

Variable T-stress and its Implication for Crack Path

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ABSTRACT. *Fatigue crack paths for inclined cracks are studied through experiments and computations under different mixed-mode loading. The elaborated theoretical model is applied for modeling crack growth trajectories in the most popular in experimental fracture mechanics specimen geometries. For the particular specimen geometries considered, the T-stress distributions are calculated along the curved crack path. It is shown that there is a greater variation of T-stress along the crack trajectories under mixed mode fracture for specimen different geometries. The experimental data for mixed mode fracture trajectories during crack growth are compared with theoretical predictions. Discrepancies in fatigue crack path have been observed in various specimen configurations. The results presented in this study for fracture specimens seem to indicate the relevance of crack tip constraint parameter, the T-stress, to fatigue crack path behavior that conventional LEFM fails to explain.*

INTRODUCTION

Mixed mode fracture in both brittle and ductile materials has been studied and a wide range of experiments are performed and several fracture models are proposed. In recent years it has been observed by a number of researches [1-3] that there is discrepancy between mixed mode fracture predicted by majority of crack growth direction criteria and experiments, with the largest errors occurring at tensile and shear combinations. They proposed that probably one reason for the discrepancy was the presence of the T-stress and crack growth direction can depend on the specimen geometry. The T-stress is defined as the constant term in the asymptotic stress expansion in front of the kink or the unbranched crack which acts parallel to and along the main flat crack.

Pisarenko and Lebedev [4] developed the most general empirical criterion which represents a superposition of the elastic (the Coulomb-Mohr) and plastic (the von Mises) classical limiting state theories. Shlyannikov [5] generalized this criterion and extended to the crack growth direction problem under elastic-plastic mixed mode fracture. The generalization consisted in accounting for the T-stress and a fracture process zone r_c [6].

In order to study the influence of the different fracture specimen geometries and its loading conditions on material fracture resistance characteristics it is necessary to calculate the fracture parameters, namely mode I and II stress intensity factors K_I and

K_{II} (SIF) and T-stress for full range of mixed mode conditions. In the present study it is stated that the T-stress is not constant and demonstrated how it changes depending on crack length and crack angle combinations. The T-stress based the generalized Pisarenko-Lebedev criterion is modified, and is applied to a crack path prediction in various fracture specimen configurations.

DETERMINING T-STRESS AND STRESS INTENSITY FACTORS

Subjects for studies are cruciform specimen under biaxial loading, center cracked plate and compact tension-shear specimen (Fig.1). Different degrees of mode mixity from pure mode I to pure mode II are given by combinations of the far-field stress level σ , biaxial stress ratio η and inclined crack angle α .

The principal feature of our modeling is the evaluation of elastic T -stress along curvilinear crack paths. The T -stress has been recognized as a measure of constraint for the small-scale yielding conditions. This study explores direct use of FEM analysis for calculating T -stress on the base of crack flank nodal displacements [3]. Using this technique, the T -stress distributions in various specimen geometries was determined from numerical calculations. On this basis the solutions for mode I and mode II stress intensity factors K_I and K_{II} for each specimen geometry have been obtained.

Figure 2 shows a flow chart for computing T-stress, stress intensity factors K_I and K_{II} , J-integral components and inherent stress biaxiality ratio B . Although the algorithm is relatively simple, the analysis can be very time consuming, since a large number of crack length and crack angle combinations is required. The sequence, which defines the order of the fracture parameters determination, is the same for each considering specimen configuration.

