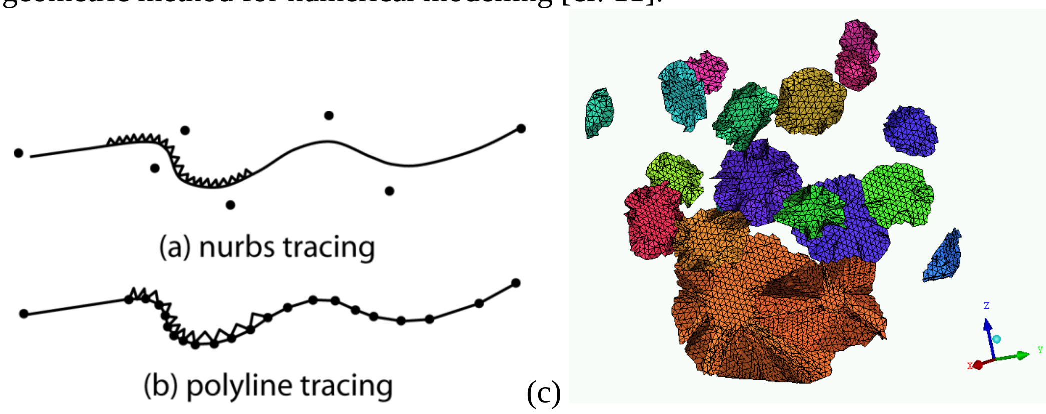


Figure 2. Mesh complexity of fracture representation in 2D and 3D. (a) Low cost NURBS representation of the fracture, (b) Equivalent polyline representation, (c) 3D mesh of fractures.

NURBS representation

A NURBS-based parametric representation provides a framework for the representation and growth of fractures which is independent of the utilized numerical method. For mesh-driven FEM growth algorithms in which fracture geometry is kept independent of the mesh [12], NURBS are a well suited solid modelling approach [3]. NURBS have been suggested as the ideal candidate for geometric housekeeping of fracture growth in the context of mesh-free modeling [13] and have recently been used in the context of crack propagation for XFEM [14].

Their representation is economical and, for cubic splines, C2 continuous: two important factors that allow the geometry to evolve with a strong resolution gradient (more geometric detail at the tips and folds; coarse elsewhere). As the resolution of the NURBS surface is fixed, the storage required to represent it does not increase as a function of fracture surface area (see Figure 3). Therefore, there is no resolution attached to the representation of the fracture, and it can be discretized as a function of local density and proximity to other features. Resolution independence is key to capture stress singularities, which change location during growth.

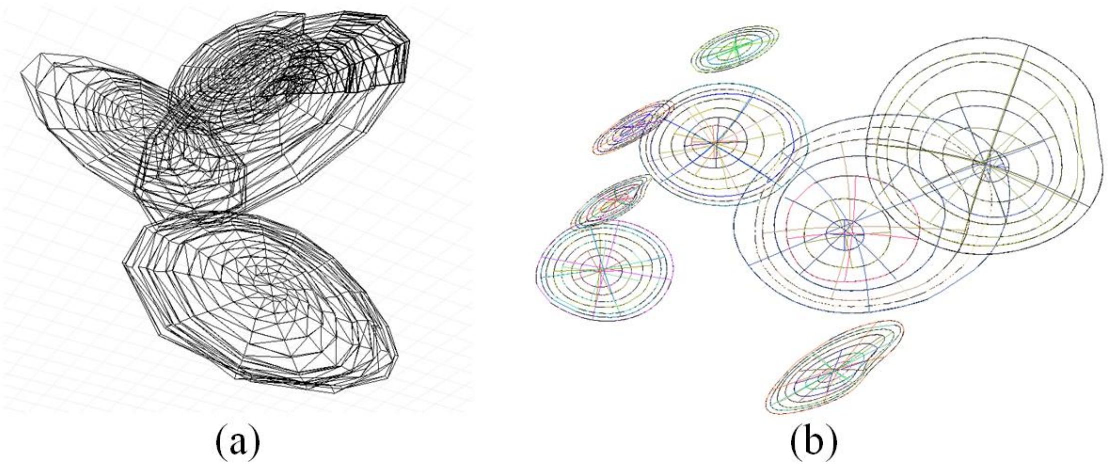


Figure 3. Geometric representation of fractures: (a) polyfacetic, meshed, triangulated surfaces, and (b) smooth, parametric, low cost NURBS. In both cases, the geometry is the result of several iterations of growth.

