Fatigue Endurance and Crack Propagation on Polymeric Material Under Ultrasonic Fatigue Testing

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ABSTRACT. *General concepts for the two principal theories of crack initiation and propagation on polymeric materials are initially presented; then, creep strength and ultrasonic fatigue testing on the polymeric material Nylon 6 are developed. Specimen was calculated numerically to fit the resonance condition and to reduce its dimension with aim to limit the temperature gradient at the specimen narrow section of this non heat conducting material. Temperature at narrow section was maintained lower than 45º C using a cooling system with cooling air; under this condition the ultrasonic fatigue tests were performed. Experimental tests were carried out at low loading range (9 – 12.5 % of yield stress of Nylon 6 in order to control the highest temperature and to avoid that specimen was out of resonance. Normalized failure function Fa was obtained in the range of applying load and it was observed that crack growth rate increases with Fa under testing conditions.*

INTRODUCTION

Polymeric materials combine inertia effects under high loading rates due to intrinsic low sound velocity and low toughness, in regard to metallic alloys, with large non-linear viscoelastic behaviour (time dependent behaviour), particularly for the low loading rates or at temperatures close to glass or phase transition temperatures. Two principal theories have been developed to approach the crack initiation and propagation in viscolelastic materials; the first one is related to the energy based criteria [1-4]; the second is the fracture mechanics approach to viscoelastic materials [5-8].

Energy based criteria for viscoelastic materials

It postulates that the work developed by external forces on a viscoelastic material is converted into potential energy (retained energy) and dissipated energy; the time of failure is determined by a threshold value of retained energy. Strain dependence on time for a viscoelastic material under arbitrary loading $\sigma(t)$, may be approached by the equation 1.

Here, D_0 and D_1 are related to compliance properties of viscoelastic material, n is an exponential constant and τ_0 represents the time unity (sec, min, hours or day). Retained energy is calculated by equation (2), proposed by Hunter [9].

$$\varepsilon(t) = D_0\sigma(t) + D_1\int_0^t \left(\frac{t-\tau}{\tau_0}\right)^n \frac{\partial\sigma(\tau)}{\partial\tau} d\tau \tag{1}$$

$$W_r(t) = \varepsilon(t)\sigma(t) - \frac{1}{2}D_0\sigma(t)^2 - \int_0^t \int_0^t D_1\left(\frac{2t-\tau_1-\tau_2}{\tau_0}\right)^n \frac{\partial\sigma(\tau_1)}{\partial\tau_1}\frac{\partial\sigma(\tau_2)}{\partial\tau_2} d\tau_1 d\tau_2 \tag{2}$$

Negative terms on equation (2) represent the dissipated energy on the viscoelastic material under loading σ(t). Total energy communicate by loading σ(t) to viscoelastic material is:

$$W_t(t) = \int_0^t \sigma(\tau)\frac{\partial\varepsilon(\tau)}{\partial\tau} d\tau \tag{3}$$

Time to failure under constant stress (strain) ratio is espressed according different criteria [10]:

Reiner–Weissenberg Criterion (R–W), with: $W_r(t) \le \frac{D_0}{2}\sigma_I^2$:

$$\left(\frac{t_f}{\tau_0}\right) = \left(\frac{(n+1)(n+2)}{2(n+2)+(2-2^{n+2})}\right)^{1/n}\left(\frac{D_0}{D_1}\right)^{1/n}\left(\frac{1}{\gamma}-1\right)^{1/n} \tag{4}$$

Maximum Work Stress Criterion (MWS), with: $W_t(t) \le \frac{D_0}{2}\sigma_I^2$:

$$\left(\frac{t_f}{\tau_0}\right) = (2+n)^{1/n}\left(\frac{D_0}{D_1}\right)^{1/n}\left(\frac{1}{\gamma}-1\right)^{1/n} \tag{5}$$

Maximun Strain Criterion (MS), with: $\varepsilon(t) \le D_0\,\sigma_I$:

$$\left(\frac{t_f}{\tau_0}\right) = (1+n)^{1/n}\left(\frac{D_0}{D_1}\right)^{1/n}\left(\frac{1}{\sqrt{\gamma}}-1\right)^{1/n} \tag{6}$$

And the Modified Reiner-Weissemberg Criterion (MR-W), with: $W_t \le (D_0/2)\,\sigma_I\,\sigma(t)$:

$$\left(\frac{t_f}{\tau_0}\right) = \left(\frac{(n+1)(n+2)}{2(n+2)+(2-2^{n+2})}\right)^{1/n}\left(\frac{D_0}{D_1}\right)^{1/n}\left(\frac{1}{\sqrt{\gamma}}-1\right)^{1/n} \tag{7}$$

In equations 4-7, σ_I is the strength under instantaneous conditions, t_f is final time to failure and $\gamma = (R\, t_f/\sigma_I)^2$ with R the stress rate.

