New Box-Counting Method as Interpretation of Crack Paths and Mechanical Properties of Concrete with Interface Layer

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ABSTRACT. *This paper elaborates a new method, which is named box-counting method, for predicting the crack paths and the mechanical properties of concrete beam including a placing joint made with a casting afterwards or an interface between substrate concrete and repaired materials. The authors developed models for FEM analysis which represent the features of the specimens in terms of the fineness modulus of the coarse aggregate and the existence of the interface. The model is unique because the coarse aggregates are modelled to hexagons with the same size though the aggregates are the mixed ones with various sizes. The results from FEM analysis of the models are consistent to the experimental ones in terms of the load-displacement behaviour and mechanical properties. The crack paths are also consistent to the observed ones in the experiment. The box-counting method is the one which counts the elements in the weakest path employing the same model as the one for FEM, which virtually accounts for the resulted crack paths and the resulted mechanical properties achieved by FEM analyses and experiments as well.*

INTRODUCTION

Adhesive performance of repaired materials to the substrate concrete is the most important issues for the repairing and strengthening materials. The crack path near the interface between such materials and substrate concrete would be the meandering one into substrate concrete, if the mechanical performance of the adhesion should be high. Then it is as important to construct a model and a method for predicting the mechanical properties of the interface as to conduct experiments.

There are many literatures that studied the modeling and the analytical procedure for FEM (finite element method). Wittmann [1] presented a new concept called “numerical concrete” with using 2D FEM model which can predict structural performance as well as size effect and crack paths. Tajima analyzed fracture process of plain concrete with using 2D or 3D lattice model [2]. Asai analyzed some RC members also using 2D or 3D lattice model [3]. Nagai analyzed the fracture process of compression member based on 3D particle model where the location and the size of coarse aggregates are copied from the real concrete specimen [4, 5]. These studies based on the meso-scale model in which coarse aggregates and ITZ (interfacial transition zone) around them are modeled. It is natural that the goal of the modeling is to copy the features of real concrete as faithfully as possible, as Nagai [4, 5] copied the real image and the location of coarse aggregates one by one.

In comparison to such a materialized modeling, there is another stance of modeling where an abstract and theoretical model is sought. Yoshikawa conducted FEM analysis with using the homogeneous model of concrete which incorporates the heterogeneity in it by introducing the local difference of strength [6].

The authors' stance for modeling is the intermediate of the above-mentioned two opposite ones. The authors constructed "KAT model" [7] in a scale of meso-level where aggregates are modeled to a single-sized particles dispersed uniformly in the modeled region. KAT model consists of triangular elements of which the height and the length of the base is 1mm, where the components of concrete (i.e. aggregate, ITZ, mortar and so on) are composed of congregated triangular elements. The coarse aggregate and its surrounding ITZ are typified to function as an inducer for cracking and a resistant dowel against cracking as well, characterizing the model as half-materialized and half-abstract one to be used for FEM analysis.

This study aims at elucidating the mechanism for controlling the crack path and the resulted mechanical performance with using KAT model. The authors present the results from newly developed box-counting method (BCM) analysis and FEM analysis both of which employ KAT model, and discuss the crack extension mechanism which is applied to BCM.

EXPERIMENT AND RESULTS

The authors prepared three types of concrete specimens with an interface layer and also monolithic specimens for a reference. Two types of them have a varied type of placing joint made from different roughening and another has an interface with repair mortar. Table 1 and Figure 1 show the attributes and details of specimens respectively, and other detailed information is referred to a reference [8]. The number of specimens was three for each case, which have a section of 100 mm by 100 mm and a length of 400mm. After 24 hours from the 1st cast of concrete in the half part of mold, the joint surface was roughened in the case of CR-C and CE. Then concrete was cast in the remained half of mold as depicted in Fig. 1. The used repair material is the commercial polymer cement mortar containing powdered acrylic resin with epoxy primer coated before patching of the repair.

The specimens were cured in water at 20°*C* for 28 days after the final cast of concrete. A 50mm depth notch was incised at the center of the specimen before the fracture mechanics test. After the fracture mechanics test, TSD (tension softening diagram) was achieved from the load-displacement curve of specimen with employing multi-linear approximation method which was standardized by JCI [9].