NURBS description of a single fracture

We assume that the initial description of the fracture can be described by four boundary NURBS curves, which are interpolated using a Coons patch which approximates the fracture surface using a blending function that defines the space between them [15, cf. 16]. The Coons patch provides an initial description of the fracture surface which does not rely on trimming. Therefore, the surface can be manipulated without the concern that topological changes might disturb the trimming function, and deviate from the underlying trimmed patch. The linearly blended Coons patch can be represented as

$$P(u,v) = (1-u)P_0(v) + uP_1(v) + (1-v)Q_0(u) + vQ_1(u) - (1-u)(1-v)P_{0,0} - u(1-v)P_{1,0} - (1-u)vP_{0,1} - uvP_{1,1} \quad (1)$$

where $P_{i,j}$ is given by $P_i(j)$ or $Q_j(i)$, $(0 \leq u \leq 1)$, and $(0 \leq v \leq 1)$.

As opposed to a bilinear interpolation which only uses the intersection corner points as input, the Coon's patch blends the definition of the four boundary curves. Thus, there are four initial curves which represent the tip boundary. These are Bezier curves, which in turn can be expressed as splines, i.e. NURBS. It follows that a Coons patch can also be expressed as a NURBS surface by creating the internal control points of the patch [17].

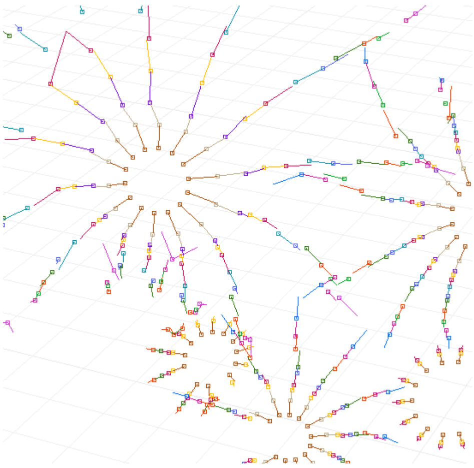


Figure 4. Propagation vectors, $\overrightarrow{p_v}$, for five growing fractures.

The geometry of the fracture is stored in three dimensions as a NURBS surface, which can be defined rationally as the tensor product of two NURBS curves, of polynomial orders n and m, and basis functions N and M [cf. 18]:

$$S(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} P_{i,j} N_{i,p}(u) M_{j,q}(v) = \sum_{i=0}^{n}\sum_{j=0}^{m} P_{i,j} R_{i,k,j,l}(u,v) \quad (2)$$

where $S(u,v)$ is a point on the surface as a function of parameters u and v, P_i is the vector of control points, and

$$R_{i,k,j,l}(u,v) = \frac{w_{i,j} N_{i,k}(u) N_{j,l}(v)}{\sum_{r=0}^{n}\sum_{s=0}^{m} w_{r,s} N_{r,k}(u) N_{s,l}(u)} \quad (3)$$

where $N_{i,k}$ are the normalized B-Spline basis functions of k-degree, defined recursively as

$$N_{i,k} = \frac{u - t_i}{t_{i+k} - t_i} N_{i,k-1}(u) + \frac{t_{i+k+1} - u}{t_{i+k+1} - t_{i+1}} N_{i+1,k-1}(u) \quad (4)$$

and

$$N_{i,0}(u) = \begin{cases} 1, \text{ if } t_i \le u \le t_{i+1} \\ 0, \text{ else} \end{cases} \quad (5)$$

where t_i are the knots, which form a knot-vector. Together, knots, control points and weights define the smooth shape of the surface. For more details on their specific roles, the reader may consult Farin [16].

COMPUTATION OF THE DEFORMATION CONSTRAINT

The fracture tip is inherently defined by the boundaries of the curve. Discretization is given by the underlying parametric space of the surface, also known as the *control net* [19] of the surface. The surface boundary is discretized into a set of sequential tips, composed by the boundaries of the underlying NURBS subnet for which a local stress intensity factor is computed. The method to do so will depend on the underlying numerical method being used to compute deformation.

Growth is characterized by a set of three laws: failure (e.g. sub-critical, Rankine, Coulomb laws), propagation (e.g. Paris, Walker laws) and growth angle (e.g. maximum circunferential stress). For a given stress intensity factor and a set of growth laws a set of polydisperse propagation vectors for each tip is defined [cf. 20, 12], each vector corresponds to the propagation of a specific tip region of the crack (see Figure 4). The growth constraint is implicitly defined by a set of propagation vectors, $\overrightarrow{p_v}$, which generate a new fracture tip, which is rarely coplanar to the previous and lies at a varying distance from the original.

