LEFM-based simulation of fatigue crack growth under non-proportional mixed-mode loading

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***ABSTRACT**. 3-dimensional fatigue crack growth simulation under non-proportional mixed-mode loading is a new and challenging topic in engineering research field. The present paper proposes an algorithm based on linear elastic fracture mechanics to discuss this problem. The crack growth in 4 thin-walled, hollow cylinders with notch under combined non-proportional cyclic tension and torsion are calculated and analysed. Different material, torsion and tension loading ratio M_T/F and out-of-phase angle are considered, R ratio equals to -1. The crack initiation locations and the crack growth paths as well as the fatigue lives are identified. The simulation results are compared to the results of the previous experiments, showing an acceptable agreement.*

INTRODUCTION

Research on fatigue crack growth under non-proportional mixed-mode loading has attracted some attention in resent years. Brüning et al. [1,2,3] provide an experimental data base for the crack initiation and growth behaviour of the fine-grained steel S460N and aluminium alloy AlMg4.5Mn under predominantly mixed-mode I and II proportional and especially non-proportional loading. Endo and McEvily [4] investigated the crack growth and threshold behaviour of small cracks initiated from small defects under in-phase and out-of-phase combined tension and torsion loading. Weick and Aktaa [5] obtained the experimental results of the fatigue life under non-proportional multiaxial cyclic loading for tubular specimens. In these references the experimental work is emphasized.

The present paper proposes an algorithm for simulating fatigue crack growth under non-proportional mixed-mode loading.

EXPERIMENT

In order to validate this algorithm, the simulation results were compared with the experimental data obtained by Brüning [1]. The thin-walled, hollow cylindes of two materials, aluminium alloy AlMg4.5Mn and steel S460N have been tested with the geometry shown in Figure 1. Two different combined non-proportional cyclic tension and torsion loading sequences with an R ratio of -1 are shown in Figure 2. Most of Brüning's experiments were performed in the large scale yielding region. The results of the 4 specimens A7, A8, S7 and S13 could be used here for comparison purpose because the loading was low enough to meet small scale yielding requirements. The basic mechanical properties of the two materials are shown in Table 1.

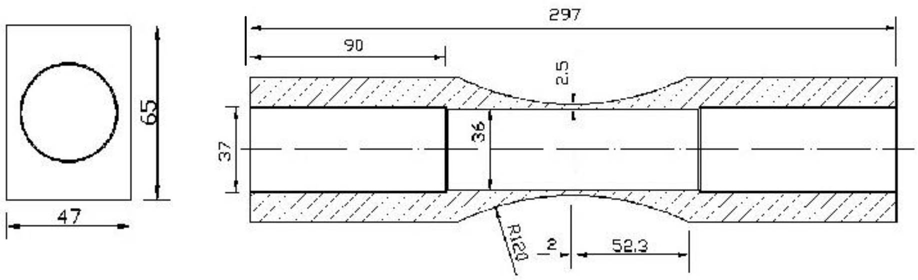

Figure 1. Specimen geometry (dimensions in millimetres)

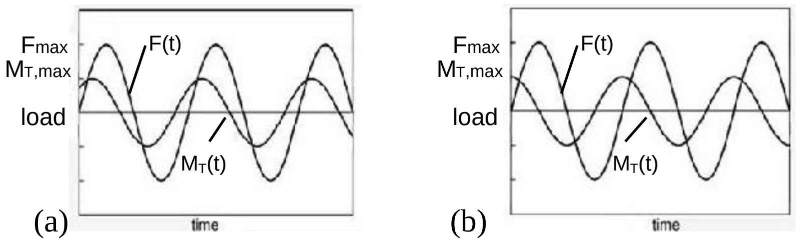

Figure 2. Loading types: (a) phase angle of 45°, (b) phase angle of 90°

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Table 1 Mechanical properties [4]

