Mixed Mode Crack Growth for Titanium Alloy in Specimen Various Geometries

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ABSTRACT. *Crack path prediction for inclined crack has been carried out making use the equivalent crack size conception. The subjects for studies are a titanium center cracked plate and a compact tension-shear specimen. The criteria of maximum tangential stress, strain energy density and generalized Pisarenko-Lebedev theory are applied to the problems of crack paths modeling. The predicted directions of crack extension and crack paths are compared with experimental results conducted on the titanium specimens. It is stated that three criteria differ in the whole range of fatigue mixed mode fracture. The angle of crack propagation and crack path under mixed mode fracture, predicted according to these criteria, presents an improved fit to the experimental data, when the influence of the second order term in the expansion of stresses in series of crack tip distance is taken into account.*

INTRODUCTION

There are many opportunities in practice for cracks in engineering components to exist in orientations that induce mixed mode crack tip displacement. Cracks that have grown under cyclic loading usually change direction in response to applied stresses. Main feature of mixed-mode fracture is that the crack growth would no longer take place in a self-similar manner and does not follow a universal trajectory that is it will grow on a curvilinear path. For mixed mode crack propagation, the crack front is continuously changing shape and direction with each loading cycle. As a result, the angle of crack propagation θ^* changes continuously. A number of criteria is available for the prediction of both brittle and ductile fracture and direction of initial crack extension. Most of them are defined using either some aspects of the stress-strain field existing prior to start of propagation or some modification thereof occurring as a consequence of the extension. This study deals with an application of three such criteria to the problem of crack path prediction under mixed mode loading. All considered criteria are generalized to take into account the effects of both the T-stress and a fracture process zone size. The computed crack paths based on these criteria are then compared with the experimental results for titanium specimens subjected to mixed mode loadings.

CRACK REORIETATION CRITERIA

For the determination of the crack path, neglecting the effect of any redistribution of stresses caused by the propagation, it is necessary to have a knowledge the entire stress field including the near-tip distribution. The formulae for all cartesian components for the biaxially loaded inclined crack, containing the non-singular second term, are given in ref [1]:

$$\begin{bmatrix} \sigma_{xx}\sigma_{xy} \\ \sigma_{yx}\sigma_{yy} \end{bmatrix} = \frac{K_1}{\sqrt{2\pi r}} \begin{bmatrix} f_{1,xx(\theta)} f_{1,xy(\theta)} \\ f_{1,yx(\theta)} f_{1,yy(\theta)} \end{bmatrix} - \frac{K_2}{\sqrt{2\pi r}} \begin{bmatrix} f_{2,xx(\theta)} f_{2,xy(\theta)} \\ f_{2,yx(\theta)} f_{2,yy(\theta)} \end{bmatrix} + \begin{bmatrix} T & 0 \\ 0 & 0 \end{bmatrix} \quad (1)$$

where T is the non-singular second order term or the T-stress, r and θ are polar coordinates centered at the crack tip. The following equations

$$K_1 = \frac{\sigma\sqrt{\pi a}}{2}\left[(1+\eta)-(1-\eta)\cos 2\alpha\right]Y_1(a/w); \quad \mathrm{K}_2 = \frac{\sigma\sqrt{\pi a}}{2}\left[(1-\eta)\sin 2\alpha\right]Y_2(a/w) \quad (2)$$

are prescribed the mode I and II stress intensity factors K_1 and K_2, respectively. In these equations σ is the nominal stress applied in the Y-axis direction, α is the inclined crack angle referred to the Y-axis, η is the biaxial stress ratio, a is the crack length, w is characteristic specimen size. From equations (1), for the nominal stress biaxial ratio $\eta=0$, one finds the corresponding expressions for the uniaxially loaded inclined crack.

In the present study the solution of the problem of crack path prediction under mixed mode conditions based on the use the following criteria. All considered criteria are generalized so that to take into account the effects of both the T-stress and a fracture process zone size r_c.

