Mixed mode cohesive crack growth at the bi-material interface between a dam and the foundation rock

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ABSTRACT.*One of the weakest points in concrete dams occurs at the rock/concrete interfaces at the base of the dam. This has prompted interest in studying the interface laws that can best represent the interaction between damage due to normal stresses and damage caused by tangential stresses (traction-separation law) in the process zone, within the framework of the cohesive crack model. Karihaloo and Xiao [1] proposed considering the Coulomb friction between the crack faces, instead of a tangential cohesive relationship. This model is called the cohesive-frictional crack model,and it is different from the model of frictional contact of crack faces, because the friction operates when the crack faces are open. In their paper,the above mentioned authors published an asymptotic expansion of a crack propagating along a joint between homogeneous materials. In the present paper, a new asymptotic expansion is presented and applied for the case of a crack propagating at a bi-material interface.*

INTRODUCTION

Cohesive crack models are an important means of describing localisation and failure in engineering structures,with reference to quasi-brittle materials. When these models are adopted, the stresses acting on the non-linear fracture process zone are considered as decreasing functions of the displacement discontinuity. These functions are assumed to be material properties through the use of a pre-defined softening law. When the model is applied at a real structure scale, the process zone is fully developed, and the displacement discontinuity shows at least two components: the normal component and the tangential one. In these conditions, the equilibrium iterations encounter difficulties in converging. This is a sign that more than one incremental solutions exists. The best way of dealing with these problems is that of considering an analytical solution of the mechanical problem in a pre-defined sub-domain. This method is called the Generalised FEM (GFEM). Karihaloo and Xiao [1] have obtained an asymptotic expansion of the stress and strain fields that arise around the faces of a fictitious crack growing in Mixed Mode (Mode I and II) conditions at the interface between two identical materials. A new asymptotic expansion, which can be applied at a bi-material interface,is presented in this paper.

THE MODEL

Theoretical investigations on the problem of interface cracks between dissimilar media date back to the late fifties. Williams [2] performed an asymptotic analysis of the elastic fields at the tip of an open interface crack and found that the stresses and displacements behave in a oscillatory manner.Malyshev and Salganik [3] discussed the implications of the oscillatory fields and made the following comment:"For opposite faces of the cut, the result is physically absurd that is they are penetrating each other.The fault of the mathematical model can be corrected if it is supposed that the opposite faces taking mutually convex shapes start to press into each other forming contacting areas". They also argued that, if the length of the cohesive zone in a Barenblatt-Dugdale type model is greater than the region of stress oscillations,the latter can be disregarded near the crack tip.

Polynomial cohesive law for quasi-brittle materials

In order to obtain a separable asymptotic field at a cohesive crack tip, in terms of r and θ functions, (see Figure 1) in quasi-brittle materials, the softening law has been reformulated into the following polynomial form:

$$\frac{\sigma_y}{\sigma_0} = \frac{\tau_{xy}}{\mu_f \sigma_0} = 1 + \sum_{i=1}^{L} \alpha_i \left(\frac{w_{eff}}{w_{eff,c}}\right)^{\frac{(2i-1)}{3}} - \left(1 + \sum_{i=1}^{L} \alpha_i\right) \left(\frac{w_{eff}}{w_{eff,c}}\right)^{\frac{2L+1}{3}} \tag{1}$$

where $\left(\sigma_0, -\mu_f \sigma_0\right)$ is a point on the failure envelope, α_i,$i = 1...L$,are fitting parameters and σ_y is the stress normal to the cohesive crack faces;w_{eff} and $w_{eff,c}$ are the effective opening displacement of the cohesive crack faces and its critical value,respectively. Eq.(1) can represent a wide variety of softening laws, and it satisfies the following requirements:for $w_{eff}/w_{eff,c} = 0$ one obtains $\sigma_y/\sigma_0 = 1$ at the tip of the cohesive crack (fictitious crack tip,shortening FCT) ; and for $w_{eff}/w_{eff,c} = 1$ one obtains $\sigma_y/\sigma_0 = 0$ (see Figure 1) at the tip of the pre-existing traction-free macrocrack(real crack tip). In the present paper,the softening law proposed in [1] has been used with the coefficients:$\alpha_1 = 0.096$,$\alpha_2 = -10.063$,$\alpha_3 = 28.738$,$\alpha_4 = -37.847$ and $\alpha_5 = 23.955$ (see Figure 2).

