Direction of the Maximum Variance of the Resolved Shear Stress and Orientation of Stage I Crack Paths

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ABSTRACT. *Our formalisation of the Maximum Variance Method (τ-MVM) postulates that, in ductile materials subjected to fatigue loading, the plane where the crack initiation phenomenon takes place, i.e. the so-called Stage I plane, is the one containing the direction along which the variance of the resolved shear stress reaches its maximum value. From an engineering point of view, the most interesting implication of the above assumption is that the τ-MVM can successfully be used to address problems involving not only constant but also variable amplitude uniaxial/multiaxial fatigue loading. Further, thanks to its particular features, from a computational point of view, after calculating the variance and covariance terms associated with the considered load history, the effective time needed to locate the critical plane does not depend on the length of the load history itself. Such a computational efficiency makes the τ-MVM an appealing engineering tool suitable for being used to perform the fatigue assessment of real components. In this scenario, the present paper aims to investigate whether, independently from the degree of multiaxiality and non-proportionality of the applied loading path, the direction experiencing the maximum variance of the resolved shear stress is capable of correctly predicting the orientation of Stage I crack paths.*

INTRODUCTION

Examination of the state of the art [1, 2] shows that the most successful criteria suitable for estimating medium/high-cycle fatigue damage under multiaxial time-variable loading are those based on the use of the so-called critical plane concept. In order to efficiently apply the above criteria, one of the trickiest aspects is correctly defining the orientation of the critical plane, where such a problem must be optimised not only in terms of modelling the physical processes taking place within the process zone [3], but also in terms of computational time required to determine the orientation of that material plane on which fatigue damage reaches its maximum value [4].

In this complex scenario, we have recently proposed a novel approach taking as a starting point the assumption that the critical plane can be determined through that direction experiencing the maximum variance of the resolved shear stress - the so-called Shear Stress-Maximum Variance Method (τ-MVM) [5]. In more detail, such an approach locates the critical plane by addressing the problem in terms of variance and co-variance of the stress components damaging the assumed critical location. From a fatigue design point of view, the most interesting peculiarity of the τ-MVM is that it is very efficient, the computational time required to reach convergence being independent from the length of the post-process load history [6]. Having said that, a very cruel question arises: Is the direction experiencing the maximum variance of the resolved shear stress capable of correctly predicting also the orientation of Stage I crack paths independently from the degree of multiaxiality and non-proportionality of the applied loading path? This paper then attempts to quantitatively answer the above question.

STAGE I AND STAGE II CRACKS UNDER FATIGUE LOADING

Back in the 60s, by performing an accurate experimental investigation, Forsyth has suggested that the process resulting in the initiation and subsequent initial propagation of micro/meso cracks can be subdivided into two different stages [7]: Stage I cracks grow along those crystallographic planes experiencing the maximum shear, their propagation being mainly Mode II dominated; Stage II cracks instead take over from Stage I propagation and their growth is Mode I governed. In other words, the formation of Stage I cracks is controlled by the microscopic shear stress/strain relative to those easy glide planes subjected to the maximum shear. The Stage I crack length is seen to vary as both the material morphology and the amplitude of the applied stress vary, the maximum length of Stage I cracks being of the order of a few grains [8].

By carefully investigating the cracking behaviour of uniaxially fatigued ductile materials, Tomkins came then to the following ground-breaking conclusion: *"Stage II propagation occurs due to plastic de-cohesion on the planes of maximum shear strain gradient at the crack tip… the same mechanism is operative also in Stage I growth, but de-cohesion occurs on only one of the available shear planes"* [7].

The initiation and initial propagation of micro/meso crack is governed by the same mechanisms also when ductile engineering materials are subjected to multiaxial fatigue loading. As to the observed cracking behaviour under complex loading paths, initially it is worth remembering here that micro/meso-cracks can propagate either on the component surface (Case A) or inwards (Case B), where Case B is seen to be much more damaging than Case A [9]. If attention is focussed solely on Case A, it is common opinion that, under multiaxial fatigue loading at room temperature, fatigue cracks always initiate on Stage I planes and it holds true independently from the degree of multiaxiality of the stress/strain field acting on the fatigue process zone (see Ref. [2] and the references reported therein). In particular, in some materials the crack initiation phenomenon is characterised by the formation of Stage I cracks whose length cover several grains [10]. On the contrary, in some other cases, Stage I cracks are so small that the overall cracking behaviour at a mesoscopic level is mainly Mode I governed [11].