The maximum tensile stress criterion (MTS)

In accordance to the MTS criterion, the crack is considered to extend in the radial direction θ^* given by point of maximum tangential stress at a critical radius r_c in front of the crack tip [2]:

$$\bar{\sigma}_{\theta\theta}^{\max} = 0.5\sqrt{\frac{1}{32}\left(\frac{a}{r_c}\right)}\left[\frac{1}{2}F_{K_1}\left(3\cos\frac{\theta}{2}+\cos\frac{3\theta}{2}\right)-\frac{3}{2}F_{K_2}\left(\sin\frac{\theta}{2}+\sin\frac{3\theta}{2}\right)\right]+\bar{T}\sin^2\theta\,,$$

$$\frac{\partial\bar{\sigma}_{\theta\theta}}{\partial\theta}=0; \quad \frac{\partial^2\bar{\sigma}_{\theta\theta}}{\partial\theta^2}\langle 0\,. \quad (3)$$

The minimum strain energy density criterion (SED)

The SED criterion proposes a radial extension in the direction θ^* corresponding to the location of minimum strain energy density at a critical distance r_c ahead of the crack tip [3]:

$$\left(\overline{W}\right)=\frac{d\overline{W}}{dV}=\left[\left(\frac{a}{r_c}\right)\overline{S}_1+\sqrt{\frac{a}{r_c}}\overline{S}_2+\overline{S}_3\right]; \tag{4}$$

$$\overline{S}_1\left(\overline{T}\right)=\frac{1+\nu}{2}\left(a_{11}F_{K_1}^2+a_{12}F_{K_1}F_{K_2}+a_{22}F_{K_2}^2\right);\ \overline{S}_2\left(\overline{T}\right)=\frac{1+\nu}{2}\left(b_{11}F_{K_1}+b_{22}F_{K_2}\right);$$

$$\overline{S}_3\left(\overline{T}\right)=\frac{1+\nu}{8}\overline{T}\cdot C_T;\ \frac{\partial\overline{W}}{\partial\theta}=0;\ \frac{\partial^2\overline{W}}{\partial\theta^2}\rangle 0$$

The Pisarenko-Lebedev criterion (PL)

This criterion represents a superposition of elastic and plastic limiting state theories [4]

$$\chi\theta_1^*(\sigma_e)+(1-\chi)\theta_2^*(\sigma_{\theta\theta})=\theta^*. \tag{5}$$

The linear-elastic material behavior is described by the maximum tensile stress criterion (Eq. 3), while the plastic limiting state is related to the von Mises theory in the form of effective stress σ_e

$$\begin{aligned}\overline{\sigma}_e^2&=(a/2r_c)\left[F_{K_1}^2\left(f_{1,xx}^2+f_{1,yy}^2-f_{1,xx}f_{1,yy}+3f_{1,xy}^2\right)+F_{K_2}^2\left(f_{2,xx}^2+f_{2,yy}^2+f_{2,xx}f_{2,yy}+3f_{2,xy}^2\right)+\right.\\&+F_{K_1}F_{K_2}\left.\left(-2f_{1,xx}f_{2,xx}+2f_{1,yy}f_{2,yy}-f_{1,xx}f_{2,yy}+f_{2,xx}f_{1,yy}+6f_{1,xy}f_{2,xy}\right)\right]+\\&+\overline{T}\sqrt{a/2r_c}\left[F_{K_1}\left(-2f_{1,xx}-f_{1,yy}\right)+F_{K_2}\left(2f_{2,xx}-f_{2,yy}\right)\right]+\overline{T}^2\end{aligned} \tag{6}$$

The angle of the crack propagation θ* has to be determined according to the following conditions:

$$\begin{cases}\dfrac{\partial\sigma_e}{\partial\theta}=0; & \dfrac{\partial^2\sigma_e}{\partial\theta^2}\rangle 0\\[2ex] \dfrac{\partial\sigma_{\theta\theta}}{\partial\theta}=0; & \dfrac{\partial^2\sigma_{\theta\theta}}{\partial\theta^2}\langle 0\end{cases}.$$

The stress intensity factor functions containing to equations (3-5) can be written as

$$F_{K_1}=0.5\left[(1+\eta)-(1-\eta)\cos 2\alpha\right]\cdot Y_1\left(\alpha,a/w,\overline{T},\eta\right); \tag{7}$$
$$F_{K_2}=0.5(1-\eta)\sin 2\alpha\cdot Y_2\left(\alpha,a/w,\overline{T},\eta\right).$$