Table 1 also shows the resulted flexural strength (Fb), which is used as a substitute for the load capacity not as a literal edge-stress, and fracture energy (GF). Ft is tension softening initial stress which is achieved from TSD and the same as tensile strength.

Table 2 shows the mechanical properties used for FEM analysis and BCM analysis. The values in Table 2 for mortar and aggregate are the averaged experimental results in the case of #1 and the highest ones in the case of #2 and #3 [7]. The values for ITZ and interface are the averaged ones in the case of #1, whereas the reference ones in the case of #2 and #3. Hereafter the sign of the case number is referred to '-' instead of '#'. For example; TN1-2 means the model is TN1 and the case number is #2.

Table 1. Attributes and mechanical properties of specimens.

Name	Type and the age at 2nd cast	*Curing	Fb (MPa)	GF (N/mm)	Ft (MPa)
CN	Monolith as a reference	Sealed, 28days	4.37	0.110	7.07
CR-C	Placing joint at 1day	Sealed, 29days	0.983	0.010	3.49
CE	Wash-out joint at 1day	Sealed, 29days	1.32	0.021	4.40
CR-A	Repaired joint at 28days	Sealed, 56days	3.23	0.055	4.66

*Curing: Age refers to the days after the first placing (for the half previously cast) of concrete. The age of the opposite half is 28days when fracture toughness test is done.

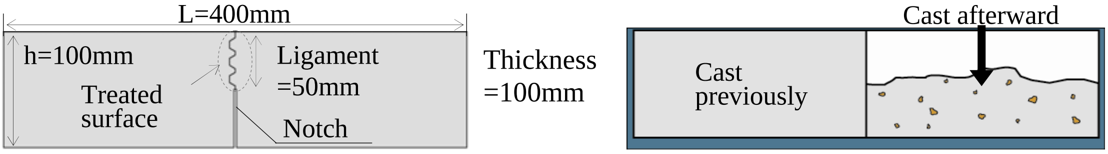


Figure 1. Detail of specimen (left) and method for producing specimen (right).

Table 2. mechanical properties for FEM and BCM analyses.

	Tensile element model				Bending model			
Materials	Case	*E (GPa)	Ft (MPa)	GF (N/m)	Type	*E (GPa)	Ft (MPa)	GF (N/m)
Mortar	#1	18.7	7.07	99.0	CN	18.7	7.07	99.0
Aggregate		39.0	8.06	74.5		39.0	8.06	74.5
ITZ		18.7	5.66	71.0		18.7	5.66	71.0
Interface		18.7	6.23	32.0				
Mortar	#2	18.7	7.21	59.8	CR-C CE	18.7	7.07	99.0
Aggregate		39.0	16.3	138.4		39.0	8.06	74.5
ITZ		18.7	3.90	125.4		18.7	5.66	71.0
Interface		18.7	3.15	11.4		18.7	3.49	18.0
Mortar	#3	18.7	7.21	59.8	CR-A	18.7	7.07	99.0
Aggregate		39.0	16.3	138.4		39.0	8.06	74.5
ITZ		18.7	2.81	25.9		18.7	5.66	71.0
Interface		18.7	3.15	11.4		18.7	5.06	85.0
Repair						18.7	4.05	85.0

*E: Young's modulus

ANALYSIS OF TENSION MEMBER

Analysis of tensile member

Figure 2 shows four types of tensile model for FEM analysis. The size of aggregate and ITZ are the same but the loacation of it is different. TN model is monolithic but TR model has an interfacial layer at the center of the model. Gradually increasing displacement was applied to the right side of the member while the opposite side is fixed.

Commercially available FEM program was used for the analysis. All the elements were plane stress elements, where distributed crack model and rotating crack model were applied. The FEM program can deal with the unloading after crack of the element. Though it can not deal with the localization of crack, the effect of this imperfection of the program is not large because crack extends in the weakest path shown later in this chapter.

The examples of resulted load-displacement curves are depicted in Fig. 3 where the adapted combination of mechanical properties were the case #2. The response for TR1 and TR3 are much the same, but TN1 and TN5 have different response reflecting the crack path.