Asymptotic fields at the tip of a cohesive crack

The adopted mathematical formulation closely follows that used by Karihaloo and Xiao [1]. Muskhelishvili showed that, for plane problems, the stress and displacements in the Cartesian coordinate system can be expressed in terms of two analytic functions, $\phi(z)$ and $\chi(z)$, of the complex variable $z = re^{i\theta}$

$$\sigma_x + \sigma_y = 2[\phi'(z) + \overline{\phi'(z)}] \tag{2}$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = 2[\overline{z}\phi''(z) + \chi''(z)] \tag{3}$$

$$2\mu(u + iv) = k\phi(z) - z\overline{\phi'(z)} - \overline{\chi'(z)} \tag{4}$$

where a prime denotes differentiation with respect to z and an overbar denotes a complex coniugate. In Eq.(4), $\mu = E/[2(1 + \nu)]$ is the shear modulus; the Kolosov constant is $\kappa = 3 - 4\nu$ for plane strain and $\kappa = (3 - \nu)/(1 + \nu)$ for plane stress; E and ν are Young's modulus and Poisson's ratio, respectively. For a general mixed mode I+II problem, the two analytic functions $\phi(z)$ and $\chi(z)$ can be chosen as series of complex eigenvalue Goursat functions (Sih and Liebowitz [5])

$$\phi_1(z) = \sum_{n=0} A_n z^{\lambda_n} = \sum_{n=0} A_n r^{\lambda_n} e^{i\lambda_n\theta},\ \chi_1(z) = \sum_{n=0} B_n z^{\lambda_n+1} = \sum_{n=0} B_n r^{\lambda_n+1} e^{i(\lambda_n+1)\theta} \quad (5)$$

$$\phi_2(z) = \sum_{n=0} G_n z^{\lambda_n} = \sum_{n=0} G_n r^{\lambda_n} e^{i\lambda_n\theta},\ \chi_2(z) = \sum_{n=0} H_n z^{\lambda_n+1} = \sum_{n=0} H_n r^{\lambda_n+1} e^{i(\lambda_n+1)\theta} \quad (6)$$

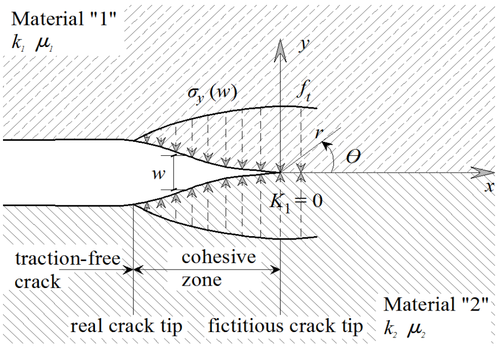

Figure 1. A traction free-crack at a bi-material interface.

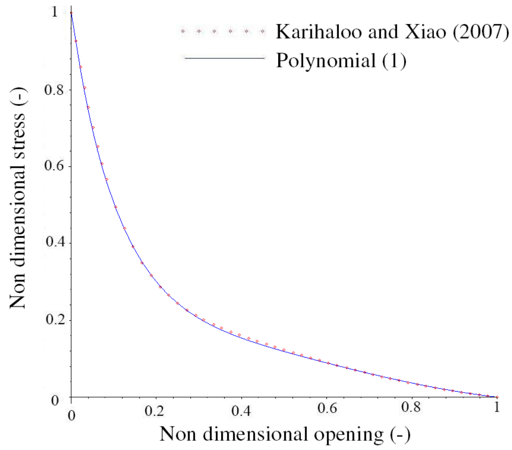

Figure 2. Cohesive law comparison [4].

