Strength Analysis of Attachment Lugs under Cyclic Loading

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ABSTRACT. *A computational model for estimating the residual fatigue life of attachment lugs is proposed. In strength analysis, the lug with single quarter-elliptical corner crack as well as with single through-the-thickness crack are examined. Stress intensity factor, as an important parameter for fatigue life estimation, is determined by applying analytical and numerical methods. The model is verified using experimental fatigue crack growth data. Predictions of fatigue crack propagation behaviour are in a good agreement with experimental observations.*

INTRODUCTION

Vital engineering components often must be linked by lug type joints in order to perform their function. The lug type joint consists of two or three parts connected with only one fastener. In the lug type joint, the combination of high concentration and fretting could potentially lead to appearance of the crack initiation, and then crack growth under cyclic loading. Fatigue, as a complex process, could be so dangerous and even to cause failure of lug, i.e. components that are connected by lug type joint. Due to previous reasons it is very important to assess, analyze and/or predict the crack initiation and crack growth behavior of lugs.

In general, when analyzing crack growth phase, the engineering practice experience has showed that most often it is possible to identify corner cracks, as well as through-the-thickness crack in the lugs. From the engineering point of view corner crack are usually approximated by quarter-elliptical crack. The present paper tackles crack growth analysis of attachment lugs with quarter-elliptical corner crack and through-the-thickness crack.

For reliable prediction of crack growth rates and fracture strengths of attachment lugs accurate stress analysis is needed. Over the years, various methods were evolved to estimate the stress intensity factor such as analytical method [1], the alternating method [2], the finite element alternating method [3-6], the finite element method [7,8] as well as the hybrid finite-element method [9].

In the present paper, the authors formulated a computational model/procedure for fatigue crack growth analysis of attachment lugs subjected to cyclic tensile loading. The paper investigates the pin-loaded lug with single quarter-elliptical corner crack as well as with single through-the-thickness crack. The validity of the estimation is discussed by comparing the present results with available experimental results.

CRACK GROWTH ANALYSIS

Developing an appropriate computational procedure for crack growth analysis is one of the key issues for the assessment of the reliability of components and structures.

In general, to accurately assess fatigue growth of quarter-elliptical corner crack in the lug it is necessary to analyze fatigue growth behavior at the point of maximum crack depth and at the point of surface crack interaction with the surface. Due to previous reason, the crack propagation process can be described by two coupled equations for crack growth rate as follows:

$$\frac{da}{dN} = C_A K^2_{\mathrm{Im}axA} \Delta K_{IA} \tag{1a}$$

$$\frac{db}{dN} = C_B K^2_{\mathrm{Im}axB} \Delta K_{IB} \tag{1b}$$

where C_A and C_B are material constants experimentally obtained, ΔK_A, ΔK_B, K_{maxA}, K_{maxB} are the ranges and maximum values of stress intensity factor at the depth A and surface B points, respectively.

Final number of loading cycles for the lug with corner crack can be estimated for both directions if expressions for crack growth rate are integrated i.e.
for depth direction:

$$N = \int_{a_0}^{a_f} \frac{da}{C_A K^2_{\mathrm{Im}axA} \Delta K_{IA}} \tag{2a}$$

and for surface direction:

$$N = \int_{b_0}^{b_f} \frac{db}{C_B K^2_{\mathrm{Im}axB} \Delta K_{IB}} . \tag{2b}$$

Since relationships for stress intensity factors are complex functions, numerical simulations have to be performed to compute fatigue life of attachment lugs up to failure for both directions.

STRESS INTENSITY FACTOR OF THE ATTACHMENT LUG

The attachment lugs, due to the fact that they connect vital engineering components, demand careful crack growth analysis and a damage tolerance analysis to aid structural integrity. For structural safety, the evaluation of stresses in the vicinity of cracks is very important. In fracture mechanics, the stress analysis is based on knowledge of the stress intensity factor at the tip of the crack. The stress intensity factor is a primary parameter for crack growth analysis due to the fact that it employs geometry, material and loading conditions.