Figure 4 shows the principal stress at

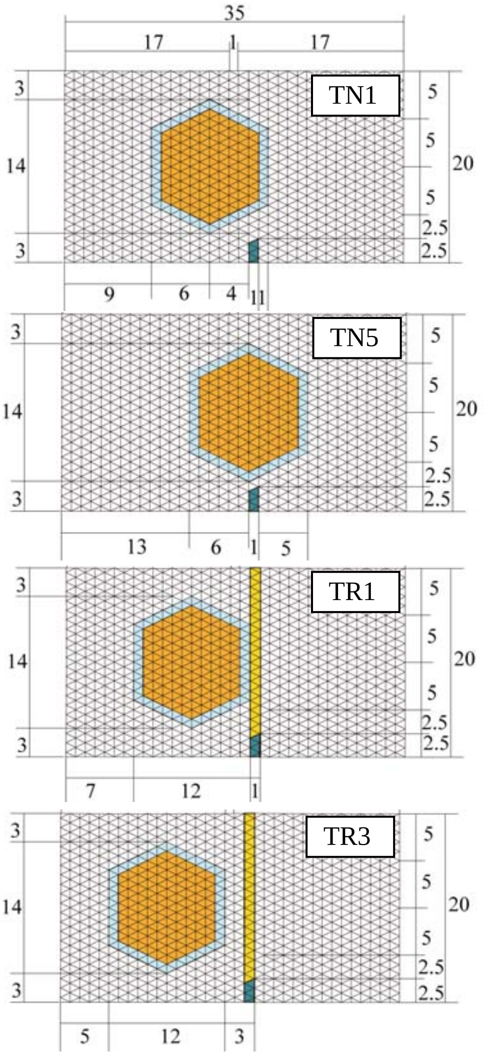


Figure 2. Four types of tension member.

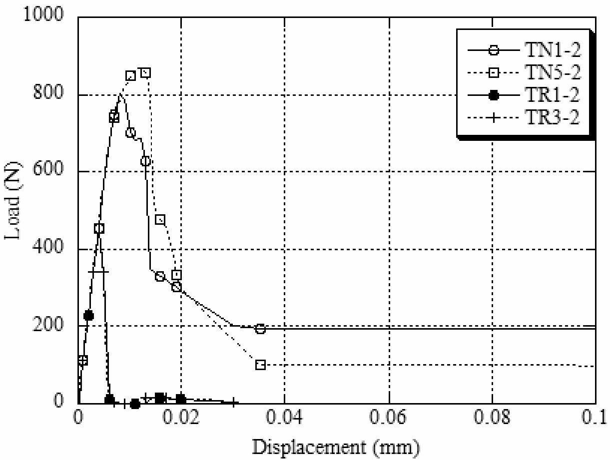
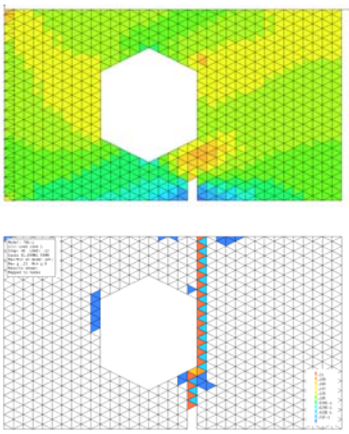


Figure 3. Analyzed load-displacement curves.



(a) Principal stress

(b) Crack strain

Figure 4. Analyzed results (TN1-1).

maximum load (upper) and crack strain for the final stage of the TN1-1. There are two region of stress concentration on the right side of the aggregate: the bottom one derives from the crack tip and the top one from the bend of the stress line. Also it is shown that the region of stress concentration by the crack tip is about 4mm by 4mm.

Principles of crack extension

The authors analyzed 12 cases of the tension members: four types of members with three different cases of mechanical properties. By comparing the crack path determined on a certain assumption with the ones achieved from FEM analysis, the principles of crack extension were established as listed below.

[1] Crack extends in the weakest path in a region enclosed 4mm by 4mm. For example, if the crack tip is at a (Fig. 5(1)), the numbers of elements in the assumed paths (solid line, dotted line and so on) multiplied by their strength were compared and the path which has the minimum value is selected to be the right path. The procedure should be repeated in every region (4mm by 4mm) till the end.

[2] The crack path extends from the bottom of the ITZ to the top of it without diversion reflecting the stress concentration. (See Fig.5 (2).)