Material	E [MPa]	¬ [-]	$R_{p0.2}$ [MPa]	R_m [MPa]	A_5 [%]	$\pm_y$' [MPa]	C (R=-1) da/dn in [mm/cycle] ΔK in [MPa$\sqrt{mm}$]	m
AlMg4.5Mn	68000	0.33	169	340	20.2	341	3.32e-16	4.25
S460N	208500	0.3	500	643	26.2	410	6.46e-14	2.92

SIMULATION ALGORITHM

The simulation algorithm is based on linear elastic fracture mechanics (LEFM), Figure 3 shows a illustration of the algorithm's idea and the simulation procedure. The main modules of this procedure can be considered as: (a) determining of the crack initiation position; (b) calculating of the maximum equivalent stress intensity factor K_{eq} in one load cycle, which is taken as the crack driving force parameter; (c) crack growth process. Module (b) and (c) are repeated until the crack growth path can be presented clearly.

Location of crack initiation

The finite element model was created using the ABAQUS software [6]. The specimen was modelled with hexahedral and tetrahedral elements. The degrees of freedom were fixed at the clamping region of one end of the specimen. At the other end the normal force F and torsional moment M_T were introduced via a reference point. The degrees of freedom of the reference point are coupled with the degrees of freedom of the neighbouring clamping region in such a way that for the clamping region only rigid body translations along and rotations around the specimen axis are allowed. The middle portion of the model is considered as the crack growth region with refined mesh as shown in Fig 4 (a). Along the notch root, there are four potential crack initiation sites numbered 1, 2, 3 and 4 clockwise, shown in Fig 4 (b).

