Short crack and long crack propagation in metals based on damage mechanics concepts

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ABSTRACT. *The fatigue assessment of metallic structural components under uniaxial cyclic loading is traditionally tackled through experimental methods such as the S-N curve approach. For more complex variable stress states, such as multiaxial stress histories, the fatigue safety can be analysed by employing a physics-based damage mechanics approach. On the other hand, fatigue failure can be recognized as the result of a stable crack propagation up to a critical condition and, in this context, the availability of suitable laws to properly describe and quantify the crack propagation is a crucial aspect.*

In the present paper, a fatigue crack propagation law for both short (Low-Cycle-Fatigue) and long crack regime (High-Cycle-Fatigue) is discussed based on damage mechanics concepts. Fatigue crack growth law and damage mechanics approach are compared in order to determine both the damage value according to a given fatigue crack growth (FCG) law and the crack length associated to a given mechanical damage of the fatigued material. Such two methods are shown to be different formulations of the same physics-based approach to fatigue phenomena.

INTRODUCTION

Mechanical fatigue failure in a material is a complex phenomenon characterized by crack nucleation followed by crack propagation up to the final collapse. The total fatigue life can be theoretically computed as the sum of the number of loading cycles for crack nucleation and that for crack propagation [1-4], but approaches based on experimental observations are still widely used in practical applications [5]. The total fatigue life is usually crack-nucleation dominated in smooth components (defect-free structures), while it is propagation-dominated in initially flawed structures.

Damaged-based fatigue evaluations have been proposed by several researchers [6-9] with the aim to quantify the degree of material deterioration at a given point or in a limited region of a cyclically stressed material. The damage approach does not take into account any connection between the mechanical deterioration level and the extent of the growing cracks, while this connection is examined by the crack propagation approach (for example, the Paris law [10, 11]). In the present paper, the relationships between such two approaches to fatigue phenomena are discussed for both long and short cracks [12].

DAMAGE MODEL

Damage assessment

In order to evaluate the damage increment in a given region of an isotropic material, the current stress state conveniently expressed through the stress invariants can be used. Damage quantification can be obtained by means of a so-called endurance surface $E = 0$ depending on the current stress state. A necessary condition to get damage increasing is given by $E > 0$, while $E < 0$ corresponds to a not-damaging stress state.
Such an endurance function can be written as follows [8, 9]:

$$E(\boldsymbol{\sigma}, \mathbf{s}_e) = \left[a_1 \cdot I_1(\boldsymbol{\sigma}) + a_2 \cdot I_2^{1/2}(\boldsymbol{\sigma}) + a_3 \cdot I_3^{1/3}(\boldsymbol{\sigma}) + a_4 \cdot J_2^{1/2}(\mathbf{s}_e) + a_5 \cdot J_3^{1/3}(\mathbf{s}_e)\right] - \sigma_0 = 0 \qquad (1)$$

$$\text{with } \mathbf{s}_e = (\mathbf{s} - \mathbf{s}_b)$$

where $a_1, a_2, ..., a_5$ and σ_0 are material constants to be determined for a given material, whereas the stress tensor invariants, I_1, I_2, I_3, and the deviatoric stress invariants, J_2, J_3, are functions of the stress tensor $\boldsymbol{\sigma}$ and the effective deviatoric stress tensor $\mathbf{s}_e$, respectively. Note that $\mathbf{s}_e = \mathbf{s} - \mathbf{s}_b$, where $\mathbf{s}_b$ is the deviatoric stress tensor, $\mathbf{s} = s_{ij} = (\sigma_{ij} - \delta_{ij} \cdot \pi)$ is the current deviatoric stress tensor, δ_{ij} is the Kroneker delta function, $\pi = \sigma_{ii}/3$ is the hydrostatic stress. The deviatoric stress tensor $\mathbf{s}_b$ allows the endurance surface to evolve in the stress space. Moreover, damage increment takes place only if $dE > 0$ [8, 9]. It can be observed that damage is a positive non-decreasing scalar parameter, i.e. at each load step of the fatigue process the damage increment dD is greater than or equal to zero ($dD \geq 0$). The endurance function $E(\boldsymbol{\sigma}, \mathbf{s}_e)$ can evolve depending on the change of the deviatoric back stress tensor $\mathbf{s}_b$ during the stress history, and such a change can be expressed as follows [9]:

$$d\mathbf{s}_b = \begin{cases} C \cdot dE^h \cdot (\mathbf{s} - \mathbf{s}_b) & \text{if } dD > 0 \\ 0 & \text{if } dD = 0 \end{cases} \qquad (2)$$

where C and h are material parameters, and dE is the endurance function increment.