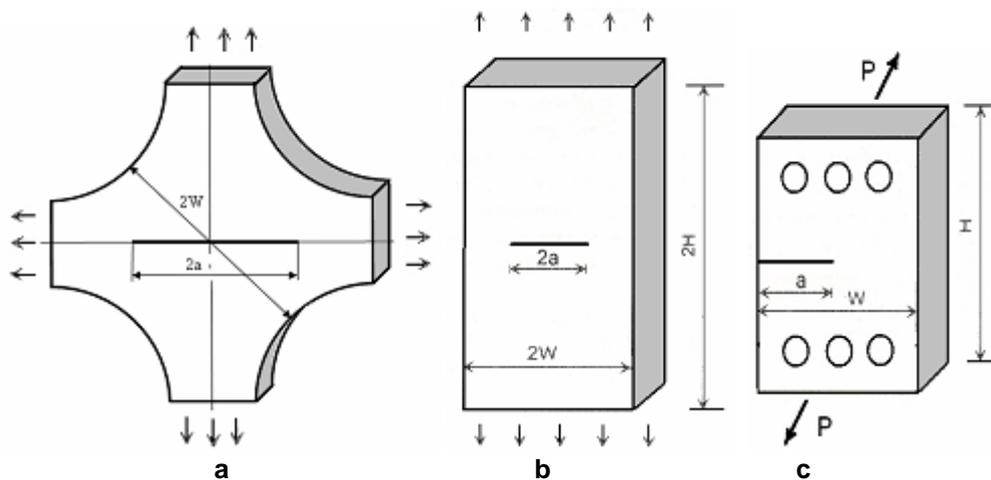


Figure 1. (a) Cruciform specimen (CS), (b) center cracked panel (CCP) and (c) compact tension-shear specimen (CTS).