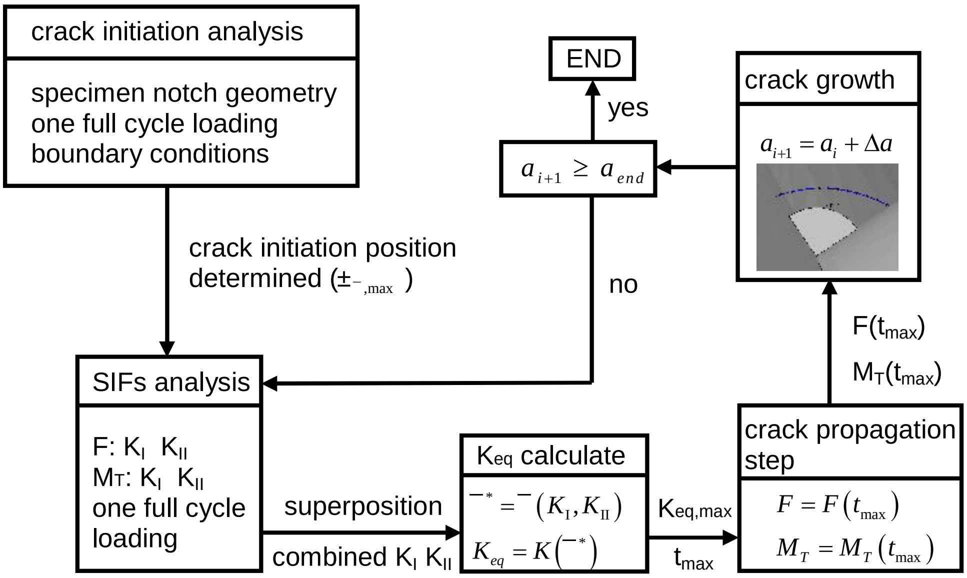

Figure 3. Simulation algorithm procedure

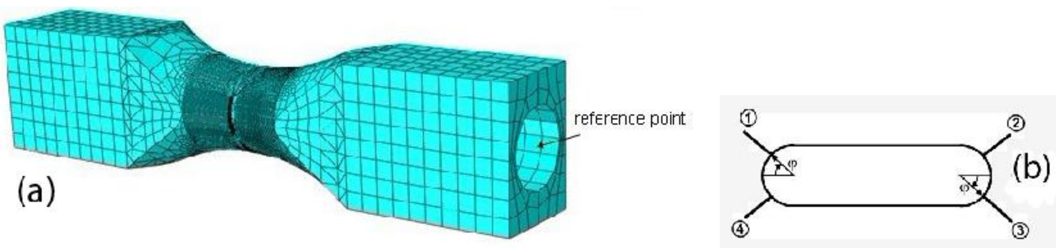

Figure 4. (a) Finite element model, (b) Crack initiation sites

One full load cycle was analysed for the uncracked specimen. Solutions for unit load cases F=1 and M_T=1 are provided, $\pm_F$ and $\pm_M$. The tangential stress ± along the notch root, location s , at each time t is

$$\pm(s,t) = \pm_M(s)\cdot M_T(t) + \pm_F(s)\cdot F(t). \tag{1}$$

In Figure 5 (a) each curve describes the variation of the tangential stress corresponding to one node in the notch root during one full cycle load. The mode showing the maximum stress is taken as the crack initiation site.

Equivalent stress intensity factor calculation

In linear elastic fracture mechanics, stress intensity factor SIF is the most important parameter to predict crack growth direction and crack growth rate. In mixed-mode loading, the calculated mode I and mode II SIFs, K_{I} and K_{II}, are used to obtain the equivalent SIF, K_{eq}, which determines the crack growth rate.

Erdogan and Sih [7] proposed that the equivalent stress intensity factor according to the maximum circumferential stress criterion can be described by:

$$K_{eq} = \frac{1}{4}\left(3\cos\frac{\theta^*}{2} + \cos\frac{3\theta^*}{2}\right)K_{\mathrm{I}} - \frac{3}{4}\left(\sin\frac{\theta^*}{2} + \sin\frac{3\theta^*}{2}\right)K_{\mathrm{II}} \tag{2}$$

θ^* represents the direction where the circumferential stress in the vicinity of crack tip is maximum.

$$\theta^* = 2\arctan\left(\frac{1}{4}\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}} \pm \frac{1}{4}\sqrt{\left(\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}}\right)^2 + 8}\right) \tag{3}$$

By application of the parameter K_{eq} the fatigue life can be calculated based on Paris law as equation (4), where C and m are material constants, see Table 1.

$$\frac{da}{dN} = C \cdot \left(\Delta K_{eq}\right)^m \tag{4}$$

In this paper the Fracture Analysis Code 3D (FRANC3D) [8] is used to calculate stress intensity factors and simulate crack growth. It is designed to simulate crack growth in engineering structures with arbitrary component or crack geometry, loading and boundary conditions. FRANC3D adaptively remeshes a finite element model created by ABAQUS.

On the basis of crack initiation analysis, quarter-elliptical cracks are used for simulating the initial cracks in the notch root, which are inserted into the crack initiation positions. The stress intensity factors for mode I and mode II are calculated by FRANC3D software for tension F=1 and torsion M_{T}=1. The mixed-mode K_{I} and K_{II}, shown in Figure 5 (b) are obtained from the superposition of tension and torsion K_{I} and K_{II} values, as equation:

$$\begin{aligned} K_{\mathrm{I}}(\mathrm{t}) &= K_{\mathrm{I},F} \cdot F(\mathrm{t}) + K_{\mathrm{I},M} \cdot M_T(\mathrm{t}) \\ K_{\mathrm{II}}(\mathrm{t}) &= K_{\mathrm{II},F} \cdot F(\mathrm{t}) + K_{\mathrm{II},M} \cdot M_T(\mathrm{t}). \end{aligned} \tag{5}$$

The equivalent stress intensity factors K_{eq} are calculated by equation (2) and (3). K_{II} is always very small compared to K_I, therefore the disparity between K_I curve and K_{eq} curve are not obvious in Figure 5 (b). The time at the maximum K_{eq} in a full cycle, t_{max}, is considered to be the crack propagation moment. The load for crack propagation can be determined, $F(t_{max})$and $M_T(t_{max})$.

