Under cyclic loading, plastic dissipation in heat at the crack tip modifies the stress intensity factor

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ABSTRACT *Plastic dissipation at the crack tip under cyclic loading is responsible for the creation of an heterogeneous temperature field around the crack tip. A thermomechanical model is proposed in this paper for the theoretical problem of an infinite plate with a semi-infinite through crack under mode I cyclic loading both in plane stress or in plane strain condition. It is assumed that the heat source is located in the reverse cyclic plastic zone. The analytical solution of the thermomechanical problem shows that the crack tip is under compression due to thermal stresses coming from the heterogeneous stress field around the crack tip. The effect of this stress field on the stress intensity factor (its maximum and its range) is calculated for the infinite plate. The heat flux within the reverse cyclic plastic zone is the key parameter to quantify the effect of dissipation at the crack tip on the stress intensity factor.*

INTRODUCTION

During a cyclic loading of a crack, the plasticity is located in the reverse cyclic plastic zone near the crack tip which was first explained by Paris in 1964 [1] and studied later by Rice in 1967 [2]. This effect is now well known and participates for instance in the explanation of the crack closure phenomenon which was noted by Elber in 1970 [3]. In metals, during plastic strain, a significant part of the plastic energy (around 90% [4, 5]) is converted into heat. The dissipated energy in the reverse cyclic plastic zone also generates an heterogeneous temperature field which depends on the intensity of the heat source associated with the plasticity and the thermal boundary conditions of the cracked structure. Due to the thermal expansion of the material, the temperature gradient near the crack tip creates thermal stresses which contribute to stress field in this region. The objective of this work is to quantify the effect on the stress intensity factor of this heterogeneous temperature field. However, there are two significant problems in order to estimate the thermal stresses: the first is the quantification of the heat source associated with the plasticity near the crack tip and the second is to make a good estimation of the boundary conditions of the thermal problem (convection from the surface of the cracked structure for example). The proposed paper is focused on the theoretical problem of an infinite plate with a semi-infinite through crack loaded in fatigue in mode I (Figure 1). In the associated thermal problem, the thermal losses due to convection and radiation are neglected.

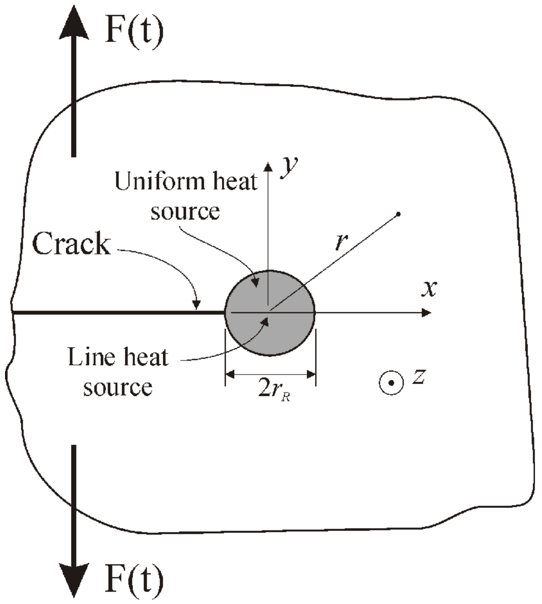


Figure 1: Schematic of the thermomechanical problem of a semi-infinite crack in an infinite plate under cyclic tension (mode I) caused by a remotely mechanical loading F(t)