Crack nucleation-dominated fatigue life

Initially undamaged structures have a crack initiation-dominated life while the crack propagation phenomenon can be assumed to be negligible. In such situations, the damage increment dD can be assumed to be independent of the current damage level D (associated to the crack length) since the crack is not present till the final failure. Therefore, damage can be simply expressed as a function of E and dE [9]:

$$dD = \begin{cases} A \cdot E^B \cdot dE & \text{if} \quad E \geq 0 \text{ and } dE > 0 \\ 0 & \text{if} \quad E \leq 0 \text{ or } dE < 0 \end{cases} \tag{3}$$

where A and B are two material constants.

Crack propagation-dominated fatigue life

For initially flawed structures, the damage increment can be assumed to depend on the current damage D, that is, on the previously accumulated damage (and the crack growth rate depends on the current crack size, according to the Paris law). In other words, a sort of 'memory effect' takes place and, therefore, the damage increment dD should be expressed as a function of E, dE and the current damage D, and a possible expression could be:

$$dD = \begin{cases} D^q \cdot \left(A \cdot E^B \cdot dE\right) & \text{if} \quad E \geq 0 \text{ and } dE > 0 \\ 0 & \text{if} \quad E \leq 0 \text{ or } dE < 0 \end{cases} \tag{4}$$

where q is a material constant.

The presence of stress raisers can facilitate the early nucleation of surface cracks and, therefore, the total life for notched structural components can be assumed to be crack propagation-dominated instead of crack nucleation-dominated. Since the stress state in such situations is characterised by a significant gradient in a small region around the notch hot spot H, such a non-homogeneous stress field can be taken into account by inserting a reducing factor $G < 1$ in the damage increment [8]:

$$\begin{aligned} dD &= D^q \cdot \left[A \cdot E^B \cdot dE\right] \cdot G = \\ &= D^q \cdot \left\{\left[a_1 \cdot I_1(\boldsymbol{\sigma}) + a_2 \cdot I_2^{1/2}(\boldsymbol{\sigma}) + a_3 \cdot I_3^{1/3}(\boldsymbol{\sigma}) + a_4 \cdot J_2^{1/2}(\mathbf{s}_e) + a_5 \cdot J_3^{1/3}(\mathbf{s}_e)\right] - \sigma_0\right\}^B \cdot dE \cdot \underbrace{\left(e^{-V \cdot \gamma}\right)}_{G} \end{aligned} \tag{5}$$

where V is a material constant and γ the stress field parameter, representing the local absolute value of the stress field gradient computed at point H. In such a situation, the principal stress direction can be assumed to be be almost constant in a small region around the hot spot H. In the following, for the sake of simplicity, structures without stress raisers are examined, i.e. $\gamma = 0$, $G = 1$. Note that, for initially undamaged structures, the damage increment dD can be computed by simply setting $q = 0$ in Eq.(4).

Evaluation of the damage model parameters

The quantities involved in the presented damage model (such as E, $\mathbf{s}_b$ and the damage increment dD) depend on several parameters ($a_1, a_2, ..., a_5, A, B, C, h, V, q, \sigma_0$). As is shown in Refs [8, 9], some of the above parameters can be obtained from analytical relationships for simple uniaxial cyclic loading, whereas the remaining parameters can be estimated through experimental results and a best-fit approach based on the genetic algorithm (GA) method [13-15].

The case of crack propagation-dominated fatigue life (such as in initially damaged structural components) can be assumed to be correctly described by the Paris law, suitable for stable crack propagation in the LEFM regime, or by the Donahue et al. law [16], suitable also for short cracks.

For a cyclic loading with zero mean value and constant stress amplitude (R = -1) and assuming absence of stress gradient effect, the stress invariants and the deviatoric stress invariants can be easily determined, and the damage increment given by Eq. (4) becomes [8]:

$$dD = D^q \cdot \left(A \cdot E^B \cdot dE\right) = D^q \cdot A \cdot [W \cdot \sigma - \sigma_0]^B \cdot dE = \\ = D^q \cdot A \cdot \left[\underbrace{a_1 \cdot \sigma + \frac{a_1}{p} \cdot \sigma + a_5 \cdot \sqrt[3]{\frac{2}{27}}\,\sigma}_{W \cdot \sigma} - \underbrace{\frac{a_1}{p} \cdot \sigma_{af-1}}_{\sigma_0} \right]^B \cdot dE \qquad (6)$$