It is possible to conclude by observing that, according to the considerations briefly summarised above, the maximum shear stress is then an engineering quantity which is closely related to the initiation and initial propagation of fatigue cracks [2]. This implies that such a stress component can successfully be used to estimate fatigue damage, provided that, the critical planes used to perform the fatigue assessment are capable of correctly modelling the formation of Stage I cracks.

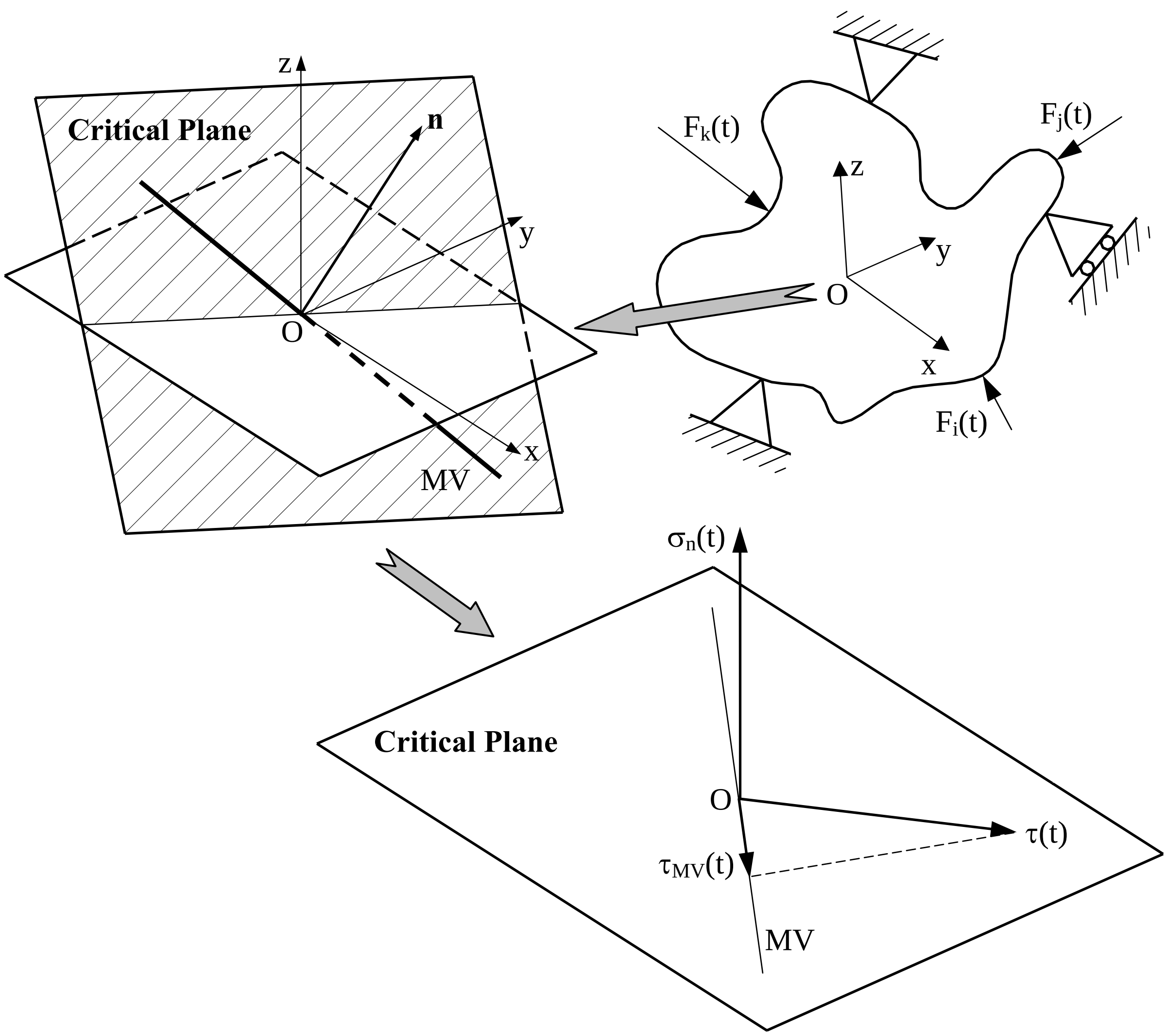

Figure 1. Critical plane and direction, MV, experiencing the maximum variance of the resolve shear stress.