DETERMINING T-STRESS AND STRESS INTENSITY FACTORS

In order to use all generalized criteria to facilitate prediction of crack path it is necessary to determine at each successive position of the crack front, the stress intensity factors (SIF), K_1 and K_2, and T-stress. However, for the actual bent crack geometry, the expressions for the SIF cannot be easily determined. To overcome this difficulty an

Table 1. Stress components and SIF functions on the crack flanks

stress components	center cracked plate	compact tension-shear specimen
$\underline{\theta=+\pi}$ $\sigma_{xx}=-2\frac{K_2}{\sqrt{2\pi r}}+T$, $\sigma_{yy}=0$, $\sigma_{xy}=0$	$Y_2\left(\frac{a}{w}\right)=\frac{(T-\sigma_{xx})}{\sigma}\sqrt{\frac{2r}{a}}\frac{1}{(1-\eta)\sin 2\alpha}$	$Y_2\left(\frac{a}{w}\right)=\frac{(T-\sigma_{xx})}{2\sigma}\sqrt{\frac{2r}{a}}\frac{1}{\cos\alpha}$
$\underline{\theta=-\pi}$ $\sigma_{xx}=2\frac{K_2}{\sqrt{2\pi r}}+T$, $\sigma_{yy}=0$, $\sigma_{xy}=0$	$Y_2\left(\frac{a}{w}\right)=\frac{(\sigma_{xx}-T)}{\sigma}\sqrt{\frac{2r}{a}}\frac{1}{(1-\eta)\sin 2\alpha}$	$Y_2\left(\frac{a}{w}\right)=\frac{(\sigma_{xx}-T)}{2\sigma}\sqrt{\frac{2r}{a}}\frac{1}{\cos\alpha}$

approximate procedure has been independently proposed by authors [5,6]. Essentially, the procedure involves replacing the bent crack with a straight-line crack approximation. The present work explores direct use of FEM analysis for calculating *T*-stress on the base of crack flank nodal displacements [7]. Using this technique, first of all the *T*-stress distributions in various specimen geometries was determined from numerical calculations. Then on this basis the solutions for mode I and mode II stress intensity factors K_I and K_{II} for each specimen geometry have been obtained. Table 1 present equtions describing the stress components and SIF functions on both the upper and lower crack flanks.

Subjects for numerical studies are central notched plate and compact tension-shear specimen (Fig.1). All investigated configurations contain an internal crack of length *2a* for CNS or *a* for CTS. The initial crack makes on angle α with the loading direction. By changing α, different combinations of modes I and II are achieved. In the CTS α = 90º correspond to pure mode I and pure mode II can be achieved when α = 0º. For the CNS α = 90º correspond to pure mode I.