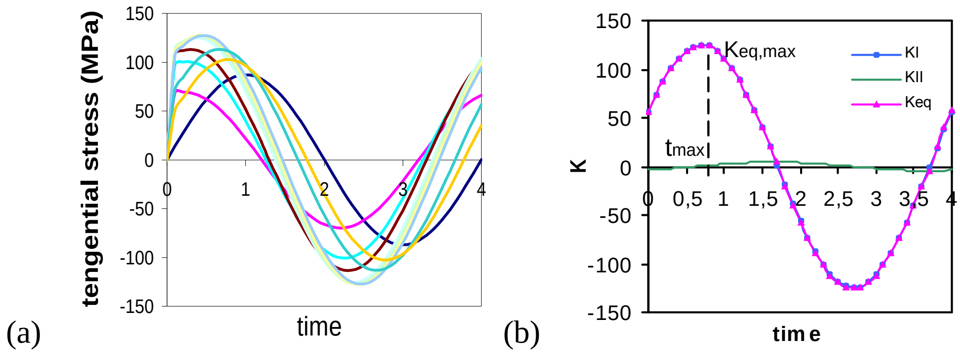

Figure 5. (a) tangential stress curve, (b) K_I K_{II} K_{eq} curve

Crack growth process

The load corresponding to the $K_{eq,max}$, $F(t_{max})$ and $M_T(t_{max})$ are applied to the specimen with the actual cracks. Stress analysis is carried out in ABAQUS. The SIF solutions for the cracks are again obtained from FRANC3D based on the finite element stress analysis. Although the solutions for K_I, K_{II} and K_{eq} are already at hand for $t = t_{max}$, the re-evaluation of this loading case combination can now be used to let the FRANC3D software create the crack propagation increment Δa.

The ABAQUS/FRANC3D interface was implemented to complete the crack growth process. The finite element modelling and the stress analysis are performed in ABAQUS, FRANC3D is used to calculate crack growth parameters and updates the crack geometry and mesh. This process is continued until the crack has grown to a certain length.

RESULTS

Non-proportional loading for phase angle of 45 °

For out of phase loading with a phase angle of 45°, specimen A8 (AlMg4.5Mn; F=6.5Kn; M_T=72Nm) and S13 (S460N; F=22.5Kn; M_T=272Nm) are simulated. According to the stress analysis for the uncracked specimen, the local circumferential stress in site 1 and 3 are 1.6 times larger than the stress in site 2 and 4. The cracks should occur in site 1 and 3. Simulation results show that for these two different specimens the crack growth paths are very similar, the crack propagates along an angle of approximately 45° with the longitudinal axis. After the crack reaches a certain length, it has a tendency to turn into a plane, which is perpendicular to the longitudinal axis.

The simulated crack growth paths have a good agreement with the experimental results, as shown in Figure 6. A small deviation between the calculated and experimental fatigue life can be observed for specimen A8, while the number of cycles are 369000 and 370000, as shown in Figure 8 (a). However, a clear over estimate of fatigue life obtained from the simulation compared to the experimental data is found for model S13, shown in Figure 8 (b) The simulation showed a higher fatigue life (127000 cycles) than the actual specimen (67000 cycles).