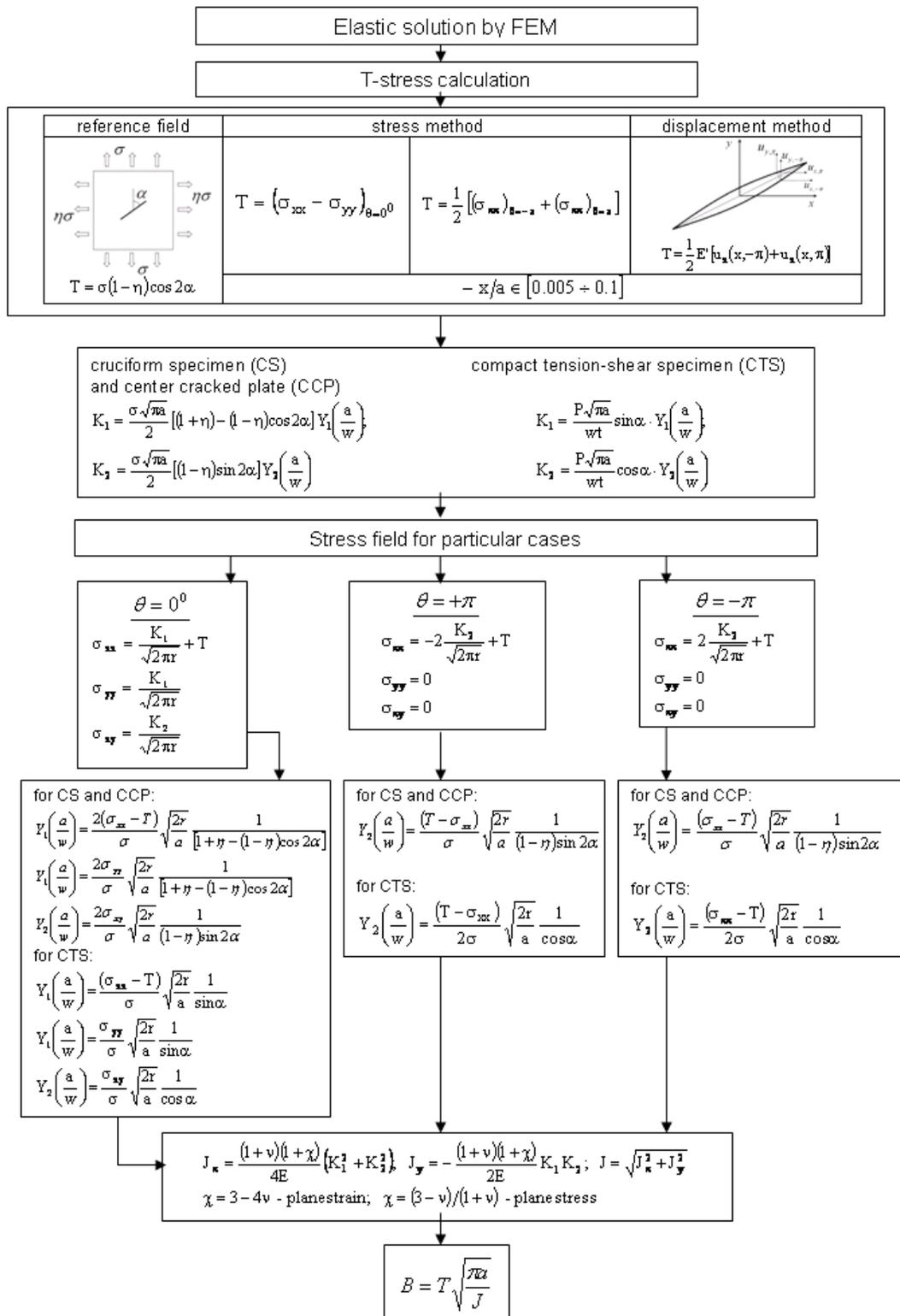


Figure 2. Flow chart for cracked body stress field analysis with T-stress.

All investigated configurations contain an internal crack of length $2a$ for CS and CCP or a for CTS, which is angled to the edges of the specimens. The cruciform specimen is subjected by uniform stress of magnitude σ and $\eta\sigma$ along the remote edges parallel to the Y and X axes, respectively. The initial crack makes an angle α with the loading direction. By changing α , different combinations of modes I and II are achieved. For example, it is clear that for the biaxially loaded CS $\alpha = 0^\circ$ or $\alpha = 90^\circ$ correspond to pure mode I, while pure mode II can be achieved when $\alpha = 45^\circ$ and $\eta = -1$. In the CTS $\alpha = 90^\circ$ correspond to pure mode I and pure mode II can be achieved when $\alpha = 0^\circ$. Finally, for the CCP $\alpha = 90^\circ$ correspond to pure mode I.

The normalized T-stress distributions of various fracture specimen geometries under mixed mode loading conditions are determined from finite element calculations. Figure 3 is a plot of the T-stress ahead of the crack-tip ($\theta=0^\circ$) as a function of an initial crack angle α and relative crack length a/w for different specimen geometries. Note that the deviation of current value of T from the corresponding original value for $a/w=0.1$ (or $a/w=0.5$ for CTS) increases with increasing relative crack length at fixed crack angle position. All configurations maintain approximately the same positive level of constraint under mixed mode conditions. As discussed in the literature, a positive T-stress in the elastic case generally leads to high constraint mixed mode loading, while geometries with negative T-stress lose constraint.