Eq.(5) is applied to material 1 in Fig. 1 ($0 \leq \theta \leq \pi$) while Eq.(6) is applied to material 2 ($-\pi \leq \theta \leq 0$), where the complex coefficients are $A_n = a_{1n} + ia_{2n}$,$B_n = b_{1n} + ib_{2n}$,$G_n = g_{1n} + ig_{2n}$ and $H_n = h_{1n} + ih_{2n}$. The eingenvalues, λ_n and coefficients $a_{1n}, a_{2n}, b_{1n}, b_{2n}, g_{1n}, g_{2n}$,$h_{1n}$ and h_{2n} are real. By substituting the complex functions (5) in Eqs(2),(3) and (4), the complete series expansion of the displacements and stresses near the tip of the crack can be written exactly as in Karilahoo and Xiao [1].The coefficients a_{1n},a_{2n},b_{1n} and b_{2n} are used in the case of material 1. The coefficients g_{1n},g_{2n},h_{1n} and h_{2n} are used for material 2.For more details see Alberto A.,Barpi F. and Valente S. [6].

The conditions at the bi-material interface

The opening displacement (COD) of the crack faces can be written as $w = v\Big|_{\theta=\pi} - v\Big|_{\theta=-\pi}$:

$$w = \sum_{n=0} \frac{r^{\lambda_n}}{2} \left[\frac{k_1 + \lambda_n}{\mu_1} a_{1n} + \frac{\lambda_n + 1}{\mu_1} b_{1n} + \frac{k_2 + \lambda_n}{\mu_2} g_{1n} + \frac{\lambda_n + 1}{\mu_2} h_{1n} \right] \sin \lambda_n \pi \quad (7)$$

and the sliding displacement (CSD) can be written as $\delta = u\Big|_{\theta=\pi} - u\Big|_{\theta=-\pi}$:

$$\sum_{n=0} \frac{r^{\lambda_n}}{2} \left[\frac{\lambda_n - k_1}{\mu_1} a_{2n} + \frac{\lambda_n + 1}{\mu_1} b_{2n} + \frac{\lambda_n - k_2}{\mu_2} g_{2n} + \frac{\lambda_n + 1}{\mu_2} h_{2n} \right] \sin \lambda_n \pi \tag{8}$$

The formulation of the problem shown in Figure 1 can be assessed by establishing continuity, in terms of stress and displacement, when $\theta = 0$ (the two materials are bonded along the line of the crack extension), and when $\theta = \pm\pi$ (cohesive crack surfaces). The stresses at the cohesive crack tip are non-singular (because the stress intensity factors are $K_1 = K_2 = 0$). The above mentioned conditions can be summarised as follows.

Cohesive frictional crack with normal cohesive separation
The following conditions need to be satisfied ($\theta = \pm 0$, two materials are bonded):

$$u|_{\vartheta=0^+} = u|_{\vartheta=0^-} \ , \ v|_{\vartheta=0^+} = v|_{\vartheta=0^-} \ , \ \sigma_y|_{\vartheta=0^+} = \sigma_y|_{\vartheta=0^-} \ , \ \tau_{xy}|_{\vartheta=0^+} = \tau_{xy}|_{\vartheta=0^-} \qquad (9, 10, 11, 12)$$

Eqs (9) , (10) , (11) and (12) give:

$$\frac{1}{\mu_1}[(k_1 - \lambda_n)a_{1n} - (\lambda_n + 1)b_{1n}] = \frac{1}{\mu_2}[(k_2 - \lambda_n)g_{1n} - (\lambda_n + 1)h_{1n}] \tag{13}$$
$$\frac{1}{\mu_1}[(-k_1 + \lambda_n)a_{2n} + (\lambda_n + 1)b_{2n}] = \frac{1}{\mu_2}[(-k_2 + \lambda_n)g_{2n} + (\lambda_n + 1)h_{2n}] \tag{14}$$
$$a_{1n} + b_{1n} = g_{1n} + h_{1n} \tag{15}$$
$$(\lambda_n - 1)a_{2n} + (\lambda_n + 1)b_{2n} = (\lambda_n - 1)g_{2n} + (\lambda_n + 1)h_{2n} \tag{16}$$

The continuity of u guarantees that of ε_x. For each value of λ_n, the asymptotic fields in material 1 are characterized by a vector of 4 unknowns [a_{1n},a_{2n},b_{1n},b_{2n}]; similarly, in material 2 they are characterized by a second vector [g_{1n},g_{2n},h_{1n},h_{2n}].