FUNDAMENTALS OF THE MAXIMUM VARIANCE METHOD

Since the most important features of the τ-MVM have already been discussed in Refs [5, 12] in great detail, considering also the numerical aspects involved in the solution of the problem [6], only the fundamental idea on which such a method is based will be considered below.

The τ-MVM takes full advantage of the fact that, in a time-variable load history, the variance of the stress process damaging the component being assessed is proportional to the amplitude of the signal and not to its mean value [13]. In particular, the variance of a time-variable signal is the expected value of the square of the deviation of that signal from its mean value, so that it quantifies the amount of variation of the signal itself within the two extremes defining the maximum range.

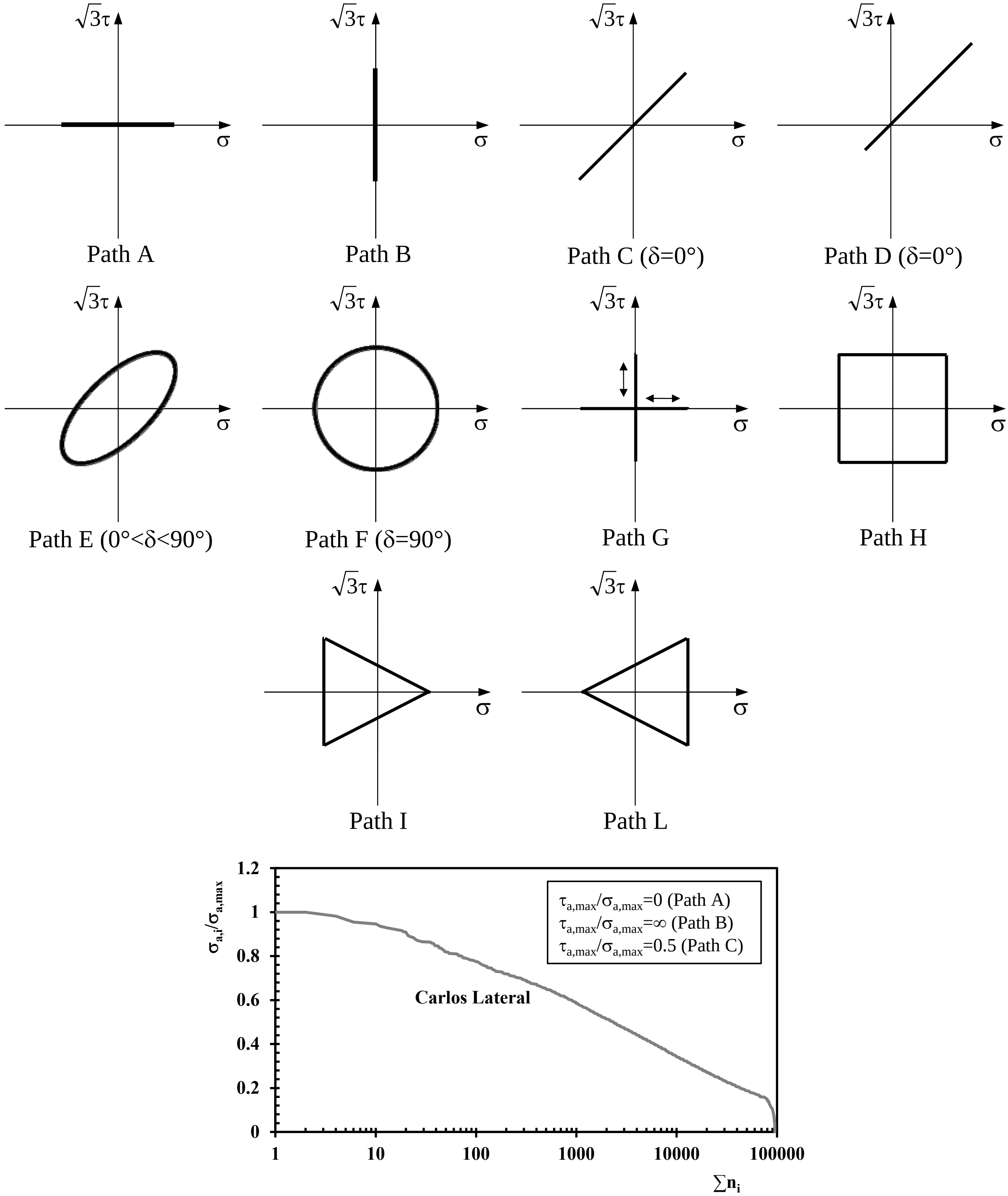

Figure 2. Investigated constant and variable amplitude loading paths.