THE TEMPERATURE FIELD ASSOCIATED WITH FATIGUE CRACK PROPAGATION

During fatigue crack growth under cyclic loading, the cyclic plastic strains at each cycle are confined within the reverse cyclic plastic zone. A proportion of the plastic strain energy is dissipated in heat and generates a temperature variation. Generally, the size of this reverse cyclic plastic zone is very small. In order to determine the temperature field, it is possible to consider the thermal problem associated with the fatigue crack propagation as a line heat source centered in the reverse cyclic plastic zone along the crack tip in an infinitely thick body. Ranc et al. [6] have compared the numerical solution (by finite element analysis) of the thermal problem in the case of a uniform heat source in a cylinder with a radius equal to the radius of the reverse cyclic plastic zone (Figure 1) and the analytical solution of the thermal problem with a line heat source. The temperature variation field obtained with the line heat source and the uniform heat source hypothesis are very close together outside the reverse cyclic plastic zone. Therefore inside this zone the temperature can be very differently distributed, but this is not the aim of this paper. This study is focused on the effect of the temperature gradient on the stress state outside this plastic zone in order to calculate its consequence on the stress intensity factor.

The dissipated power per unit length of crack front is assumed to be proportional to the surface area of the reverse cyclic plastic zone and the loading frequency *f* [7]:

$$q = f\,\xi = f\,\eta r_R^2, \tag{1}$$

with ξ the dissipated energy per unit length of crack front during one cycle, r_R the radius of the reverse cyclic plastic zone and η a material dependent proportionality factor.

In the plane stress and plane strain cases, the reverse cyclic plastic zone radius are respectively

$$r_R = \frac{\Delta K_I^2}{8\pi\sigma_y^2} \quad \text{and} \quad r_R = \frac{\Delta K_I^2}{24\pi\sigma_y^2} \tag{2}$$

where ΔK_I is the range of variation of the mode I stress intensity factor and σ_y is the cyclic yield stress of the material. For instance if we choose the following typical values $\sigma_y = 500\ MPa$ and $\Delta K_I = 5\ MPa\sqrt{m}$, the value of the radius of the reverse cyclic plastic zone in plane stress is 4 µm which remains small compared to the specimen size usually used in fracture mechanics tests. The dissipated power per unit length of crack front is therefore proportional to the variation of the stress intensity factor to the power four:

$$q = q_0 \Delta K^4 \tag{3}$$

These results have been already shown analytically [7] and numerically [8].

A constant heat source will be considered in this paper, in such case an analytical solution for our problem exists. Furthermore, note that in general the fatigue crack velocity is small, especially when the stress intensity range ΔK is close to the threshold value ΔK_{th}. Since q is proportional to ΔK^4, for a slow moving crack, ΔK and also the heat source q can be assumed constant. Moreover, in such case the heat source associated with the fatigue crack propagation can also be considered to be motionless. This assumption can be justified by the calculation of the Peclet number, noted *Pe*, which compares the characteristic time of thermal diffusion with the characteristic time associated to the heat source velocity (i.e. the velocity of the reverse cyclic plastic zone at the crack tip). In this case the Peclet number is expressed by

$$Pe = \frac{Lv}{a} \tag{4}$$

where L is the characteristic length of crack propagation, ν the crack velocity and a the thermal diffusivity. For a crack length of around 1 mm, a crack velocity of 0.1 $mm.s^{-1}$ and a thermal diffusivity of 1.5×10^{-5} m^2s^{-1} (typical value for steel) the Peclet number is 6×10^{-3}. This value remains small compared to unity and therefore the heat source can also be considered as motionless.

Within all these assumptions, the thermal problem is axisymetric and if the line heat source is along the z axis which is the normal direction to the surface of the plate (figure 1a), the associated heat transfer equation is

$$\rho C \frac{\partial T}{\partial t} = q\,\delta(0) + \lambda \frac{\partial^2 T}{\partial r^2} \qquad (5)$$

with ρ the density of the material, C its heat capacity, λ its heat conductivity and *δ(r)* the Dirac function.