The following conditions need to be satisfied along the cohesive zone ($\theta = \pm\pi$):

$$\sigma_y|_{\vartheta=\pi} = \sigma_y|_{\vartheta=-\pi} \neq 0 \ , \ \tau_{xy}|_{\theta=\pi} = \tau_{xy}|_{\theta=-\pi} = -\mu_f \sigma_y|_{\vartheta=\pi} \neq 0 \qquad (17, 18)$$

where μ_f equals the positive or negative value of the of kinetic friction coefficient, which is assumed to be constant, and to depend on the relative sliding direction of the two crack edges. In other words, $\mu_f > 0$ when $\delta < 0$ and $\mu_f < 0$ when $\delta > 0$.

Eqs (17) and (18) give:

$$(a_{2n} + b_{2n} + g_{2n} + h_{2n}) \sin((\lambda_n - 1)\pi) = 0 \tag{19}$$
$$[(\lambda_n - 1)(a_{1n} + g_{1n}) + (\lambda_n + 1)(b_{1n} + h_{1n})] \sin((\lambda_n - 1)\pi) = 0 \tag{20}$$
$$\{[g_{2n} + h_{2n} + \mu_f(a_{1n} + b_{1n})]\lambda_n + [-g_{2n} + h_{2n} + \mu_f(a_{1n} + b_{1n})]\} \cos((\lambda_n - 1)\pi) +$$
$$\{[g_{1n} + h_{1n} + \mu_f(a_{2n} + b_{2n})]\lambda_n + [-g_{1n} + h_{1n} + \mu_f(a_{2n} + b_{2n})]\} \sin((\lambda_n - 1)\pi) = 0 \tag{21}$$

Eqs(19), (20) and (21) show that the asymptotic solution is composed of two parts:

(a) if $\sin((\lambda_n - 1)\pi) = 0$, Eq.(21) requires:

$$\{[g_{2n} + h_{2n} + \mu_f(a_{1n} + b_{1n})]\lambda_n + [-g_{2n} + h_{2n} + \mu_f(a_{1n} + b_{1n})]\} = 0 \qquad (22)$$

This part of the solution is characterized by integer eigenvalues.

(b) if $\cos((\lambda_n - 1)\pi) = 0$, Eqs (19),(20) and (21) require:

$$(a_{2n} + b_{2n} + g_{2n} + h_{2n}) = 0 \qquad (23)$$

$$[(\lambda_n - 1)(a_{1n} + g_{1n}) + (\lambda_n + 1)(b_{1n} + h_{1n})] = 0 \qquad (24)$$

$$\{[g_{1n} + h_{1n} + \mu_f(a_{2n} + b_{2n})]\lambda_n + [-g_{1n} + h_{1n} + \mu_f(a_{2n} + b_{2n})]\} = 0 \qquad (25)$$

This part of the solution is characterized by fractional eigenvalues.

(a) *Integer eigenvalues*

Since g_{1n} and h_{1n} can be written as functions of a_{1n} and b_{1n} [6] through Eq. (13) and (15),the same expression used in the homogeneous case [1] can hold ($w = \delta = 0$). For $\lambda_0 = 1$, one obtains $\sigma_0 = a_{10} + b_{10}$, $\mu_f\sigma_0 = 2b_{20}$ and $\sigma_x|_{\vartheta=0} = 2a_{10} - 2b_{10}$.