From a structural integrity point of view, the above definition results in the fact that the variance of a stress signal can be assumed to somehow be related to the associated fatigue damage extent [5, 13].

The above preliminary considerations should make it evident that, by correctly post-processing the time-variable stress state at the assumed critical point, it is possible [2, 5, 12] to determine the orientation of the critical plane by locating the material plane containing that direction, MV, which experiences the maximum variance of the resolved shear stress, $\tau_{MV}(t)$ – see Figure 1. In terms of microscopic processes taking place within the process zone and resulting in the formation of Stage I cracks, the macroscopic direction experiencing the maximum variance of the resolved shear stress can be assumed to be coincident with that microscopic easy glide direction along which the dislocation motion is maximised [14], the resolved shear stress being the driving force of such a process [15].

To conclude, it can be said that the use of the τ-MVM results in a great simplification of the multiaxial fatigue assessment issue, because, according to the above assumptions, the resolved shear stress, $\tau_{MV}(t)$, is a monodimensional time-variable stress quantity that, over time, varies its magnitude, but not its direction (Fig. 1).

Table 1. Mechanical static properties of the investigated materials.

Material	**Ref.**	σ_y *[MPa]*	σ_{UTS} *[MPa]*
Mild Steel	[16]	221.7	373.8
Hard Steel	[16]	392.4	680.8
Grey Cast Iron	[16]	-	171.7
GGG40	[17]	334	447
GTS45	[17]	305	449
18G2A	[18]	-	535
1%Cr-Mo-V	[19]	725	828
42CrMo4	[20]	980	1100
AISI 303	[20]	330	625
CK45	[21]	410	660
30NCD16	[22]	930	1070

MAXIMUM VARIACE METHOD AND STAGE I CRACK PATHS

In a ground-braking paper dated back 1977, Kanazawa, Miller and Brown affirm [14]: *"Stage I cracks form on crystallographic planes, being slip planes within individual grains of metal. These are not necessarily the planes of maximum shear in the macroscopic sense, but rather the slip system most closely aligned to these planes. Clearly, the slip systems which experience the greatest amount of deformation are those which align precisely with the maximum shear direction, and therefore most fatigue* *cracks initiate in these grains. But slip systems with lesser degrees of shear also initiate cracks at a slower rate".*

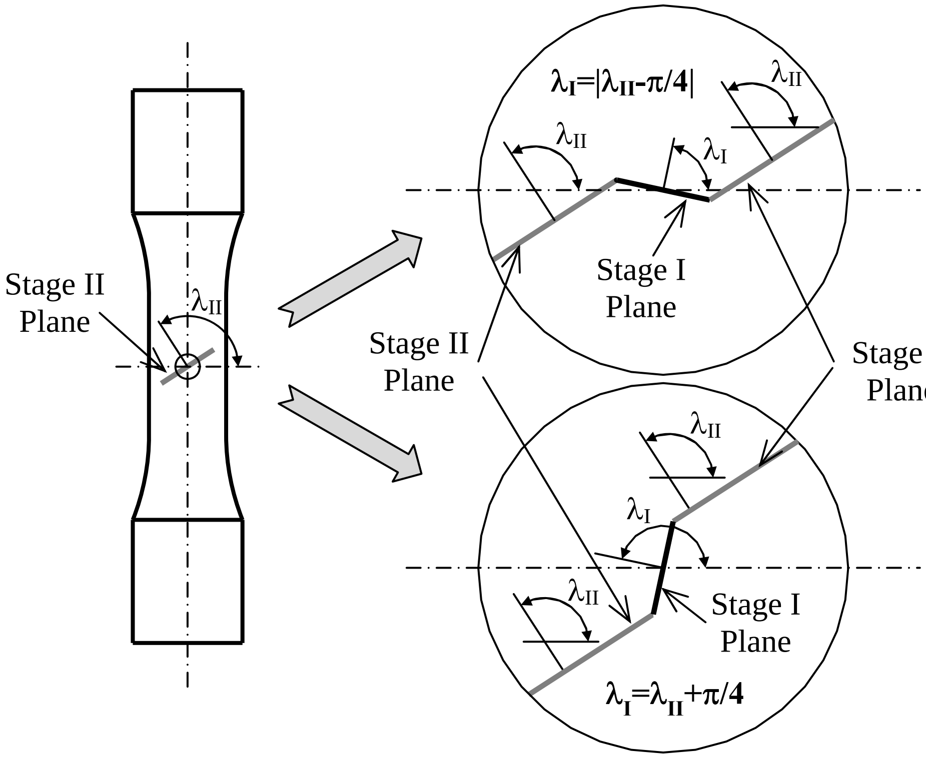

Figure 3. Assumption made to estimate the orientation of Stage I planes from the measured Stage II crack paths.