At time $t=0$, we suppose an homogeneous temperature T_0 of the plate. Between time $t=0$ and time t , the temperature variation field $\vartheta(r,t)=T(r,t)-T_0$ can be expressed by [9]:

$$\vartheta(r,t)=\frac{1}{4\pi\lambda}\int_0^t q\, e^{\frac{-r^2}{4a(t-t')}} \frac{dt'}{t-t'} = \frac{q}{4\pi\lambda}\int_{u=\frac{r^2}{4at}}^{+\infty} \frac{e^{-u}}{u} du = \frac{-q}{4\pi\lambda} Ei\left(\frac{-r^2}{4at}\right) \qquad (6)$$

with $a=\lambda/(\rho C)$ the heat diffusivity and $-Ei(-x)=\int_x^{\infty}\frac{e^{-u}}{u}du$ the integral exponential function. The temperature is proportional to the dissipated power (see equation 6).

Figure 2 illustrates the evolution of the temperature variation field for different times according to the radius r from the line heat source. For this calculation, standard thermal and physical properties for steel are used. The density, the heat capacity and the thermal conductivity are taken to be respectively $\rho=7800\ kg.m^{-3}$, $C=460\ JK^{-1}kg^{-1}$ and $\lambda=52\ Wm^{-1}K^{-1}$. The dissipated power per unit length of crack front is chosen equal to the unit ($q=1\ W.m^{-1}$). The curve on Figure 2 shows that the temperature increases abruptly when the radius tends to zero.

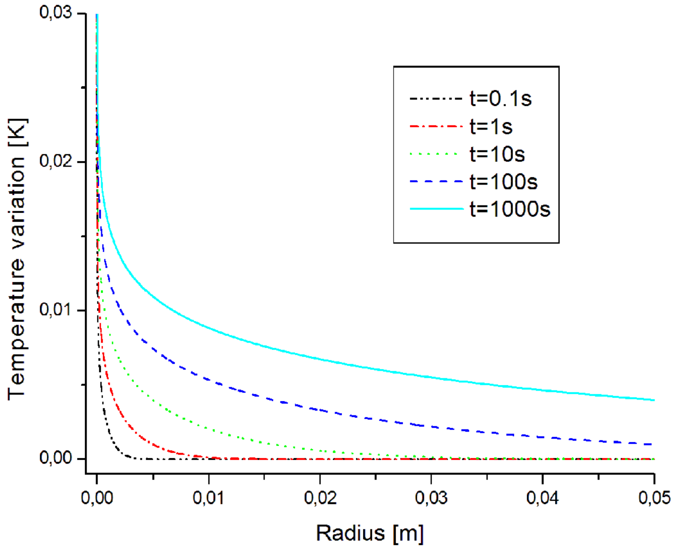


Figure 2: The temperature variation field near the crack tip for q = 1 Wm^{-1}.

THE STRESS FIELD ASSOCIATED WITH THE TEMPERATURE FIELD NEAR THE CRACK TIP

The thermo-mechanical problem

The temperature field associated with the heat source in the reverse cyclic plastic zone generates a temperature gradient varying with time outside this plastic zone and consequently thermal stresses due to the thermal expansion of the material. In order to estimate these thermal stresses the thermo-mechanical problem with the temperature field previously calculated needs to be solved. This thermo-mechanical problem is supposed to be bi-dimensional because the temperature field is axisymetric. Indeed, we consider the theoretical problem of an infinite plate with a semi-infinite through crack under mode I cyclic loading (Figure 1). The material is assumed to be homogeneous and isotropic with an elastic plastic behavior and plastic strain occurs only in the reverse cyclic plastic zone (cylinder domain with a radius r_R).

In both cases of plane stress and plane strain, there is unrestricted plastic flow through the thickness direction in the cracked specimen. With alternating plasticity in the reverse cyclic plastic zone the mean stress will tend toward to zero (i.e. mean stress relaxation). Also in the thermo-mechanical problem, only the elastic domain is considered and the boundary condition in the reverse cyclic plastic zone radius is that the radial stress is equal to zero.