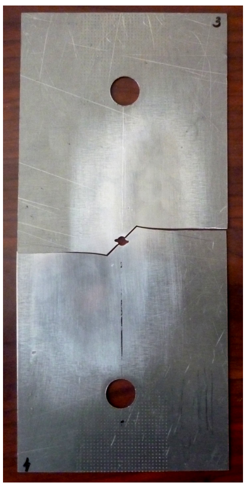

Figure 1. Central notched and compact tension shear fracture specimens

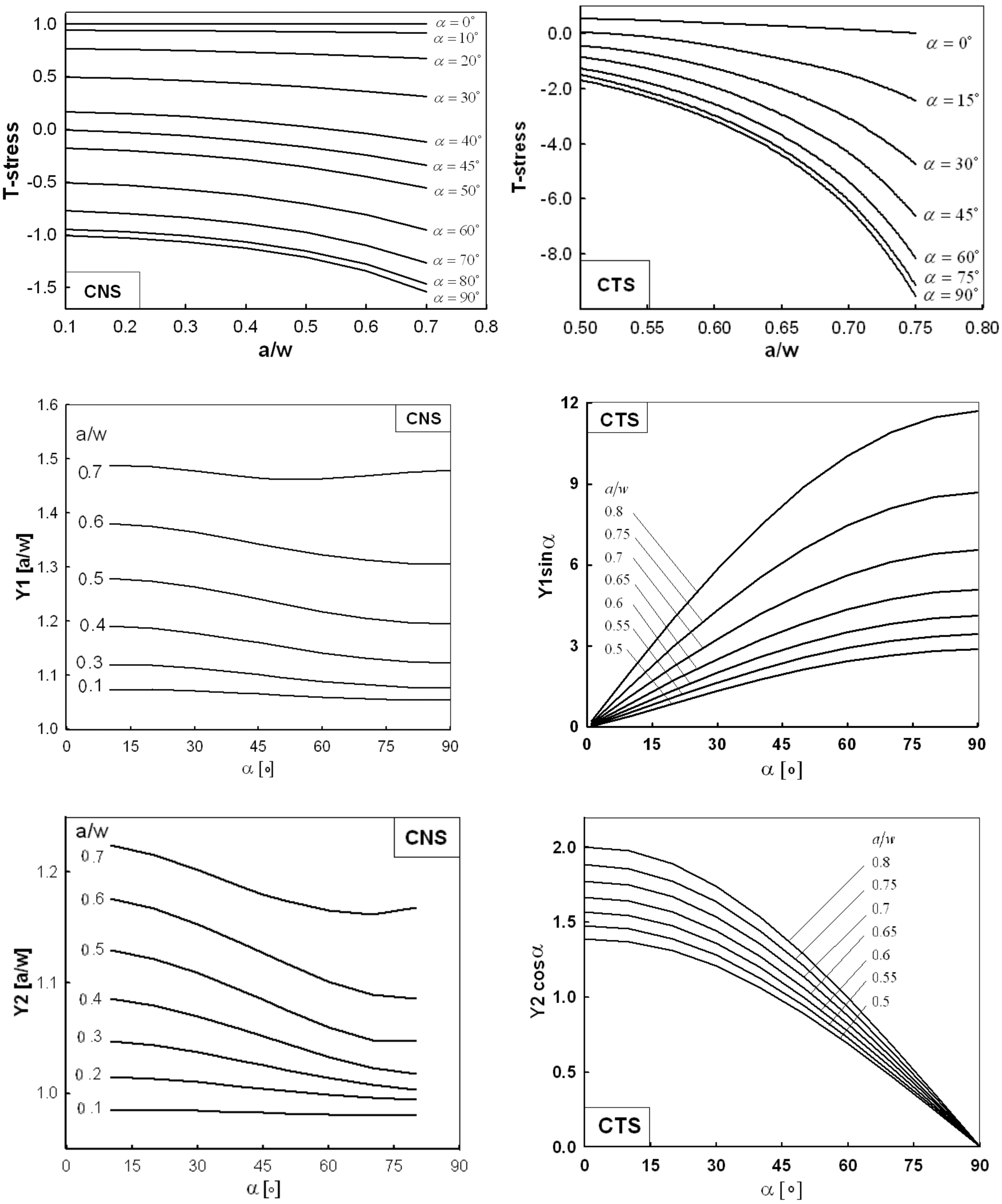

Figure 2. Mixed mode T-stress SIF distributions in CNS and CTS specimen geometries.

RESULTS AND DISCUSSION

The normalized T-stress distributions of CNS and CTS fracture specimen geometries under mixed mode loadind conditions are determined from finite element calculations.

Figure 2 presents a graph showing the results of the T-stress calculation ahead of the crack-tip (θ=0º) as a function of an initial crack angle α and relative crack length a/w for analyzed specimen geometries. It should be noted that the deviation of current value of T from the corresponding original value for a/w=0.1 (or a/w=0.5 for CTS) increases with increasing relative crack length at fixed crack angle position.

The stress approach represented by equations in Table 1 has been employed to computing mode I and mode II stress intensity factors in the specimen each configuration. Graphs showing the SIF distributions accounting for the T-stress values on the upper and the lower edges of the crack for both CNS and CTS geometries are presented in Fig 2.