Fracture mechanics approach to viscoelastic materials

Crack growth behaviour in viscoelastic material under determined configuration was studied by Christensen: the semi-infinite crack growing in a strip clamped at its edges [11-12]. Under plane-stress, steady state and isothermal conditions the energy balance on the strip clamped with unit thickness is:

$$a'\Phi - \frac{a'h\sigma^2(\infty)(1-\nu^2)}{2E(\infty)} + \int_0^h \int_{-\infty}^{\infty} \Delta dy d\xi = 0 \qquad (8)$$

Here, $a'\Phi$ is the rate of surface energy required to create new free surfaces; second term is the strain energy release rate evaluated by Griffith equation and for the plane-stress condition, but this time expressed in function of relaxed values of the far-field stress $\sigma(\infty)$ and the tensile modulus $E(\infty)$. Crack growth rate or crack velocity a' (a'= da/dt, a the crack length), is related to stress intensity factor K_v* and for polymers this parameters should be a material property, independent of loading conditions and specimen geometry. Constant strain $\varepsilon = \overline{u}/h$ is applied with $\overline{u}$ the displacement vector as shown in Figure 1; the Poisson's ratio ν is assumed to be constant. The third term in equation 8 is the rate of energy dissipation over the entire volume of the strip with length 2h.

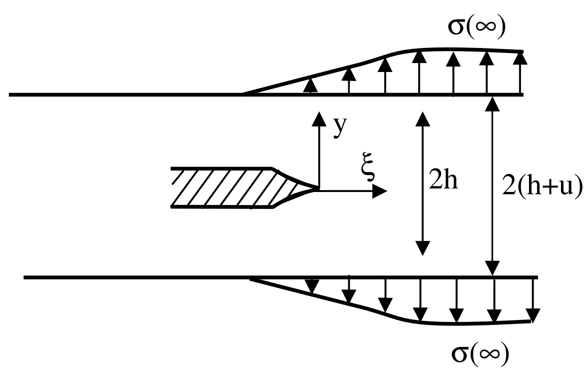


Figure 1. Christensen's model for semi-infinite crack growing in a strip clamped at its edges of viscoelastic material, under plane-stress condition.

Additionally, Φ is the surface energy corresponding to new surface generated by crack propagation and Δ is the energy rate dissipation (per unit of volume), related to viscoelastic effects. The practical use of equation 8 implies the following simplifications [13]: the energy rate dissipation Δ is derived for vicoelastic isotropic material undergoing small deformation, a single exponential form for the stress distribution close to crack tip (no stress concentration effect), and imposing a power creep law for testing material with creep exponential m = 0.5.

Furthermore, local crack growth criterion related to crack propagation models [14,15], have been developed in the last 40 years. The model developed by Schapery [15,16], assumes the energy of fracture associated with the effective crack length $\boldsymbol{a}$, and the physical crack lengt $\boldsymbol{a_p}$, Figure 2.

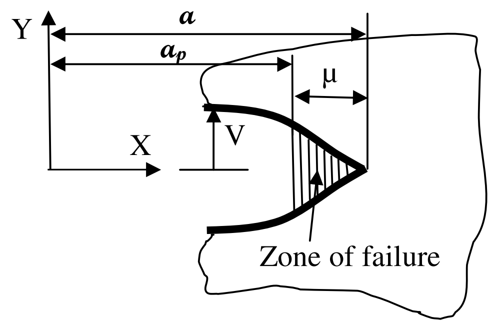


Figure 2. Schapery's model for crack propagation on viscoelastic materials

Plane-strain creep compliance is related to effective opening displacement V; it is assumed to follow the exponential law: $C_V(t) = Ct^m + C_0$, and enery inside the zone of falilure Φ is calculated by:

$$\Phi = \frac{1}{8} C_V(t_\alpha) K_{IC}^2 \tag{9}$$

Here: $t_\alpha = \lambda^{1/m}\, \alpha/\boldsymbol{a}'$, in which: $C_V(t_\alpha)$, C_0 and m are constants, K_{IC} is the stress intensity factor for mode I under crack propagation, λ is a creep compliance factor, μ is the length of failure zone, and $\boldsymbol{a}'$ is the crack velocity. Manipulating equation 9, the crack velocity is obtained as:

$$a' = \frac{\pi}{2}\left[\frac{C_V(t_\alpha)}{8\Phi(1-\frac{K_{IC}^2}{K_{IG}^2}}\right] \lambda^{1/m} \frac{K_{IC}^{2(1+1/m)}}{\sigma_m^2 I_I^2} \tag{10}$$

Where: σ_m is the maximum stress near the crack tip, I_I is a constant if the stress distribution during crack propagation rest unchanged and K_{IG} is a limiting stress intensity factor defined as: $K_{IG} = [8\Phi/C_V(0)\ ]^{0.5}$, in which $C_V(0)$ is the initial value of the plane-strain creep compliance. Theoretically, equation 10 implies that crack velocity α' increases with K_{IC}.