Further, outside of the reverse cyclic plastic zone since the constitutive behavior of the material is supposed to be elastic, it is expected in first approximation that the basic equations of thermo-elasticity will govern. The equilibrium equation is:

$$r\frac{\partial\sigma_r}{\partial r}+\sigma_r-\sigma_\theta=0 \tag{7}$$

For which σ_r is the radial normal stress and σ_θ is the circumferential normal stress. The isotropic elastic stress strain law gives:

$$\begin{aligned}
\varepsilon_r&=\frac{\partial u_r}{\partial r}=\frac{\sigma_r}{E}-\nu\left(\frac{\sigma_\theta+\sigma_z}{E}\right)+\alpha\vartheta(r,t)\\
\varepsilon_\theta&=\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_r}{r}=\frac{\sigma_\theta}{E}-\nu\left(\frac{\sigma_r+\sigma_z}{E}\right)+\alpha\vartheta(r,t)\\
\varepsilon_z&=\frac{\partial u_z}{\partial z}=\frac{\sigma_z}{E}-\nu\left(\frac{\sigma_r+\sigma_\theta}{E}\right)+\alpha\vartheta(r,t)
\end{aligned} \tag{8}$$

where E is the modulus of elasticity, ν the Poisson ratio and α is the linear coefficient of thermal expansion.

Two particular stress strain cases are considered for the elastic region: (i) plane stress where the normal stress (σ_z) is equal to zero and (ii) plane strain where the strain (ε_z) is equal to zero. In both cases the material behavior outside the reverse cyclic plastic zone is modeled by an isotropic thermo-elastic stress strain law. This set of equations is reduced to the same form, for example for plane stress they are reduced to the first two with σ_z set equal to zero. For plane strain upon solving the last stress-strain equation for σ_z and substituting it into the first two the same form is found with altered elastic and thermal constants. The plane stress form will be adopted here for simplicity and the plane strain alteration will only be noted in some final results. The plane stress case gives:

$$\frac{\partial u_r}{\partial r}\frac{u_r}{r}=\frac{\sigma_r}{E}-\nu\frac{\sigma_\theta}{E}+\alpha\vartheta(r,t) \quad \text{and} \quad \frac{u_r}{r}=\frac{\sigma_\theta}{E}-\nu\frac{\sigma_r}{E}+\alpha\vartheta(r,t) \tag{9}$$

From the second of these, multiplying by r and differentiating, results in

$$\frac{\partial u_r}{\partial r}=\frac{\partial}{\partial r}\left[r\left(\frac{\sigma_\theta}{E}-\nu\frac{\sigma_r}{E}+\alpha\vartheta(r,t)\right)\right] \tag{10}$$

Equating this to the first equation of the two above and rearranging, as well as introducing the equilibrium equation to eliminate σ_θ, gives:

$$r\frac{\partial^2\sigma_r}{\partial r^2}+3\frac{\partial\sigma_r}{\partial r}=-\alpha' E'\frac{\partial\vartheta}{\partial r} \quad (11)$$

where E'=E and α'=α for plane stress, whereas $E'=\frac{E}{1-\nu^2}$ and $\alpha'=\alpha(1+\nu)$ for plane strain.

From equation (6), we find

$$\frac{\partial\vartheta(r,t)}{\partial r}=\frac{-q e^{\frac{-r^2}{4at}}}{2\pi\lambda r} \quad (12)$$

The equilibrium equation of the thermo-mechanical problem is then

$$r\frac{\partial^2\sigma_r}{\partial r^2}+3\frac{\partial\sigma_r}{\partial r}=\frac{q\alpha' E' e^{\frac{-r^2}{4at}}}{2\pi\lambda r}. \quad (13)$$