The integration of the damage increment from the initial damage D_0 up to the generic damage D, occurring after $n < N$ cycles, can be performed [9]:

$$\int_{D_0}^{D} \frac{dD}{D} = \ln D(n) - \ln D_0 = 2n \frac{A}{B+1} \cdot \left[\left[W \cdot \sigma_\beta - \sigma_0\right]^{B+1} - \underbrace{\left[W \cdot \sigma_\alpha - \sigma_0\right]^{B+1}}_{E(\sigma_\alpha)=0} \right] = \\ = 2n \frac{A}{B+1} \cdot \left[W \cdot \sigma_\beta - \sigma_0\right]^{B+1} = 2n \frac{A}{B+1} \cdot \left[W \cdot \sigma_a - \sigma_0\right]^{B+1} \quad \text{that is,} \qquad (7) \\ \ln D(n) = \ln D_0 + \underbrace{2n \frac{A}{B+1} \cdot \left[W \cdot \sigma_a - \sigma_0\right]^{B+1}}_{\Xi} \quad \text{or} \quad D(n) = \frac{e^{\Xi \cdot 2n}}{e^{\Xi \cdot 2N}} = e^{2 \cdot \Xi (n-N)}$$

where N is the number of loading cycles up to the final collapse (i.e. when $D = 1$), and σ_β is equal to the constant stress amplitude σ_a of the cyclic loading. From the above relationship, we can note that $D(n) = D_0$ for $n = 0$, and $D(n) = 1$ for $n = N$.

CRACK GROWTH LAWS

As is well-known, the Paris law [11] describes the stable crack propagation regime for long cracks when the LEFM hypothesis holds (Fig. 1). On the other hand, the Donahue et al. [16] equation is suitable also for the near-threshold regime. Such laws are expressed by:

$$\frac{da}{dn} = C' \cdot \Delta K^m, \quad \frac{da}{dn} = C' \cdot (\Delta K - \Delta K_{th})^m = C' \cdot (Y\sigma_a \sqrt{\pi a} - \Delta K_{th})^m = C' \cdot [Y\sigma_a \sqrt{\pi(a - a_{th})}]^m \qquad (8)$$

where a is the crack length, C', m are constants depending on the material, ΔK is the effective Stress-Intensity Factor (SIF) range, and ΔK_{th} is the threshold SIF. Let us define a crack size-based damage D_P for the material :

$$D_P = \frac{a - a_{th}}{a_c - a_{th}} \quad \text{for} \quad a \geq a_{th}, \quad \text{with} \quad a_{th} = (K_{th} / \sigma_a)^2 / \pi \qquad (9)$$

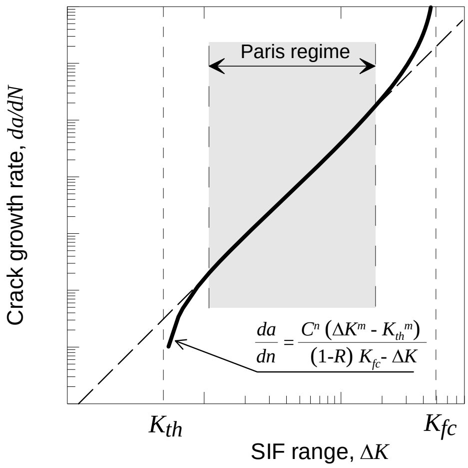

Fig. 1. Qualitative trend of the crack growth rate against the SIF range ΔK for metallic material: the Paris law holds for intermediate ΔK values (the Klesnil and Lucas relationship [17] is reported).

According to such a definition, the damage model presented above and the fatigue assessment based on the crack propagation law can be related to each other, that is, the crack size a_D related to the damage D can be determined as is described in the following. Note that a_{th} is the initial crack size below which the propagation does not take place for stress amplitude lower than or equal to σ_a.

After n loading cycles, the corresponding crack size $a < a_c$ can be easily obtained from the integration of the Donahue law ([16], see Eq. (8_2)) for the Griffith problem, i.e. for the simple case characterised by the geometric factor $Y(a) = 1, \ \forall a$:

$$n = \frac{1}{\Phi}\left[\frac{2}{m-2}\cdot\left(a*_0^{\frac{2-m}{2}} - a*^{\frac{2-m}{2}}\right)\right] \quad \text{with} \quad \Phi = 1/C'\left(\sigma_a\cdot\sqrt{\pi}\right)^m \quad \text{for} \quad m \neq 2 \qquad (10)$$