As briefly mentioned in the previous Section, the τ-MVM estimates the orientation of the critical plane through the direction experiencing the maximum variance of the resolved shear stress, such a direction being considered as coincident with that microscopic easy glide direction along which the dislocation motion is maximised. In the present section, it is investigated then whether the τ-MVM is capable of modelling the experimental reality as described by Kanazawa, Miller and Brown [14].

In order to check the accuracy of the τ-MVM in estimating the orientation of Stage I crack paths under complex fatigue loading, a systematic bibliographical investigation was carried out to select a number of appropriate experimental results, the static properties of the considered materials being summarised in Table 1. In the investigated materials fatigue cracks were generated by testing cylindrical samples under the loading paths sketched in Figure 2. By carefully observing the way the cracking behaviour of the materials listed in Table 1 was investigated, it is easy to come to the conclusion that the measured crack path orientations reported in the original sources were determined by directly measuring the orientation of the macroscopic Stage II planes [16, 20, 21]. Therefore, Stage I crack path directions were derived, as suggested indirectly by Miller

[23] and explicitely by Carpinteri and co-workers [24, 25], according to the simplified rule summarised in Figure 3.

The overall accuracy of the τ-MVM in modelling the Stage I cracking behaviour is summarised by the experimental, λ_I, vs. estimated, $\lambda_{I,e}$, Stage I crack path angle diagram reported in Figure 4 (see Figure 3 for the definition of angle λ_I).

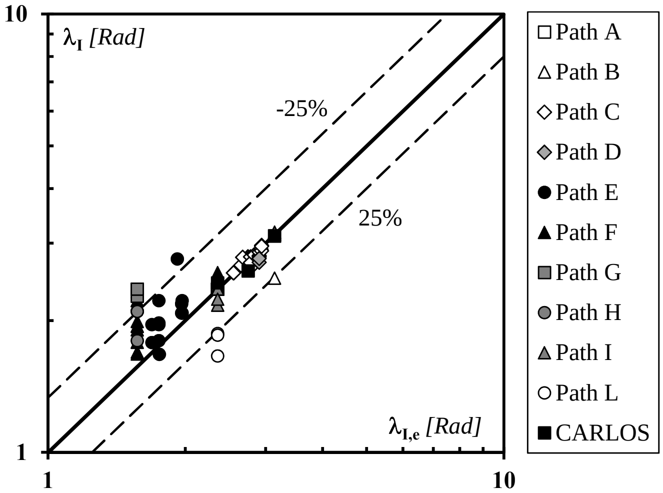

Figure 4. Accuracy of the τ-MVM in estimating the orientation of Stage I crack paths.

The above chart makes it evident that the τ-MVM is highly accurate in estimating the orientation of Stage I crack paths under proportional fatigue loading, this holding true also when variable amplitude load histories are involved. On the contrary, as far as non-proportional situations are concerned, the use of the τ-MVM results in a higher level of scattering. This can be ascribed to the fact that under non-proportional loading (i.e., when the principal directions rotate during the loading cycle) several slip systems are activated simultaneously so that microscopic Stage I cracks tend to propagate in several directions, resulting in a larger deviation of the Stage I directions [14]. On the contrary, under proportional loading, micro-cracks are seen to grow mainly along a preferential plane, resulting in smaller deviations of the propagation directions with respect to the one of maximum shear [14].

CONCLUSIONS

According to its *modus operandi*, the τ-MVM estimates the orientation of the critical plane through that direction experiencing the maximum variance of the resolved shear stress, such a plane being treated as that material plane where the initiation and initial propagation of Stage I cracks take place. This direction is calculated by assuming that metallic materials are homogenous and isotropic, that is, by disregarding the real material morphology. In spite of the above simplifying hypotheses, the sound agreement between estimated and experimental orientation of the Stage I crack paths we have obtained suggests that the τ-MVM is a design tool capable of capturing the engineering essence of the investigated phenomenon. Accordingly, the τ-MVM can safely be used, together with the appropriate multiaxial fatigue criterion [2, 5, 12], to perform the fatigue assessment in situations of practical interest.

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