This work is centered on the role of the crack reorientation criteria during mixed mode fatigue crack growth. Equations (3-7) are applied for analyzing the fatigue crack growth trajectories in specimens with the previous geometries (Fig.1). On the CTS was realized the full range of mixed mode fracture from tensile (pure Mode I) to shear (pure Mode II) loading. The CNS subjected to uniaxial tension as the initial inclination angle α is varied from 15 to 90º.

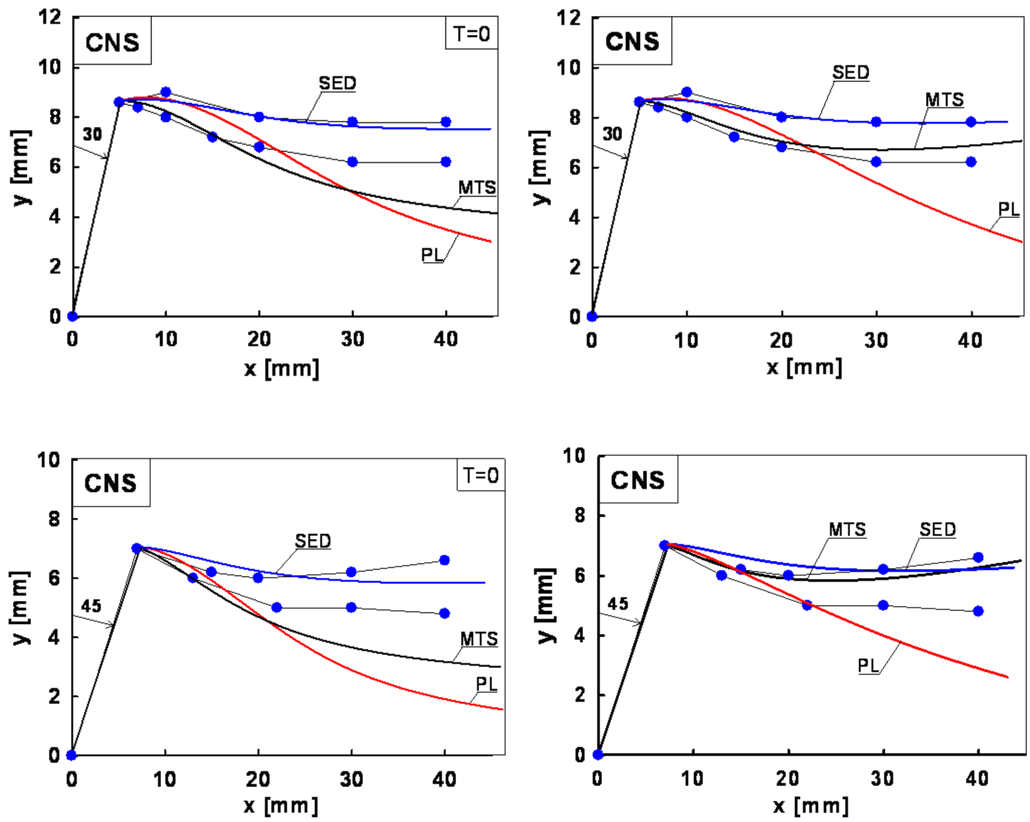

Figure 3. Comparison of crack paths based on the three criteria in the case of central notched specimen.

It is known that a "bent" crack does not propagate in its initial orientation direction. A mixed mode crack propagates along a definite trajectory which is determined by the stress state, the previous crack orientation angle and the material properties including critical distance r_c. Moreover, in mixed mode conditions the T-stress contributes to the near crack-tip stress fields. Under these conditions, in order to predict the fatigue crack propagation rate, it is necessary to determine it by means of calculations. A fatigue crack may be assumed to grow in a number of discrete steps. The initial values of α, T, a, r_c and θ* is determined by the crack growth direction criterion. After each increment of the crack growth, the crack angle α changes from the original angle and that makes the effective or equivalent length of the crack. For the next increment of crack growth, one has to consider the new crack length a_i and crack angle α_i. More details of theoretical crack path prediction are presented by the author [8,9].