where $a*_0 = a_0 - a_{th}$ and $a* = a - a_{th}$ (for long cracks: $a* \cong a$). The above relationship can be rewritten as follows:

$$\begin{aligned} n &= \underbrace{\frac{1}{\Phi}\left[\frac{2}{m-2}\cdot\left(a*_0^{\frac{2-m}{2}} - a*_c^{\frac{2-m}{2}}\right)\right]}_{N} + \frac{1}{\Phi}\left[\frac{2\cdot a*_c^{\frac{2-m}{2}}}{m-2}\cdot\left(1 - D_P^{\frac{2-m}{2}}\right)\right] = \\ &= N - \frac{1}{\Phi}\left[\frac{a*_c^{m*}}{m*}\left(1 - D_P^{m*}\right)\right] = N - \underbrace{\frac{1}{\Phi\cdot m*}\left[\frac{1}{\pi}\left(\frac{K_{fc}}{\sigma_a}\right)^2 - a_{th}\right]^{m*}}_{\Omega}\cdot\left(1 - D_P^{m*}\right) = \\ &= N - \Omega\cdot\left(1 - D_P^{m*}\right) \cong N - \frac{N\cdot a*_c^{m*}}{1 - (D_{P0}\cdot a*_c + a_{th})^{m*}}\cdot\left(1 - D_P^{m*}\right) = \begin{cases} 0 & \text{for } D_P = D_{P0} \\ N & \text{for } D_P = 1 \end{cases} \end{aligned} \qquad (11)$$

where $a*_c = a_c - a_{th}$, $m* = (2-m)/2$. From the last expression in Eqs (7), the relationship between the number of loading cycles *n* and the current damage $D(n)$ can be obtained:

$$D(n) = e^{2\cdot\Xi(n-N)} \quad \rightarrow \quad n = N + \frac{1}{2\Xi}\ln D(n) = \begin{cases} 0 & \text{for } D(n) = D_0 \\ N & \text{for } D(n) = 1 \end{cases} \qquad (12)$$

Finally, by equating the number of load cycles *n* (see Eq. (11) and Eq. (12)), we get a relationship between D_P based on the Paris law and $D = D(n)$ according to the damage model :

$$D = e^{\frac{-2N\,\Xi\, a*_c^{m*}}{1-(D_{P0}a*_c + a_{th})^{m*}}\left(1-D_P^{m*}\right)} \quad \text{or} \quad D_P = \left[-1 + \frac{\ln D}{2N\cdot\Xi\cdot a*_c^{m*}}\cdot\left[1-\left(D_{P0}a*_c + a_{th}\right)^{m*}\right]\right]^{1/m*} \qquad (13)$$

and the crack size a_D corresponding to the damage D becomes (see Eq. (9)):

$$a_D = a_{th} + (a_c - a_{th})\cdot D_P = a_{th} + (a_c - a_{th})\cdot\left[-1 + \frac{\ln D}{2N\cdot\Xi\cdot a*_c^{m*}}\cdot\left[1-\left(D_{P0}a*_c + a_{th}\right)^{m*}\right]\right]^{1/m*} \qquad (14)$$

Then, by substituting the expression $\ln D = 2\Xi\cdot(n-N)$ (obtained from Eq. (12)) in Eq. (14), the crack growth rate da_D/dn can be determined:

$$\frac{da_D}{dn} = \frac{a*_c}{N\cdot m*\cdot a*_c^{m*}}\cdot\left[1 + \frac{(n-N)}{N\cdot a*_c^{m*}}\left[1-(D_{P0}a*_c + a_{th})^{m*}\right]\right]^{\frac{(1-m*)}{m*}}\cdot\left[1-(D_{P0}a*_c + a_{th})^{m*}\right] \qquad (15)$$

Since the terms N, Ξ and a_c depend on the stress amplitude σ_a, the crack growth rate based on the present damage model is dependent on σ_a. Such a dependence is shown to not affect significantly the results.

APPLICATION

The above damage model and the crack propagation-based assessment are employed to examine the fatigue behaviour of a real metallic material. The damage model parameters connected to the Wöhler regime (referred to the *crack initiation-dominated* regime) are firstly determined through the damage increment defined in Eq. (3). Then, the parameters related to the *crack propagation-dominated* life (Eq. (4)) are obtained, and the corresponding crack growth rate (Eq. (15)) is graphically represented.

The material to be examined is the Aluminium Alloy Al 2024-T3 [18] which is characterized by the fatigue-fracture parameters reported in Tab. 1.