An analytical solution of the thermo-mechanical problem

Ranc et al. [6] have shown that the solution of the differential equation (11) is

$$\sigma_r(r,t)=\frac{\alpha' E' aq}{\pi\lambda}\left[\frac{t}{2r^2}e^{\frac{-r^2}{4at}}+\frac{1}{8a}Ei\left(\frac{r^2}{4at}\right)\right]+\frac{F}{r^2}+G \quad (14)$$

with *F* and *G* two integration constants. Equation (7) allows one to find the circumferential stress:

$$\sigma_\theta(r,t)=\frac{-\alpha' E' aq}{\pi\lambda}\left[\frac{1}{2}\frac{te^{\frac{-r^2}{4at}}}{r^2}-\frac{1}{8a}Ei\left[\frac{-r^2}{4at}\right]\right)-\frac{F}{r^2}+G \quad (15)$$

It is possible now to use the mechanical boundary conditions in order to express the two constants F and G. Because there is no thermal stress when *r* tends to infinity and because of the mean stress relaxation in the reverse cyclic plastic zone, when *t>0*, the boundary conditions are,

$$\lim_{r\to+\infty}\sigma_r(r,t)=0 \quad \text{and} \quad \sigma_r(r_R,t)=0 \quad (16)$$

The first previous condition implies G=0. With equation (14), the second condition allows one to express the constant, F, as follows, which is in fact a function depending on time to respect the boundary conditions whatever the time t :

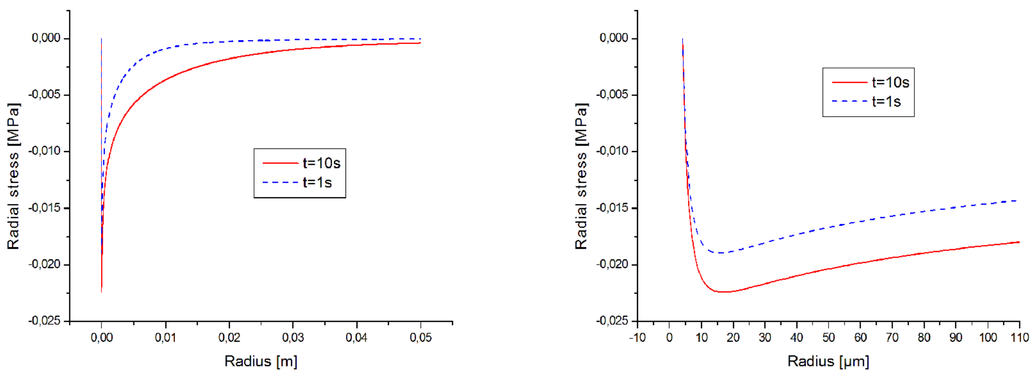
$$F=\frac{\alpha' E' a q r_R}{\pi\lambda}\left[\frac{-t}{2}\frac{e^{\frac{-r_R^2}{4at}}}{r_R}-\frac{r_R}{8a}Ei\left(\frac{-r_R^2}{4at}\right)\right] \quad (17)$$

Then, the radial and the circumferential normal stresses are:

$$\sigma_r(r,t)=\frac{\alpha' E' aq}{\pi\lambda r^2}\left[\frac{t}{2}\left(e^{\frac{-r^2}{4at}}-e^{\frac{-r_R^2}{4at}}\right)+\frac{1}{8a}\left(r^2 Ei\left(\frac{-r^2}{4at}\right)-r_R^2 Ei\left(\frac{-r_R^2}{4at}\right)\right)\right] \quad (18)$$

$$\sigma_\theta(r,t)=\frac{-\alpha' E' q a}{\pi\lambda r^2}\left\{\frac{t}{2}\left(e^{\frac{-r^2}{4at}}-e^{\frac{-r_R^2}{4at}}\right)+\frac{1}{8a}\left[-r_R^2 Ei\left(\frac{-r_R^2}{4at}\right)-r^2 Ei\left(\frac{-r^2}{4at}\right)\right]\right\} \quad (19)$$