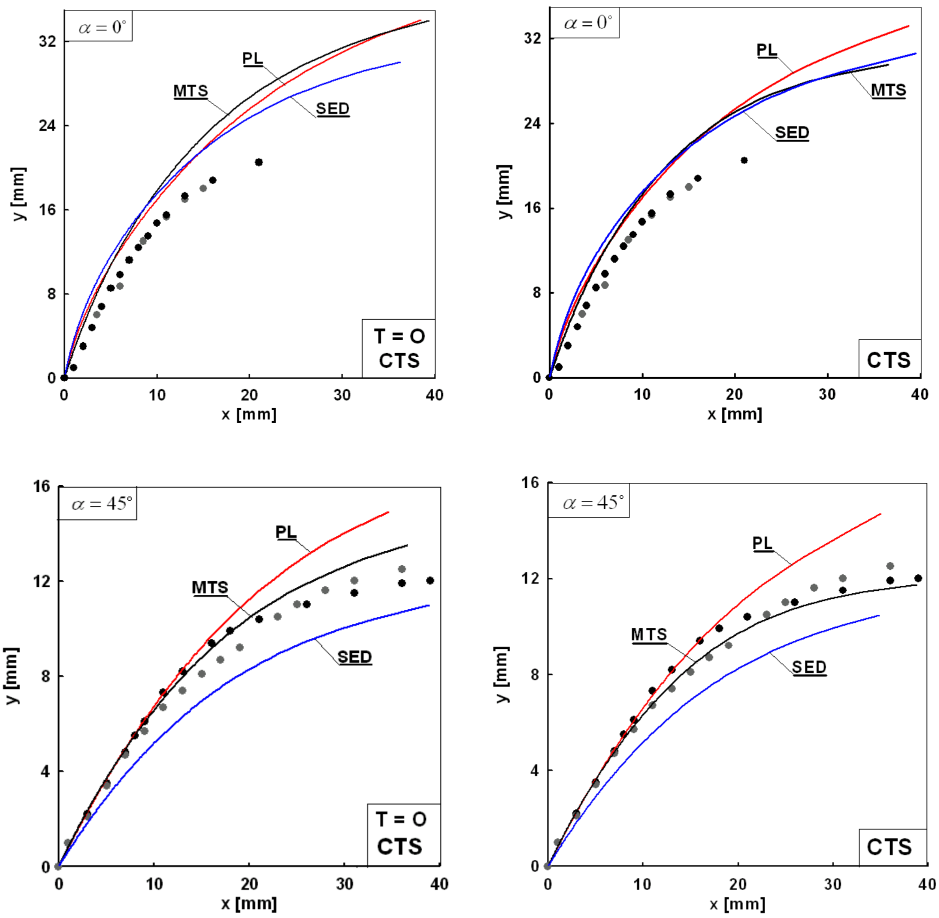

Figure 4. Comparison of crack paths based on the three criteria in the case of compact tension shear specimen.

Figures 3 and 4 present a comparison of both computational and experimental fatigue crack paths for titanium alloy. The main mechanical properties for the titanium alloy are: yield stress 508 MPa, ultimate tensile stress 529 MPa, strain hardening exponent 9.29. The comparisons of the criteria indicate that their predictions are generally different and in view of the scatter in experimental results all of them appear to have some degree of applicability. The difference is dependent on loading direction and geometric configuration. The modification in the definition of the criteria of MTS, SED and PL consisting in taking into account the T-stress and variable critical distance has a pronounced effect on the prediction of crack paths under mixed mode fracture. For each criterion the results of the corresponding original method (i.e. the one not considering the influence of the T-stress) are differ from generalized one. In particular it is observed that, the crack paths from the modified SED, compared to those other criteria, are closer to test results and the difference between original and improved SED is smaller than that for the other criteria.

Another remark concerns the crack path predictions for the various specimen geometries. As it follows from the comparison the PL criterion is the least satisfactory for the central notched specimen. However, at the same time this criterion agreeable describes experimental trajectories for the compact tension shear specimens.

The general observation is that the consideration of the influence of the T-stress which variable along crack length and take into account the critical distance in the generalized fracture reorientation criteria improved fit to the experimental data.

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