(b) *Fractional eigenvalues*

$$\lambda_n = n + \frac{3}{2}, \qquad n = 0, 1, 2, ...$$

Eqs(14) and (16) allow one to express g_{2n} and h_{2n} as functions of a_{2n} and b_{2n} [6]. Therefore, Eq.(23) gives:

$$b_{2n} = -\frac{(\mu_1 k_2 \lambda_n + \mu_1 + \mu_2 k_1 + \mu_2 \lambda_n)}{(\mu_2 \lambda_n + \mu_2 + \mu_1 k_2 \lambda_n + \mu_1 k_2)} a_{2n} \qquad (26)$$

Eqs(13) and (15) allow one to express g_{1n} and h_{1n} as functions of a_{1n} and b_{1n}[6]. Therefore, Eq.(24) gives:

$$b_{1n} = \frac{(-\mu_1 k_2 \lambda_n + \mu_1 + \mu_2 k_1 - \mu_2 \lambda_n)}{(\mu_2 \lambda_n + \mu_2 + \mu_1 k_2 \lambda_n + \mu_1 k_2)} a_{1n} \qquad (27)$$

and Eqs(26) and (27) in Eq.(25) give:

$$a_{2n} = -\frac{a_{1n}}{\mu_f} \qquad (28)$$

Substituting Eqs (26),(27) and (28) in Eqs (2) and (3) gives:

$$\sigma_y|_{\vartheta=\pm\pi} = \sum_{n=0} r^{\frac{2n+1}{2}} \left[\frac{2n+3}{2} \left(\frac{\mu_2(k_1+1) - \mu_1(k_2+1)}{\mu_2 + \mu_1 k_2}\right) a_{2n}\right] \sin \frac{2n+3}{2}\pi \qquad (29)$$

$$\tau_{xy}|_{\vartheta=\pm\pi} = \sum_{n=0} r^{\frac{2n+1}{2}} \left[\frac{2n+3}{2} \left(\frac{\mu_2(k_1-1) - \mu_1(1-k_2)}{\mu_2 + \mu_1 k_2}\right) a_{1n}\right] \sin \frac{2n+3}{2}\pi \qquad (30)$$

$$\hat{\sigma_y} = \frac{\sigma_y|_{\vartheta=\pm\pi}}{\sigma_0} = \frac{\tau_{xy}|_{\vartheta=\pm\pi}}{-\mu_f\sigma_0} = \sum_{n=0} e_n r^{\frac{2n+1}{2}} \tag{31}$$

where

$$e_n = \frac{1}{\sigma_0}\Big[\frac{2n+3}{2}\Big(\frac{\mu_2(k_1+1) - \mu_1(k_2+1)}{\mu_2 + \mu_1 k_2}\Big)a_{2n}\Big] \sin\frac{2n+3}{2}\pi \tag{32}$$

It is worthwhile noting that the e_n coefficients vanish in the homogeneous case.This is the main difference between the two cases.

Substituting Eqs(26),(27) and (28) in Eqs(7) and (8) gives

$$w = \sum_{n=0} r^{\frac{2n+3}{2}}\left[\frac{2(\mu_1 + \mu_2 k_1)}{\mu_1\mu_2}a_{1n}\right] \sin\frac{2n+3}{2}\pi \tag{33}$$

$$\delta = \sum_{n=0} r^{\frac{2n+3}{2}}\left[-\frac{2(k_1k_2 - 1)}{\mu_1 k_2 - \mu_2}a_{2n}\right] \sin\frac{2n+3}{2}\pi \tag{34}$$