It can be seen from Fig. 3 that for particular geometry considered, the maximum positive T-stress is realized at $a/w=0.5$ and $\alpha = 0^\circ$ under equi-biaxial tension-compression of the cruciform specimen, while the minimum negative T-stress is realized at $a/w=0.5$ and $\alpha = 90^\circ$ in the compact tension-shear specimen. Furthermore, for the CTS there is a greater variation of T-stress along the crack under mixed mode loading when T-stress rapidly decreases with increase of relative crack length. As follows from Fig. 3, the biaxially loaded CS specimen is more constrained by the in-plane parameter T with respect to other specimen geometries.

By fitting the numerical calculations, the constraint parameter T-stress as the function of varied crack length and crack angle for particular geometry considered was represented in the form of the approximation equation.

When applying any of fracture criteria to predict crack propagation, the point of view being that the stress-strain characteristics are not determined at the crack tip itself, but at some distance r_c from it. Many of the fracture mechanics theories are based on a critical distance local to the crack tip. In the present work the critical distance r_c ahead of the crack tip is assumed to be located where the stress strain state in the element reaches a certain critical value that can be measured from a uniaxial test. A fracture process zone size as the critical distance was introduced by Shlyannikov [7,8]. This concept in common with the Pisarenko-Lebedev criterion and T-stress equations was applied for the crack path prediction for the three geometric configurations containing the single-edge and the central initial cracks.