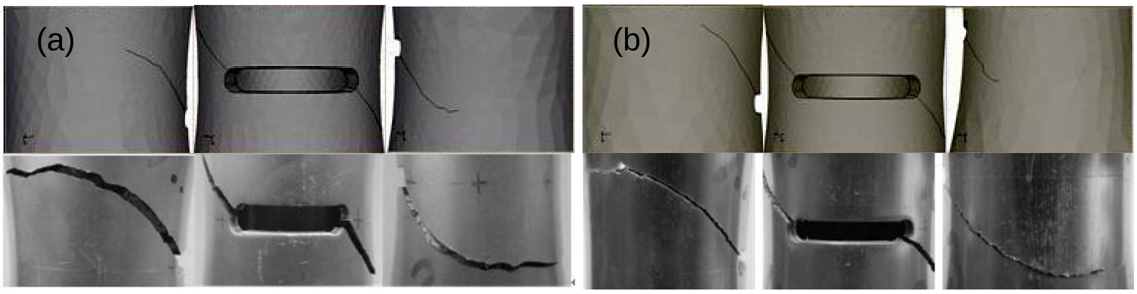

Figure 6. (a)A8 crack growth path, (b) S13 crack growth path

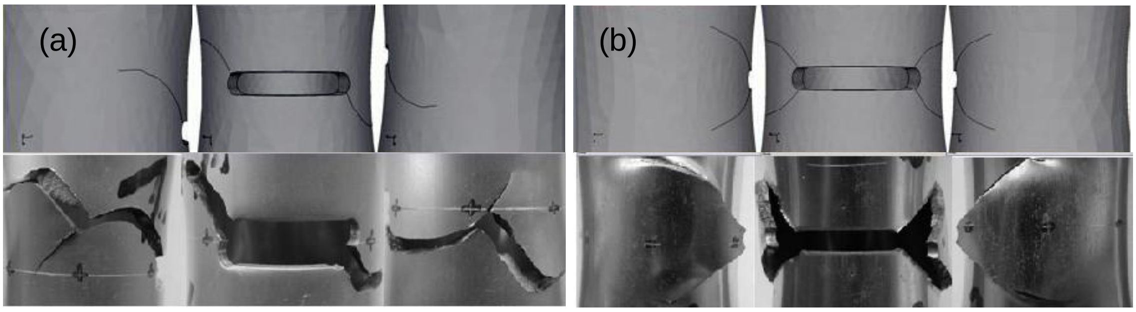

Figure 7. (a) A7 crack growth path, (b) S7 crack growth path

Non-proportional loading for phase angle of 90 °

For specimen A7 (AlMg4.5Mn; F=8kN; M_T=96Nm) both simulation and experiment displayed that the crack propagates following a very curvilinear path. In the experiment the crack branching appeared and the crack growth path continues nearly perpendicularly to the previous propagation direction. The simulation algorithm in this paper does not contain a condition for crack branching. Therefore, only the first curvilinear paths are presented as shown in Fig 7 (a). For specimen S7 (S460N; F=27kN; M_T=408Nm) 4 initial corner cracks are inserted into the notch root because the maximum local circumferential stresses are almost equal for sites 1-4.

The crack growth curves are shown in Figure 8. The calculated fatigue life of model A7 gave a reasonable approximation of the cycles to the actual specimen. The fatigue life from the simulation is 260000 cycles corresponding to the crack length is 14mm, and the cycle number is 190000 for the tested specimen when the crack grows to the same length. The calculated number of cycles for S7 is much higher than the experimental result, 61000 and 25000, respectively.