The damage model parameters ($a_1, a_2, ..., a_5, A, B, C, h, V, q, \sigma_0$), determined through the damage increments in Eqs (4, 6) neglecting the stress gradient effect ($\gamma = 0$) and assuming $h = q = 1$ – corresponding to the crack initiation-dominated fatigue (Wöhler

Tab. 1. Mechanical characteristics and damage parameters of the Al. Alloy 2024-T3

			Damage models	
	Paris law constants		*Wöhler regime*	*Paris regime*
K_{fc}	45 MPa $\sqrt{m}$	A	6.128E-14	2.170E-16
K_{th}	3.6 MPa $\sqrt{m}$	B	0.026	0.412
C'	1.86E-11 m/cycle	σ_0	177.4 MPa	152.0 MPa
m	4.05	a_1	0.402	0.034
		a_4	1.480	1.270
		a_5	-0.958	-0.819
		C	0.474	0.159

regime) and the crack propagation-dominated fatigue (Paris regime) – and by applying the genetic algorithm (GA) procedure [9], are listed in Tab. 1.

The experimental S-N curve (continuous line) and the constant amplitude fatigue life evaluated through the present damage model for the Wöhler regime (round symbols) are shown in Fig. 2a. Note that the stress amplitude σ_a values used to compute the model parameters through the GA method are indicated by square symbols. As can be observed, the present model (round symbols) satisfactorily approximates the material fatigue curve (continuous line) even near the conventional fatigue limit region.

In Fig. 2b, the crack growth rate (expressed in m/cycle) against the SIF range $\Delta K = \sigma_a \sqrt{\pi a}$ (expressed in Pa m$^{1/2}$) is displayed for both the classical Donahue law

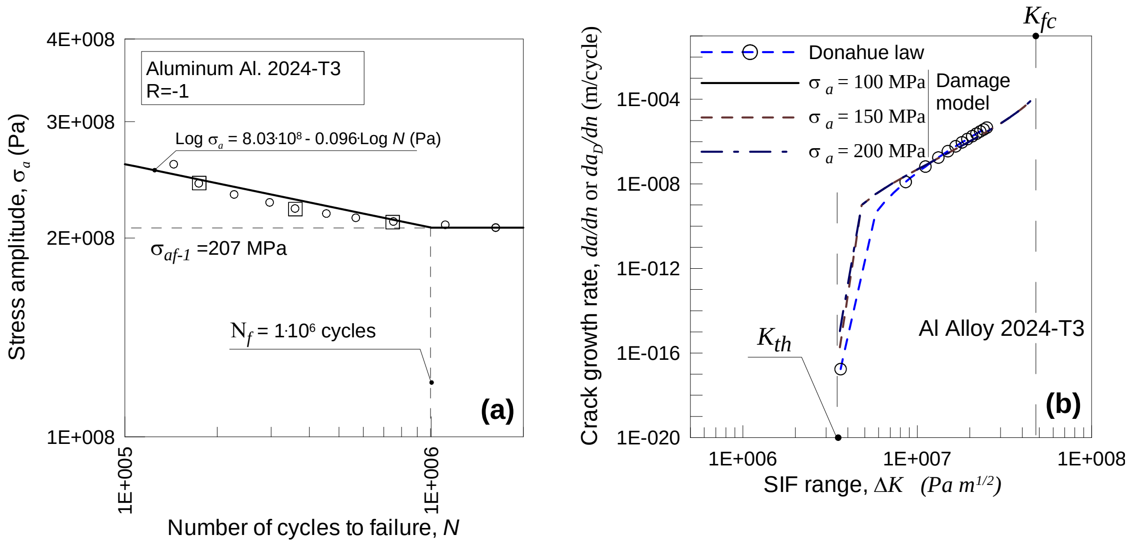

Fig. 2. (a) Wöhler curve (continuous line) and evaluation through the present damage model for the Wöhler regime according to Eq. (3) (round symbols), for Aluminum Al 2024-T3. (b) Crack growth rate curves obtained from the Paris law and the present damage model.

(dashed line with round symbols) and the damage model represented by Eq. (15) for three different values of σ_a. The damage model curves present nearly the same slope for intermediate ΔK values (Paris region), whereas $da_D / dn \to 0$ for $\Delta K \to K_{th}$. Further, an unstable crack propagation ($da_D / dn \to \infty$) can be noticed for $\Delta K \to K_{fc}$, as was experimentally observed and taken into account in the Klesnil and Lucas [17] and Forman et al. [19] relationships. Note that the stress amplitude σ_a dependence of the damage model curves is not significant since the three curves in Fig. 2b are almost superposed.

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