Different models and theories for crack propagation on viscolelastic materials are been developed for the case of constant stress (strain) ratio; plane-stress, steady state and isothermal conditions. No general theory is available for tri-dimensional crack growth behavior in viscoelastic materials; nevertheless, fatigue endurance and crack propagation were studied in a polymeric material undergoing ultrasonic fatigue testing.

MATERIAL AND TESTING CONDITIONS

Material

Polymeric material Nylon 6 was used for ultrasonic fatigue testing and crack propagation analysis. It is a cast nylon polyamide with good tensile strength, wear resistance, abrasion and vibration. Its elastic modulus is close to 3 GPa and presents low reactivity in wide variety of chemicals, alkalis, dilute acids or oxidizing agents [17]. The principal physical and mechanical properterties are shown in Table 1.

Table 1. Principal Mechanical a) and physical b) properties of Nylon 6.

ρ (g/cm^3)	**σ_y (MPa)**	**Compression σ (MPa)**	**E (GPa)**	**HR**	**Elongation (%)**
1.15	82	92	2.75	RA 105	20

a)

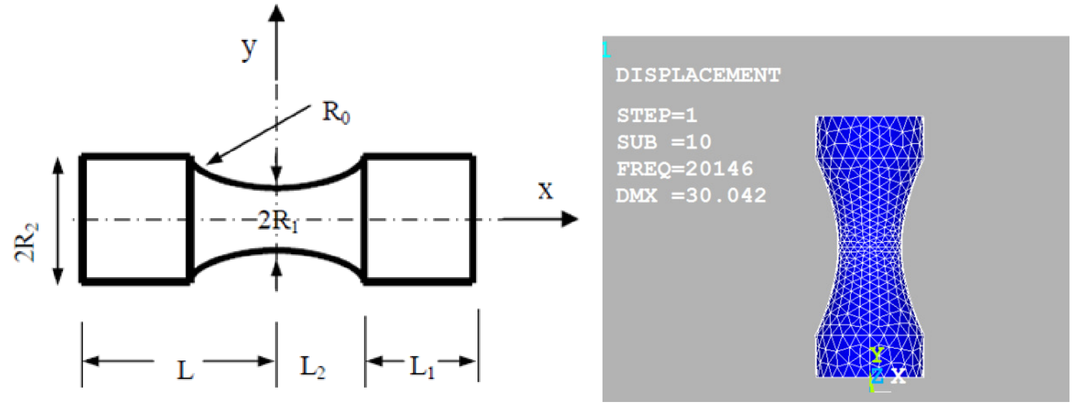
Melting Temperature (° C)	**Glass Temperature (° C)**	**Thermal Conduc. (W/m-°K)**
220	47	0.29

b)

Nylon 6 is obtained by thermal reaction of caprolactam, heated at 533° K in an inert atmosphere of nitrogen for about 4-5 hours to promote polymerization. This thermoplastic is fabricated using two types of monomers containing 6 or 12 carbon in the chain; the commercial names are respectively: nylon 6 and nylon 6/6.

Testing conditions

All ultrasonic fatigue tests were carried out at room temperature, with no control of environmental humidity, at the frequency of 20 KHz and with loading ratio R = -1. Under these conditions, the highest temperature is located at the narrow section of the hourglass shape specimen. In order to maintain the high temperature below 45° C, three actions were undertaken: 1) Ultrasonic fatigue specimen has been determined with the smallest dimensions for the narrow section, Figure 3, in order to reduce temperature gradient at this zone; 2) Appling load was comprised between 9 and 13% of the corresponding yield stress; 3) A cooling system with cool air was implemented to evacuate heat at specimen narrow section.



L= 13.5, L_1 = 4, L_2= 9.5, $2R_1$= 5, $2R_2$= 10, R_0= 19.3

Figure 3. Specimen dimensions (mm) and longitudinal natural frequency obtained by Finite Element Method to fit resonance condition under ultrasonic fatigue testing.

RESULTS

The creep life time predition is plotted in Figure 4 for this polymeric material; Table 2 shows the viscoelastic properties and strength for the high stress and short time to failure σ_R.