According to the well established literature on the mechanical behaviour of concrete joints (see Cervenka et al. [7]),softening only depends on $w_{eff} = \sqrt{w^2 + \delta^2}$:

$$\sum_{n=0} r^{\frac{2n+3}{2}} 2\Big(\frac{(\mu_1 + \mu_2 k_1)^2}{(\mu_1\mu_2)^2}a_{1n}^2 + \frac{(k_1k_2 - 1)^2}{\mu_1^2 k_2^2 - \mu_2^2}a_{2n}^2\Big)^{1/2} \sin\frac{2n+3}{2}\pi \tag{35}$$

$$\hat{w} = \frac{w_{eff}}{w_{eff,c}} = \sum_{n=0} r^{\frac{2n+3}{2}}\bar{d}_n$$

$$\bar{d}_n = 2\Big(\frac{(\mu_1 + \mu_2 k_1)^2}{(\mu_1\mu_2)^2}a_{1n}^2 + \frac{(k_1k_2 - 1)^2}{\mu_1^2 k_2^2 - \mu_2^2}a_{2n}^2\Big)^{1/2} \sin\frac{2n+3}{2}\pi \tag{36}$$

Let us consider the truncated $N+1$ terms of $\hat{w}$ (36), and denote $d_0 = \bar{d}_0$,$d_n = \bar{d}_n/d_0$ $(n > 1)$

$$\hat{w} = d_0 r^{\frac{3}{2}}\Big(1 + \sum_{n=1}^{N} d_n r^n\Big) \tag{37}$$

From this relation, we can obtain

$$\hat{w}^{\frac{(2i-1)}{3}} = \Big(\frac{w_{eff}}{w_c}\Big)^{\frac{(2i-1)}{3}} = d_0^{\frac{(2i-1)}{3}} r^{\frac{(2i-1)}{2}}\Big(1 + \sum_{n=1}^{N} d_n r^n\Big)^{\frac{(2i-1)}{3}} \tag{38}$$

$$\hat{w}^{\frac{(2i-1)}{3}} = d_0^{\frac{(2i-1)}{3}} r^{\frac{(2i-1)}{2}}\Big(1 + \sum_{n=1}^{M} \beta_{in} r^n\Big), \qquad (M \geq N) \tag{39}$$

in which:

$$\beta_{in} = \frac{f_i^{(n)}(0)}{n!}, \quad f_i(r) = \Big(1 + \sum_{n=1}^{N} d_n r^n\Big)^{\frac{(2i-1)}{3}} \tag{40}$$

where $f_i^{(n)}(0)$ denotes the nth derivative at $r = 0$. We now substitute Eq.(39) in the right hand side of Eq.(1), and Eq.(31) and a constant stress term in its left hand side. In this way we obtain:

$$\hat{\sigma_y} = \left(\frac{\sigma_y}{\sigma_0}\right) = \left(\frac{\tau_{xy}}{\mu_f \sigma_0}\right) = 1 + \sum_{n=0} e_n r^{\frac{2n+1}{2}} = 1 + \sum_{i=1}^{L} \alpha_i d_0^{\frac{2i-1}{3}} r^{\frac{2i-1}{2}} \left(1 + \sum_{n=1}^{M} \beta_{in} r^n\right) -$$

$$\left(1 + \sum_{i=1}^{L} \alpha_i\right) d_0^{\frac{2L+1}{3}} r^{\frac{2L+1}{2}} \left(1 + \sum_{n=1}^{M} \beta_{(\frac{2L+1}{2})n} r^n\right) \tag{41}$$

Through a term by term comparison applied to Eq. (41) we obtain the relations between the coefficients e_i , α_i and β_{in}.

NUMERICAL RESULTS

Figure 3 show a gravity dam model proposed as a benchmark by the Int. Commission On Large Dams [9] (dam height 80 m, base 60 m, $w_{eff,c} = 2.56mm$, $\mu_f \sigma_u = 0.85\ MPa$).

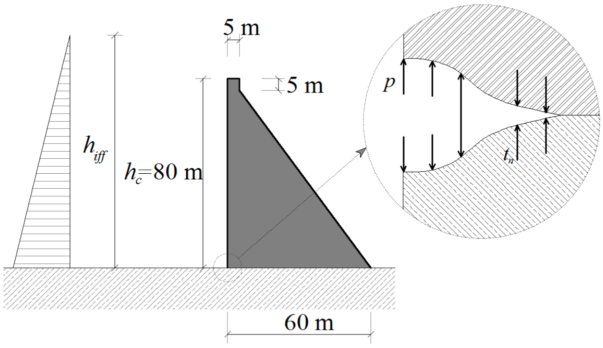

Figure 3. Gravity dam proposed as benchmark by ICOLD [9].