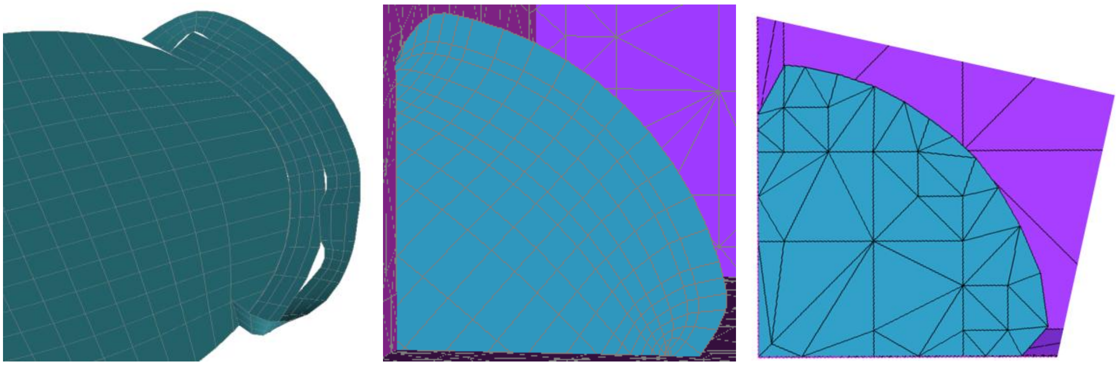
FRACTURE GROWTH

Unlike faceted fractures, smooth fracture growth cannot be achieved by adding triangles/quadrilaterals to the fracture representation. In order to faithfully capture growth within the original NURBS representation, the surface must be refitted or modified. The modification of existing surfaces [4] provides the advantage of retaining a single geometric entity to represent each fracture, avoiding possible domain inconsistencies due to stitching of lofted surface extensions (see Figure 5a), lengthy refitting operations required when re-approximating, and trimming curve definitions.

Growth Algorithm

The extension of the fracture NURBS representation is subdivided into the following steps:

1. Definition of punctual constraints
2. Knot insertion at surface boundary to enhance local level of detail [cf. 18]
3. Control point movement based on mid-range Gaussian influence of the applied constraints [21].



(a) (b) (c)

Figure 5. Parametric representation of a growing fracture, originally disc-shaped and modified in response to deformation, using (a) lofting, (b) NURBS constraint-based growth, and (c) ensuing mesh.

For each tip location, T_o, a punctual constraint described by a set of three parameters [22] is defined as

- a *space constraint*, defined by the new tip location $T_f = T_o + \overrightarrow{p_v}$, given by the computed propagation vector,
- a *parametric constraint*, closest NURBS subnet boundary point to the new tip location, which may coincide with the parametric location of the current tip location, $\left(u_{T_o}, v_{T_o}\right)$,
- a *localization constraint*, $f(i,j)$, a function that defines the influence of the growth constraint on the rest of the surface, where

$$f: [0, n-1] \times [0, m-1] \rightarrow R^+ \\ \text{for } \exists(i,j) \in [0, n-1] \times [0, m-1] \\ \text{and } R_{i,j}(u,v) f(i,j) \neq 0. \quad (6)$$

Thus, $R_{i,j}(u,v) = f(i,j)$, is the natural influence of the constraint on its vicinity, i.e. the influence given by the degree of the NURBS. It follows that the punctual constraints are applied using a sequential iterative algorithm to obtain a set of displacement vectors, which are then applied to the NURBS' control points, defined as

$$\overrightarrow{m(\iota,j)} = \frac{f(i,j)}{\sum_k^{n-1} \sum_l^{m-1} \left[R_{k,l}(u,v) f(k,l)\right]} \overrightarrow{p_v} \quad (7)$$

The above discussion sets the grounds for the implementation of NURBS-based shape functions for elements at and around the tips, which within a hybrid mesh will improve quality by allowing integration and interpolation to be computed directly on the geometry. This is in line with the novel Isogeometry developments in the FEM field [19].

CONCLUSIONS

A method has been presented to grow NURBS fracture surfaces using a set of stress-intensity factor-dependent constraints. The presented algorithm is tailored for fracture growth which follows the extension of fractures along specific boundaries, with a variation of angles, and with increasing level of detail gained by adding curvature to the growing fracture surface. The shape of the NURBS is directly modified as a function of growth, by using an iterative control point movement algorithm for stable geometric-based NURBS modification.

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