Williams and Ewing [9] and Finnie and Saith [10] also employed the T-stress to predict crack path. However, their method required the knowledge of the fracture process zone size a priori, which is difficult to determine. In contrast, the present

method is based on the continuous correlated computing of the crack growth direction and the fracture process zone size at each step of the crack length increment along the curvilinear trajectory.

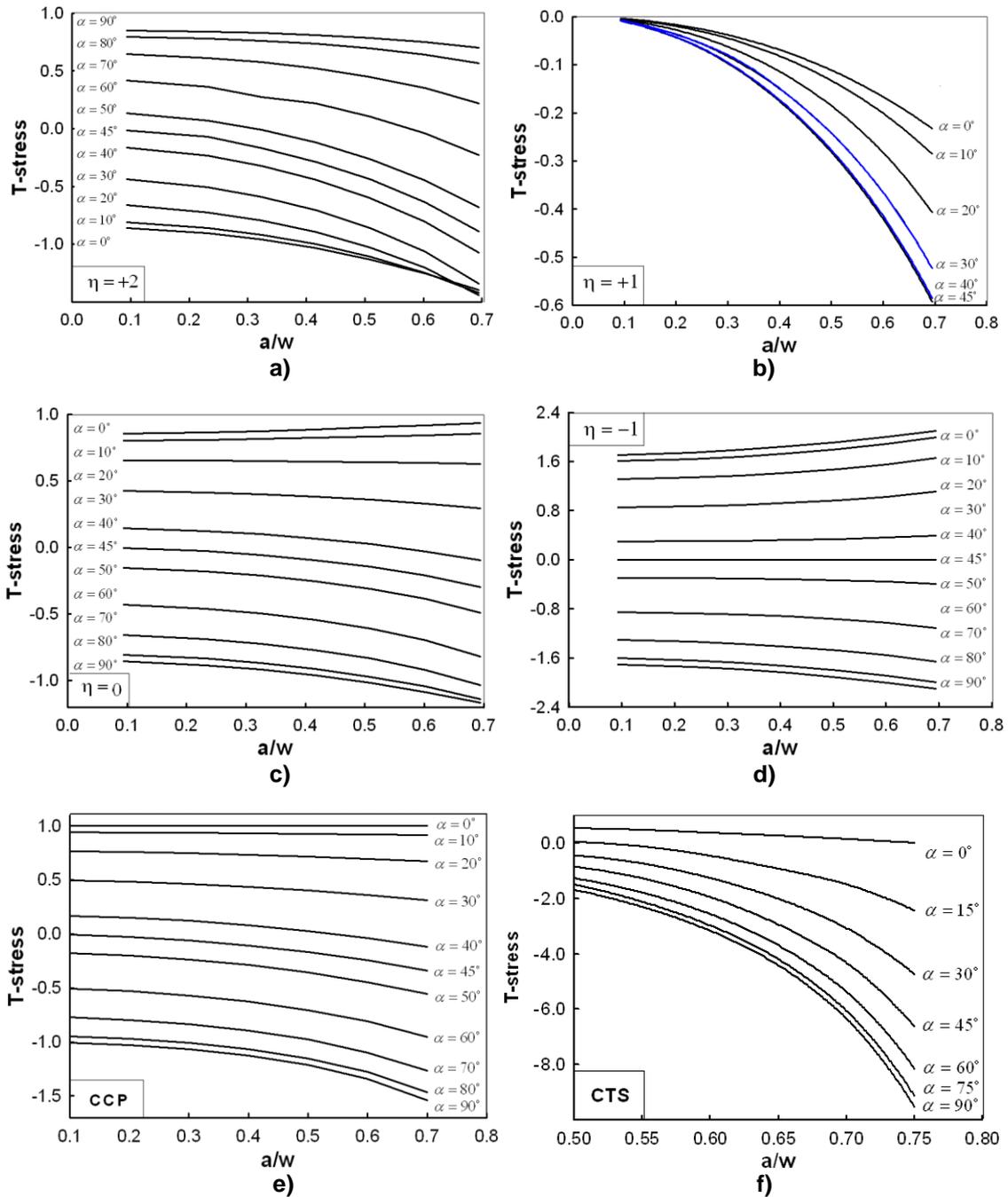


Figure 3. Mixed mode T-stress distributions in various specimen geometries (a-d)–CS, (e) – CCP, (f) - CTS.

DISCUSSION

It is helpful to verify the theoretical prediction through the T-stress and crack path behavior in the specimen different geometries. The numerical results are considered and compared with experimental data in Fig.4 for material different properties (Table 1). The left row in Fig. 4 depicts the behavior of the T-stress, whereas the right row in Fig. 4 gives us the crack paths in considered fracture specimen geometries under mixed mode loading. The constraint parameter T is plotted against the normalized crack length a/w .

The experimental study of fatigue crack trajectories in the aluminum alloy is performed on biaxially loaded cruciform specimen. All specimens for biaxial loading contained inclined through thickness central cracks. Mixed mode I/II fatigue crack path experiments on the high-strength steel and the titanium alloy used the compact tension shear and the center cracked plate specimens consequently.

Table 1. Mechanical properties

Material	Yield stress R_e (MPa)	Ultimate stress R_m (MPa)	Reduction of area (%)	Strain hardening exponent
Aluminum alloy	160	384	25	4.29
Steel	1039	2064	45	6.43
Titanium alloy	508	534	11	9.29

By substituting the experimental values of the crack length increment and the crack angle deviation in the approximation equation describing the T-stress behavior, the current values of the constraint parameter were obtained for each specimen. Graphs showing these experimental results are presented in Fig. 4 and denoted by symbols. The theoretical predictions are shown in Fig. 4 by the solid lines. As it follows from this comparison, there is good agreement between calculations and experiments in the T-stress distributions. It should be noted that in mixed mode conditions, the crack growth is not in the same plane as the initial crack. At each step of their growth the crack continuously changes its position with respect to the acting loads thus forming a curved path.

Left row of graphs in Fig.4 represents a comparison of numerical and experimental fracture trajectories in the various specimen geometries. The numerical results related to two variants of calculations. The first of them keeps the non-singular term ($T \neq 0$) in the stress expansion. The second one is ignored the T-stress ($T = 0$) and consequently it is assumed that the T-stress has no influence. This contrary to the observed effect of the T-stress contribution under mixed mode fracture especially for the central cracked panel and the compact tension shear specimens.

It can be seen from these figures that there are the discrepancies in fatigue crack path in different specimen geometries. It means that the deviation from a traditional LFM simplified analytical one term singular solution increases with increasing relative crack