[3] When counting the numbers of the elements at the top of vertical ITZ, half of the element should be reduced. (See Fig.5 (3).)

Box counting method

The authors established the BCM for the prediction of crack path and mechanical properties. There are two processes in the method.

[1] The crack path is predicted with observing the above mentioned principles.

[2] After the path is determined, the number of the boxes (i.e. triangular elements) of each material is counted. (See Fig. 6) Then the mechanical property of each material is calculated with summing up the numbers of element multiplied with each property of the material. The mechanical property is the summed values of all materials divided with the face area of the member.

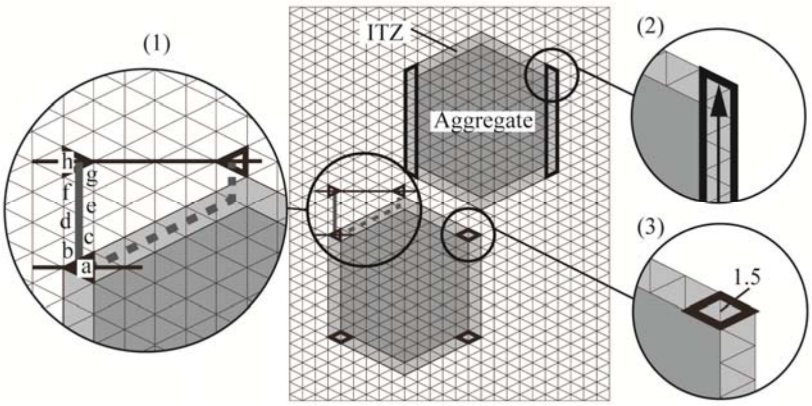


Figure 5. Principles of crack extension.

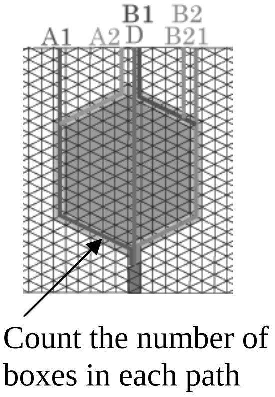
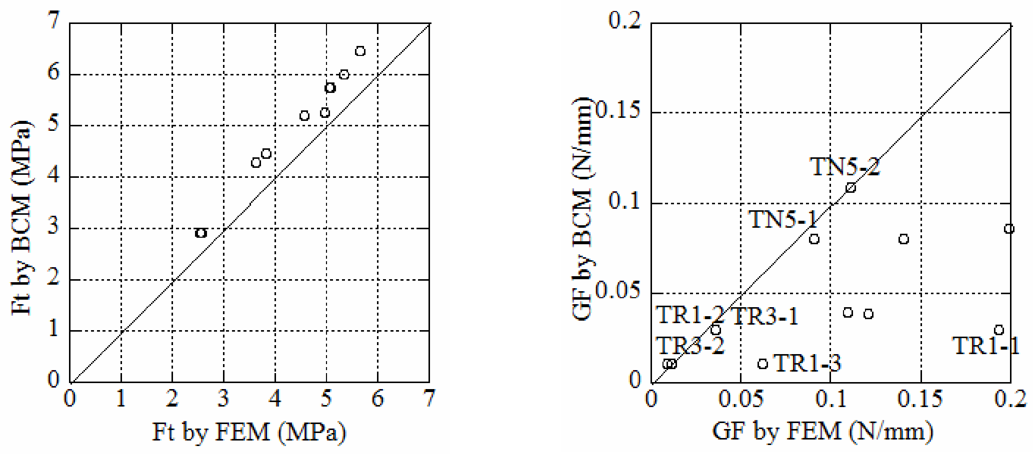


Figure 6. Box counting.



(a) Ft (b) GF

Figure 7. Comparison of the results by BCM with that of FEM.

The consistency of BCM is assured comparing the results by BCM with the one by FEM. Figure 7 shows Ft and GF of all the 12 cases of the tension member. BCM predicts Ft 10% higher than that of FEM. The reason for it is that the maximum load appears when the crack tip is middle of the crack path and the number of the element which reached Ft is limited in the case of FEM analysis. But BCM assumes all the element have Ft. Although Ft is consistently predicted, GF is not. The reason for it is that GF generates outside of the crack path, as Figure 4(b) shows that ITZ behind the path is cracking. Then the result of the member which has the simple path is consistent with that by FEM.