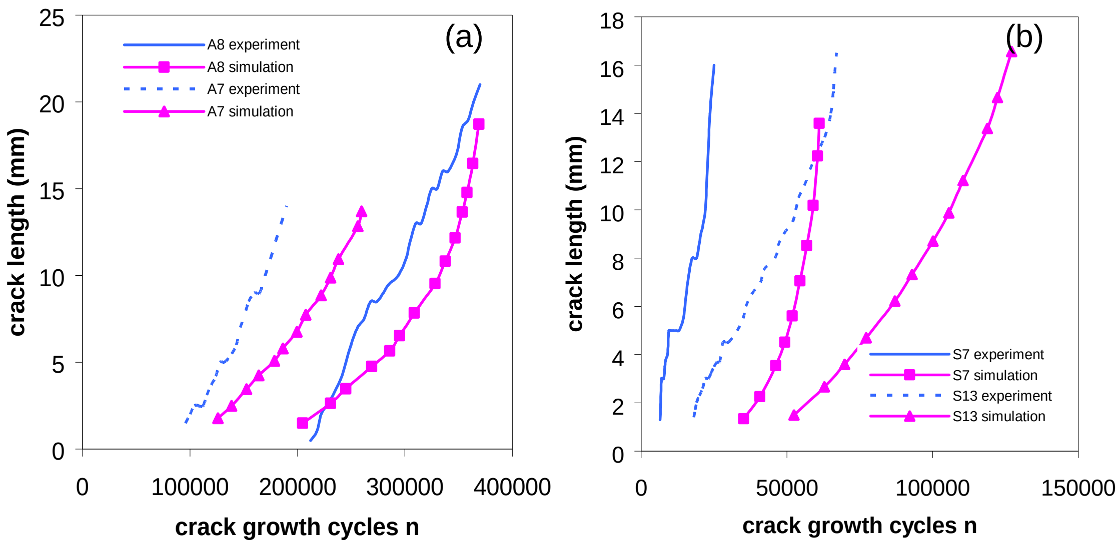

Figure 8. (a) A7, A8 fatigue life, (b) S7, S13 fatigue life

Table 2. Crack initiation and stress ration information

Phase angle	45°	45°	90°	90°
Model	A8	S13	A7	S7
f=M_T/F [mm]	11.08	12.09	12	15.11
Number of cracks	2	2	2 (4)	4
$\pm_{noch,\max}/\pm'_y$	0.378	1.168	0.453	1.577

DISCUSSION and CONCLUSIONS

The parameter torsion and tension loading ratio f is defined as f=M_T/F. It is a significant parameter for crack initiation, see table 2. Both simulation and experimental results show that when the torsion load reaches to a certain percentage of the total load, 4 cracks will grow. In specimen S7, M_T /F is 15.11 mm, the torsion load plays a more important role in crack growth compared to the other three specimens, the M_T/F values are 11.08 mm, 12.09 mm and 12 mm respectively. However, 4 cracks also initiated in specimen A7. Crack 1 and 3 are the dominating cracks, crack 2, 4 arrested at a very small crack length. The phase angle is another factor that affects the crack initiation since the load ratio is 12 for A7 and phase angle is 90°.

Table 2 shows the $\pm_{notch,max}$ /$\pm'_y$ values for the four specimens calculated based on linear elasticity. $\pm_{notch,max}$ is the maximum tangential stress along the notch root for uncracked specimen, $\pm'_y$ is the material cyclic yield stress. It was found that for specimens A7 and A8, $\pm_{notch,max}/\pm'_y$ are 0.453 and 0.378. LEFM can be considered to be applicable. The crack growth path and fatigue life calculated from the simulation show an acceptable agreement with the experimental data. For the steel specimens S7 and S13, the ratio of $\pm_{notch,max}/\pm'_y$ and $\pm'_y$ are 1.577, 1.168, the crack growth paths are similar to the experiment, but having much higher cycles to failure than the experiment. The crack growth curves in Figure 8 (b) indicate that the number of cycles which the crack spends in the notch area are clearly higher for the simulation results than in the experiment. This means that the cracks grow much faster for the early short crack in the experiment compared to the simulation. Plasticity should be a main reason to explain the acceleration of crack propagation, in this case K_{eq} used to estimate fatigue life is invalid.

In the present paper, 3 dimensional crack growth simulation is implemented by using the LEFM-based algorithm. The simulation for two different materials AlMg4.5Mn, S460N, two phase angles 45° and 90°, different loading levels and M_T/F ratio have been presented. The conclusions are summarized as:

(1) The M_T/F ratio has a great influence on the number of cracks, 4 cracks will occur when torsion loading M_T reaches a certain percentage of the total loading.

(2) The loading phase angle affects the crack growth path behaviour and also the crack initiation numbers.

(3) Compared to the phase angle of 45°, specimens under out of phase loading with a phase angle of 90°, demonstrate more variations in crack initiation and crack growth path, depending on different load level and M_T/F ratio.

(4) K_{eq} calculated from the superposition of K_I and K_{II} under tension and torsion loading is shown to be an appropriate parameter to estimate fatigue life of specimen under lower loading level.

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