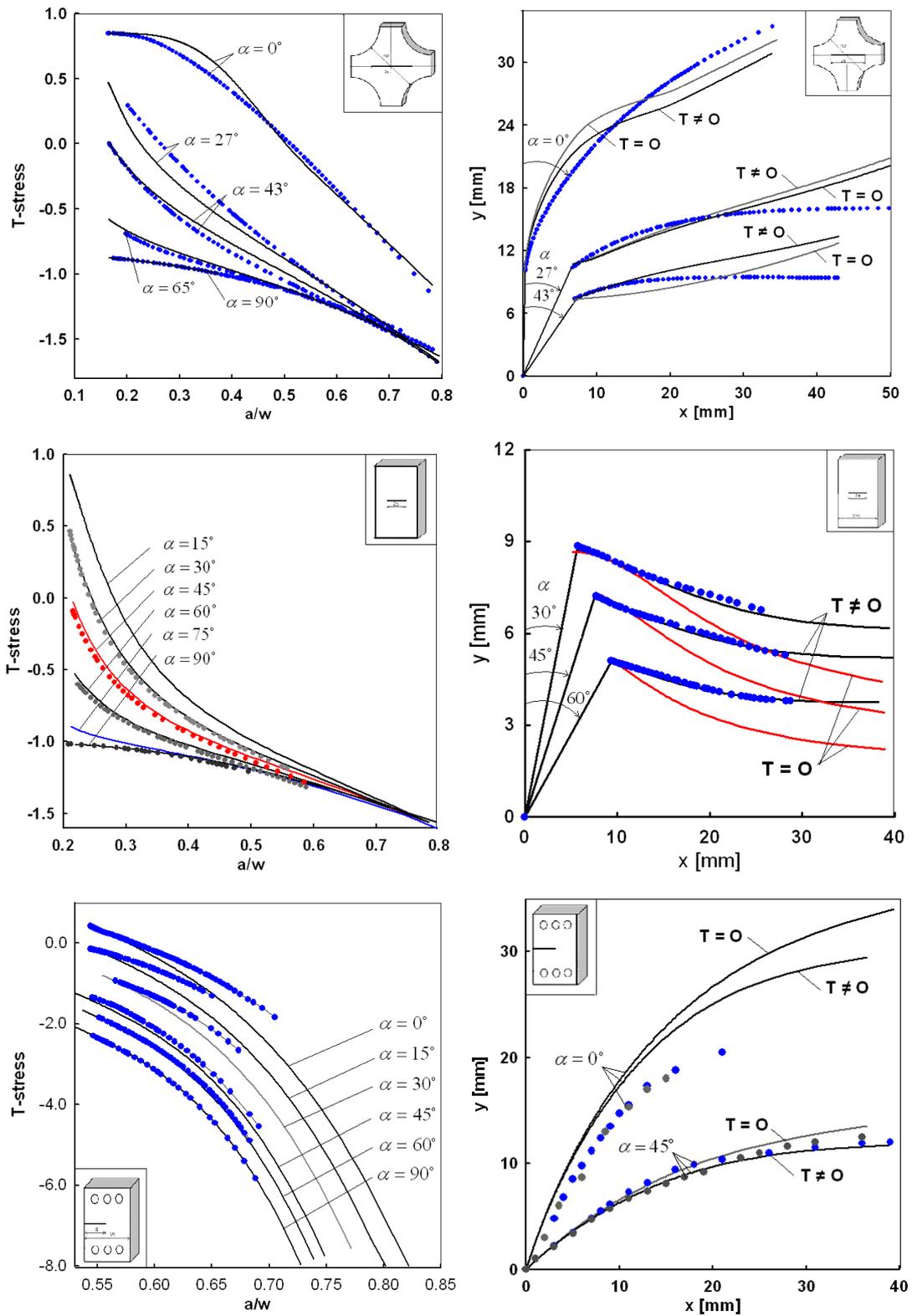


Figure 4. Comparison experimental and predicted T-stresses and crack trajectories.

length and with changing loading biaxiality. On the contrary there is a good agreement between the experimental data for specimen considered geometries and theoretical results accounting for T -stress variations with relative crack length and crack inclination.

CONCLUSIONS

Using crack flank nodal displacements technique, the T -stress distributions in various specimen geometries was determined from numerical calculations. For fracture specimen each configuration the variation pattern of the mixed mode T -stress with angle of crack inclination and relative crack length have been given quantitatively. For the cruciform specimen the distribution of the T -stress additionally is given as a function of load biaxiality. In the present study it is stated that the T -stress is not constant and demonstrated how it changes depending on crack length and crack angle combinations.

The experimental data for mixed mode fracture trajectories during crack growth are compared with theoretical predictions. Discrepancies in fatigue crack path have been observed in various specimen configurations. The results presented for fracture specimens seem to indicate the relevance of crack tip constraint parameter, the T -stress, to fatigue crack path behavior that conventional LEFM fails to explain.

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