ANALYSIS OF BENDING MEMBER

Analysis by FEM and BCM

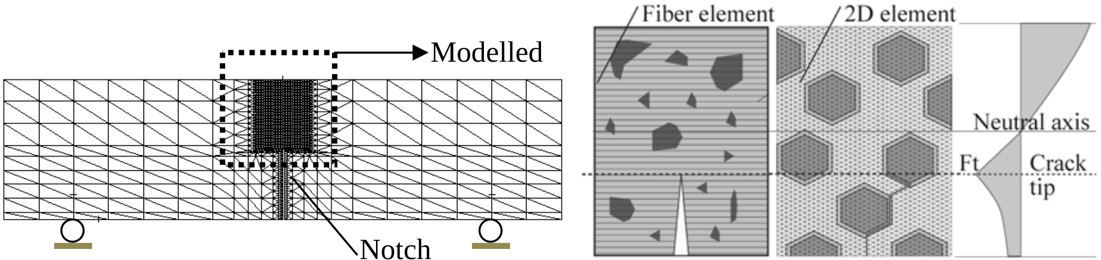
Two types of bending member were analyzed by FEM and BCM: CN (monolith) and CR-A (repaired). The whole model is shown in Fig. 8 and the detailed model of the ligament is shown in Fig.9. The resulted crack paths are shown in Fig. 9. Fig 9(a) tells that the lower half of the path in CN is predicted correctly but the upper half is not. The reason for this is due to the influence of the compressive stress near the loading point.

Figure 10 is the load-deflection relation of two types of bending member analyzed by FEM. Though there is some deviation in the case of CN, the response is consistent with the experimental results. The upper half has little effects to the behavior, because maximum load is reached when crack tip is middle of the path, and the path is consistent at that point, which makes Fb consistent. It is worthy of note that the sub-crack in upper half of FEM path is the main BCM path. Figure 9(b2) tells that both paths are consistent in the case of CR-A. The straight path in the repair material is due to the low strength of the repair material.

Prediction of Fb by BCM

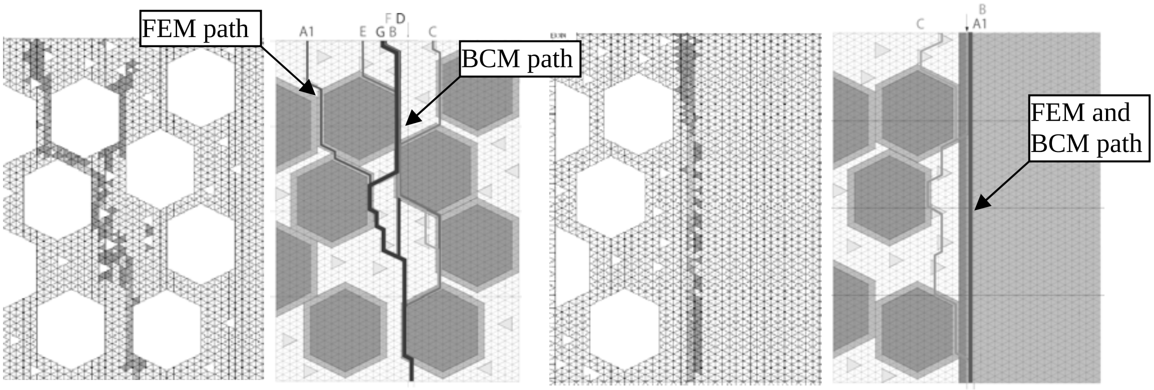
The authors presented the fiber model method by which Fb can be calculated with assuming the stress block model [10] in which Ft is the key material property as Figure 8(b) shows. It can be said that BCM is an “extended fiber model method” from linear to two-dimensional. Though BCM cannot directly predict Fb, the extended fiber model can predict it with employing the same experimental equation as the one used for fiber model method [10], as follows.

$$Fb = 0.7089Ft \tag{1}$$



(a) Whole model (b) Concept of extended fiber model

Figure 8. Bending model.



(a1) FEM (CN) (a2) BCM (CN) (b1) FEM (CR-A) (b2) BCM(CR-A)

Figure 9. Detailed model of ligament and the resulted path in bending model.