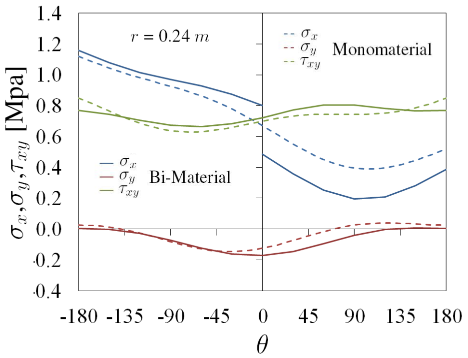

Figure 4.First term ($\lambda \leq 1.5$) of the asymptotic expansion:comparison between the monomaterial case and bi-material case

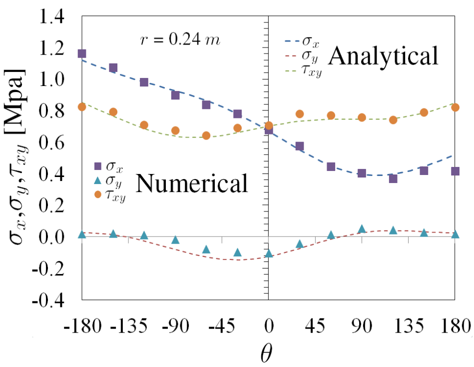

Figure 5. Comparison between an analytical ($\lambda \leq 1.5$) and numerical solution in the monomaterial case

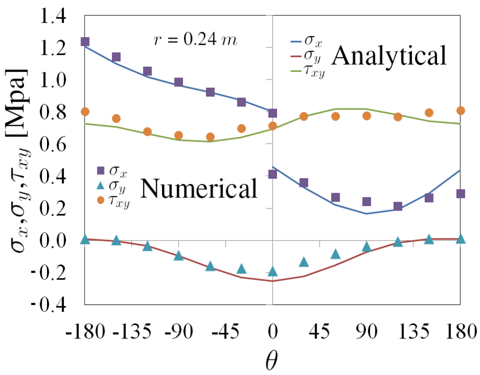

Figure 6. Comparison between an analytical ($\lambda \leq 1.5$) and numerical solution in the bi-material case

Table 1 show that the assumed material properties and the tensile strenght of the joint are considered as negligible. Therefore a large value μ_f = 50 is considered. As assumed in [8], the water penetrates into the crack where $w > w_{eff,c} * 2/9$.

Table 1. Material properties

Material	Density (kg/m^3)	Young's modulus (Mpa)	Poisson's ratio (Mpa)
Rock	2700	41000	0.1
Concrete	2400	24000	0.15

Figs 4, 5 and 6 refer to h_{iff} = h_c+4m (see Figure 3). A first analysis was based on the mean value of the parameters shown in Table 1.In this homogeneous case, the asymptotic expansion proposed in [1] is used.The results are shown in Figure 5 (a_{10} = 0.211MPa, b_{10} = $-0.203MPa$, b_{20} = 0.425MPa,a_{12} = 0.05MPa ,a_{11} = 0.05MPa and the distance of the FCT from the upstream edge is 10.8 m) A second analysis was based on the previously proposed asymptotic expansion . The results are shown in Figure 6(a_{10} = 0.1071MPa,b_{10} = $-0.0986MPa$, b_{20} = 0.369MPa , a_{11} = $-0.697MPa$ and the distance of the FCT from the upstream edge is 12 m).

CONCLUSION

The special polynomial form proposed as a cohesive law can represent most of the commonly used cohesion-separation relations.In this way , the asymptotic fields can be written in terms of r and θ functions (separable form).Thus the asymptotic fields at the tip of a cohesive crack growing at a bi-material interface are known.
The simple assumption of mean elastic values is not conservative.

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