The stress analysis can be considered by applying analytical and numerical approaches [10]. The present authors tackled both approaches for stress intensity factor evaluation of the attachment lugs. As the pin-loaded lug with single quarter-elliptical corner crack [Fig.1] is investigated, the relationship for stress intensity factor can be expressed as follows [11]:

$$\Delta K_I = \Delta S \sqrt{\frac{\pi a}{Q}} M_e \, f_1 G_1 \sqrt{\frac{1}{\cos\left(\frac{\pi D}{2w}\right)}} \, g_\phi \, , \tag{3}$$

where: ΔS represents stress range, a denotes the crack length in depth direction, D is a hole diameter and w presents the width of lug. The elastic shape factor Q [11] can be written as:

$$Q = 1 + 1.47\left(\frac{a}{b}\right)^{1.64}, \left(\frac{a}{b} \leq 1.0\right). \tag{4}$$

The factor M_e includes front-face, back-face and finite-width corrections [11] and the relationship is given by:

$$M_e = \left(M_1 + \left(\sqrt{Q\frac{b}{a}} - M_1\right)\left(\frac{a}{t}\right)^p\right) f_{w1} \tag{5}$$

where:

$$M_1 = 1.2 - 0.1\frac{a}{b}, \left(0.02 \leq \frac{a}{c} \leq 1.0\right) \tag{6}$$

$$p = 2 + 8\left(\frac{a}{b}\right)^2 \tag{7}$$

and

$$f_{w1} = \sqrt{\frac{1}{\cos\left(\frac{\pi}{2}\frac{D+b}{w-b}\right)}} \, . \tag{8}$$

The Bowie's correction for a pin-loaded lug with single crack can be expressed as follows [12]:

$$f_1 = 0.707 - 0.18\lambda + 6.55\lambda^2 - 10.54\lambda^3 + 6.85\lambda^4 , \tag{9}$$

and

$$g_\phi = 1 + \left(0.1 + 0.35\left(\frac{a}{t}\right)^2\right)(1 - \sin\phi), \tag{10}$$

where: $\phi = 0°$ for position A and $\phi = 90°$ for position B (Fig.1) and

$$G_1 = \frac{1}{2} + \frac{w}{\pi(D+b)}\sqrt{\frac{D}{D+2b}} \, . \tag{11}$$

In addition to the pin-loaded lug with the quarter-elliptical corner crack, the present authors tackle the lug with single through-the-thickness crack (Fig.1). Due to previous reason, the expression for the stress intensity factor in the case of lug with single quarter-elliptical corner crack (Eq.3) is reduced. The relationship for pin-loaded lug with single through-the-thickness crack can be written as:

$$\Delta K_{IT} = \Delta S \sqrt{\pi b}\, f_{w1}\, f_1\, G_1 \sqrt{\frac{1}{\cos\left(\dfrac{\pi D}{2w}\right)}}\,. \tag{12}$$

Furthermore, a numerical approach is employed for the stress analysis by applying the finite element method. In the package MSC/Nastran [13], quarter-point (Q-P) singular finite elements [14] are used to simulate the through-the-thickness crack growth in attachment lugs.

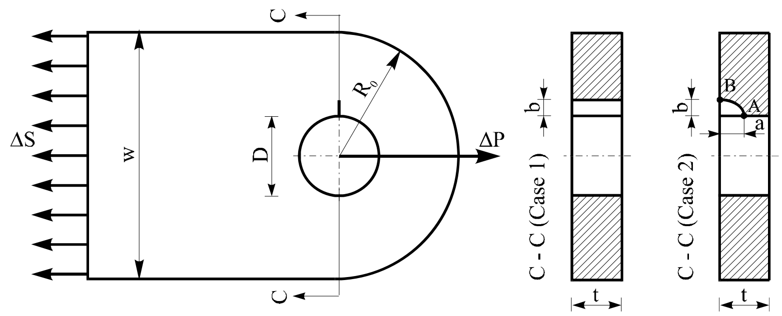

Figure 1. Geometry of the lug with single crack
(case 1 - through-the-thickness crack; case 2 – quarter-elliptical corner crack).

NUMERICAL RESULTES

To illustrate computation model for crack growth analysis of attachment lugs with one quarter-elliptical corner crack emanating from the hole or through-the-thickness crack, a few numerical examples are presented in this Section. These examples examine stress analysis as well as fatigue life estimation. In order to verify the validation of presented model for crack growth simulation obtained results are compared with experimental data.