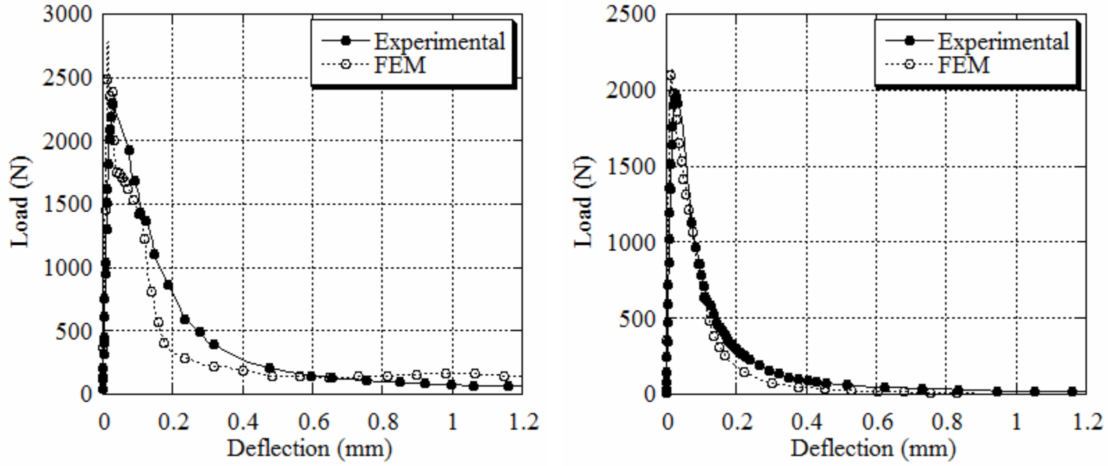
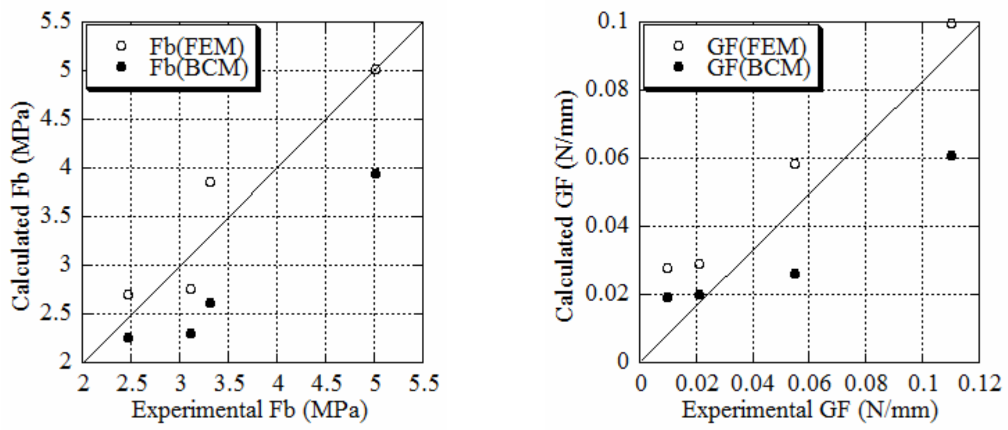


Figure 10. Load-deflection relation of bending member (left: CN, right: CR-A).

Figure 11 shows the correlations of calculated (both by FEM and BCM) results with experimental ones. Fb predicted by BCM is lower than that of what is predicted by FEM and the accuracy is inferior to that of predicted by FEM. The reason for it is Ft in Eq. (1) is the local Ft achieved from TSD whereas Ft from BCM is the averaged one throughout the path. Then if Ft is replaced with the maximum Ft in the path, the predicted values by BCM would be more consistent with the experimental ones. Predicted GF has the same tendency as seen in Fb. But the results by BCM are more consistent in the case of bending member than that in the case of tension member. This derives from the fact that the tension member has only one aggregate whereas the bending member has many aggregates which makes the properties averaged.



(a) Fb (b) GF

Figure 11. Predicted Fb and GF by FEM or BCM.

CONCLUSION

The authors presented box counting method as an interpretation of results from FEM analysis. This new method can predict crack path and mechanical properties (Ft, Fb and GF) at almost the same accuracy as FEM.

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