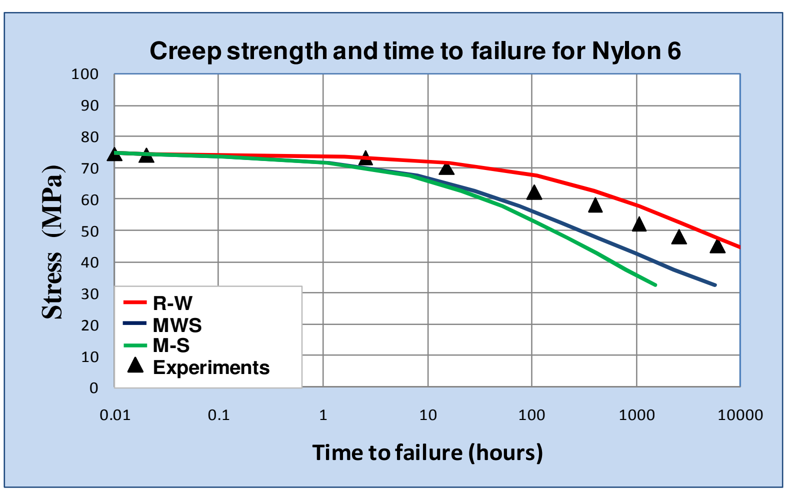


Figure 4. Creep lifetime predition and experimental results for Nylon 6.

Figure 4 presents different models to predict the creep lifetime under constant load, depending on σ_0 the applied load and the stress σ_R [18]: R-W the Reiner–Weissenberg

Criterion, MWS the Maximum Work Stress Criterion, and M-S the Maximum Strain Criterion.

Table 2. Viscoelastic properties and strength under instantaneous conditions for Nylon 6

T (° C)	D_0 (1/Mpa)	D_1 (1/Mpa)	n	τ_0	σ_R (Mpa)
22	3.4×10^{-4}	2.35×10^{-5}	0.42	1 hour	75

Experimental results under ultrasonic fatigue testing on Nylon 6 are plotted on Figure 5; vertical axis represents the ratio between the nominal stress σ_n and the yield stress σ_y.

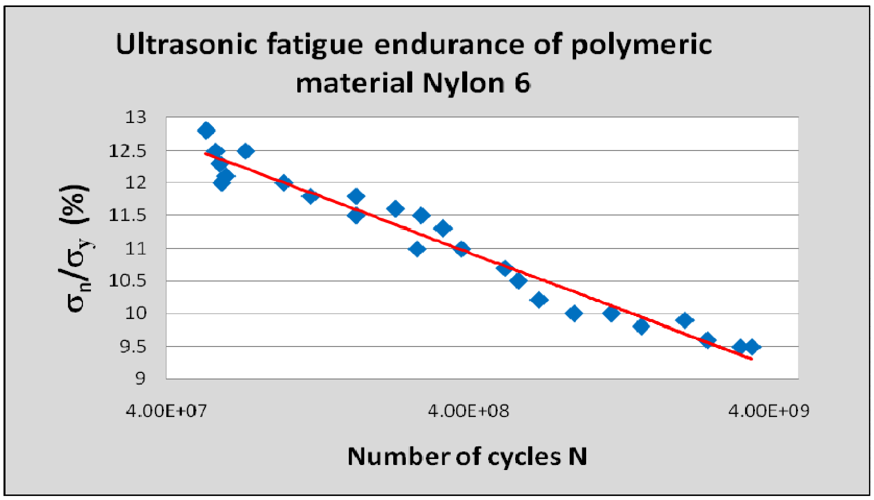


Figure 5. Ultrasonic fatigue endurance of Nylon 6, under low loading.

For the case of low loading rates [19], the very long-term creep tests until rupture could be replaced by constant strain rate tests, without losing accuracy. Time to failure under ultrasonic fatigue should be approached using the MWS Criterion:

$$t_c = C_2 \frac{(1 - F_a^2)^{1/n}}{F_a^{2/n}} \qquad (11)$$

Here $C_2 = (D_0/2D_1)^{1/n}$ and Fa is the normalized failure function applying for a specific failure mode; in this case the ultrasonic fatigue testing with defined low loading range on Nylon 6.

Fa function for results plotted on Figure 5 ranges from: 0.9455 (5×10^7 cycles) and 0.7756 (3×10^9 cycles); when t_c is calculated in hours. Crack propagation in this polymeric material accelerates with the normalized failure function, since it is related to applying loading (high values of Fa corresponding to short fatigue life); nevertheless, no full analysis was developed at this stage. Crack growth rate dependence on Fa for Nylon 6 will be a subject for next future studies.

Conclusions

- An overview for crack initiation and propagation on viscolelastic material was developed regarding the energy based criteria and the fracture mechanics approach.
- Ultrasonic fatigue testing on polymeric material Nylon 6 was carried out for low loading range: 9 – 12.5% of corresponding yield stress.
- Theoretical creep lifetime prediction curves have been obtained for this material using the Criteria: R-W, MWS and M-S. Experimental points are located between the R-W and MWS Criteria. The viscoelastic properties for creep testing have been determined for this material.
- Under described loading condition, crack growth rate increases when increasing the normalized failure function Fa; nevertheless, no detailed investigation was carried out at this stage.

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