Stress analysis of an attachment lug

In this example, stress intensity factor calculation of the lug with single through-the-thickness crack was carried out. The lug made of 7075 T7351 Aluninium Alloy was subjected by cyclic loading with constant amplitude (a maximum force P_{max}= 63716 N and stress ratio R = 0.1). Geometry characteristics of the lug with single through-the-thickness crack are: w = 83.3 mm, D = 40 mm, t = 15 mm, b_0 = 2.5 mm (the lug No.6 [15]). Material characteristics are as follows: σ_u= 432 MPa, $\sigma_{0.2}$= 334 MPa.

In addition to analytical approach for stress intensity factor evaluation, numerical approach based on finite element method is introduced in this paper. The lug with single through-the-thickness crack is tackled as contact problem. For this purpose singular six-node finite elements [15] are used. Actually, step-by-step, for each increment of crack length different meshes are modeled by using super-elements around crack tip [13].

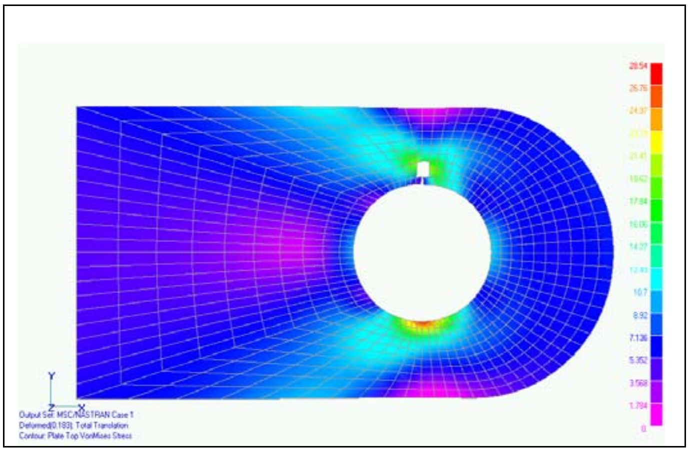

Figure 2. Stress distribution of the lug with one through-the-thickness crack (b = 5.33 mm, the lug No.6).

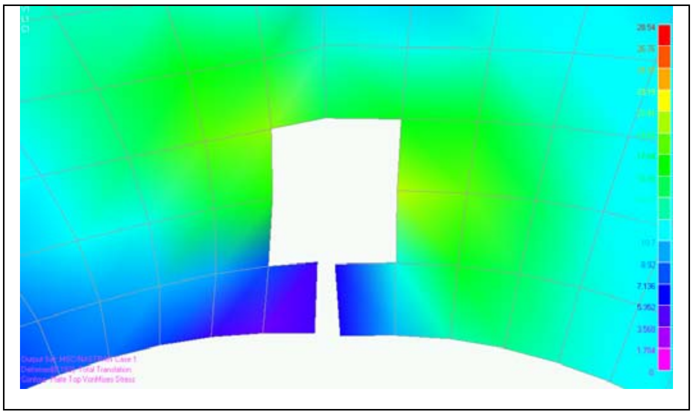

Figure 3. FE model by applying "super element" (b = 5.33 mm, the lug No.6).

The step-by-step procedure is repeated until the computed crack growth is very close to the final failure of the attachment lug. A representation of the finite element analysis for the lug with single through-the-thickness crack (b = 5.33 mm) is presented in Fig.2 and Fig.3. Moreover, for the same geometry of lug the stress intensity factor is calculated by applying analytical approach (Eqs.8-9 and Eqs.11-12). Differences between analitical and numerical (FEM) approaches are presented in Table 1.

Table 1. Comparison of the calculated stress intensity factors obtained by applying analytical and numerical approaches

$b*10^{-3}$ [m]	K_{ImaxT}^{FEM} [MPam$^{1/2}$]	$K_{ImaxT}^{Anal.}$ [MPam$^{1/2}$]	Δ [%]
5.33	21.543	21.400	0.664
7.50	22.877	22.700	0.774

Crack growth analysis of the lug with one through-the-thickness crack
This example deals with the calculation of the number of loading cycles up to failure of the lug with single through-the-thickness crack. The straight attachment lug (Fig.1, case 1) has geometry characteristics as follows: w = 114.3 mm, D = 38.1 mm, t = 12.7 mm, b_0= 0.635 mm. External loading is with constant amplitude (a far-field maximum gross stress S_{max}= 103.45 MPa and stress ratio R = 0.5). The lug is made of 7075 T651 Al alloy and material characteristics are as follows: σ_{ys}= 516.4 MPa, C_B=2.55*10^{-10} (for R=0.5).

Using the fatigue performance data, according to the lug geometry and defined crack growth model in previous Section, it is possible to calculate stress intensity factor by applying Eqs.8-9 and Eqs.11-12. Computed values of stress intensity factors for adequate crack increments are presented in Fig.4.a.

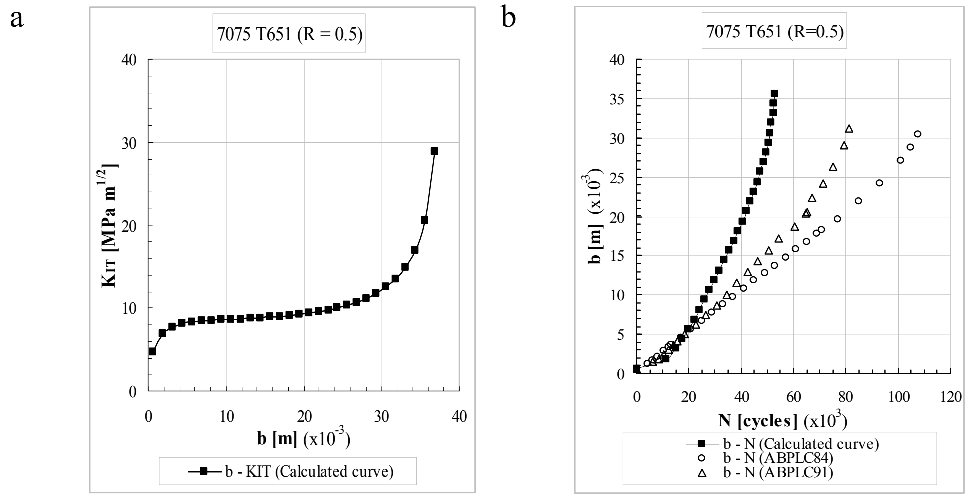

Figure 4. a) Stress intensity factor versus crack length;
b) Crack length versus number of loading cycles (experiment from Ref. [16]).

Furthermore, by using Eqs.2a-2b together with Eq.12, and Eqs.8-9 as well as Eq.11, the crack length is computed as a function of the number of loading cycles up to failure. Obtained results for crack length versus number of loading cycles up to failure are presented in Fig.4.b. In the same Figures, all computed results for number of loading cycles up to failure are compared with experimental data [16].

It is indicated in Fig.4.b that the estimated values of number of loading cycles up to failure are conservative when compared to experimental data. In engineering practice existance of conservativity in fatigue crack growth analysis is always benefitial since in this way safe residual service life of structural elements could be determined.

Fatigue life estimation of the attachment lug with simgle corner crack
This example examines the fatigue life estimation of a lug with single quarter-elliptical corner crack emanating from the hole (Fig.1, case 2). The lug is subjected to axial cyclic loading with constant amplitude (with a far-field maximum gross stress S_{max}=41.38 MPa and stress ratio R = 0.1). Geometry characteristics of the lug are as

follows: w = 57.15 mm, D = 38.1 mm, t = 12.7 mm, $a_0 = b_0$= 0.635 mm [16]. The material of the lug tackled in this example is the same as in the previos one ($C_A=C_B=4.68*10^{-10}$ for R=0.1).

Based on the known characteristics of material, corner crack geometry and loading, the values of stress intensity factors are computed by applying Eqs.3-11 for both, depth and surface directions. Calculated results for stress intensity factors for depth and surface directions as a function of crack length are shown in Fig.5.a. Thus, obtained stress intensity factors and corresponding crack increments are used for the crack path simulation of quarter-elliptical corner crack. The evaluated crack path for the considered attachment lug is shown in Fig.5.b.

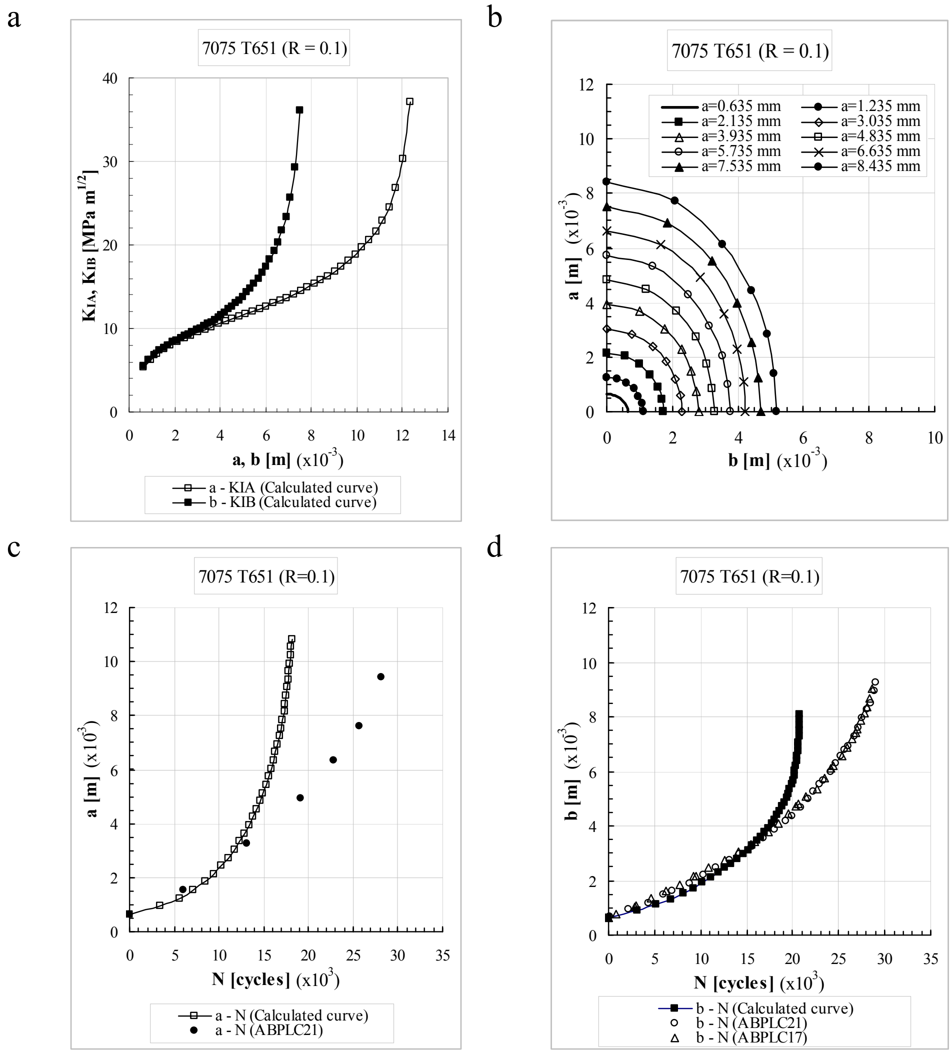

Figure 5. a) Stress intensity factor versus crack length; b) Crack path simulation; c) Crack length versus number of loading cycles depth direction (exp. from Ref. [16]); d) Crack length versus number of loading cycles surface direction (exp. from Ref. [16]).

Furthermore, by using Eqs.2a-2b with Eqs.3-11 for the stress intensity factor, it was possible to calculate number of loading cycles up to failure. All calculated results are presented in Fig. 5.c and Fig. 5.d for depth and surface directions, respectively.

CONCLUSIONS

The paper presents a computational model for the crack growth analysis of the attachment lug with single quarter-elliptical crack as well as with single through-the-thickness crack. The proposed model examines the stress analysis, the fatigue life estimation and the crack path simulation. In the stress analysis, both analytical and numerical approaches are employed to determine the stress intensity factor. In the numerical approach, finite element analyses are conducted using the packages MSC/Nastran, and quarter-point (Q-P) finite elements are employed to simulate the stress field around the crack tip. The fatigue lives up to failure are compared with experimental results available in the literature. Good correlation between numerical and experimental results is obtained.

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