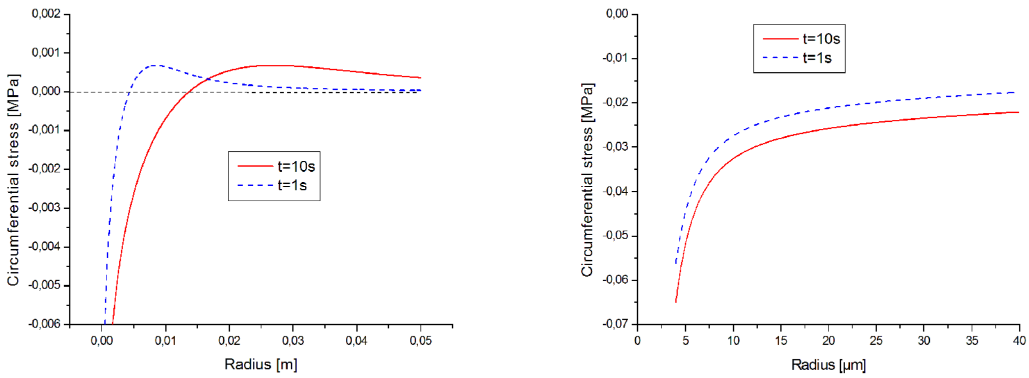
These radial and circumferential normal stresses are then calculated with the following typical values for steel: a Young modulus E=210 GPa, a Poisson ratio ν=0.29, a thermal expansion coefficient of the material $\alpha=1.2\times10^{-5}$ K^{-1}, and the line heat source is taken equal to the unity (q=1 $W.m^{-1}$). The figure 3 gives for r_R=4 μm the evolution of the radial stress as it varies with the radius for two times t=1 s and t=10 s. Note that the radial stress is always negative because the material is under compression due to the thermal expansion of the material near the crack tip. The figure 3 is an enlargement of the curve around the reverse cyclic plastic zone. This figure shows a minimum of the radial stress. After t=1 s and t=10 s, the minimum radial stresses are respectively equal to $-2.2x10^{-2}$ MPa and -1.9×10^{-2} MPa and the respective positions from the center of the reverse cyclic plastic zone (r = 0) are 17 μm and 16 μm for a unit line heat source



-a- -b-

Figure 3: The distribution of the radial normal stress for various times for a unit heat source q = 1 Wm^{-1} and r_R = 4 μm; a) general view, b) enlargement near the reverse cyclic plastic zone.

The figure 3a shows the evolution of the circumferential stress along a radial axis for two different times. Near the reverse cyclic plastic zone (r_R=4 µm) the circumferential stress is negative (Figure 4) because the temperature is high and through the circumferential direction, the material is under compression due to the thermal expansion and the constraint effect. Further from this zone, the temperature is lower and the circumferential stress becomes positive (tension) due to the confinement of the material near the crack tip (Figure 4). For times t=1 s and t=10 s the circumferential stress in the edge of the reverse cyclic plastic zone is respectively equal to -0.056 MPa and -0.065 MPa for a unit line heat source. It has to be pointed out that all the previous stress values are small because they are computed for a unit heat source q (per unit length of crack front), but the stresses are proportional to q.



-a- -b-

Figure 4: The circumferential stress distribution at various times for a unit heat source $q=1 \quad W.m^{-1}$ and r_R = 4 µm; a) general view, b) enlargement near the reverse cyclic plastic zone.

THE EFFECT OF THE THERMAL STRESSES ON THE STRESS INTENSITY FACTOR UNDER CYCLIC LOADING

Now within the heterogeneous stress field due to the thermal stresses, if we consider the previous theoretical case of an infinite plate with a semi-infinite crack along a radial line from r_R to $+\infty$, the associated stress intensity factor, $K_{I,temp}$, due to the temperature gradient can be determined from the wedge force (Green's function) solution (see [10] page 87) as:

$$K_{I,temp}(t)=\sqrt{\frac{2}{\pi}}\int_{r_R}^{\infty}\frac{\sigma_\theta(r,t)}{\sqrt{r-r_R}}dr. \qquad (20)$$

From equations (19) and (20), the stress intensity factor due to thermal stresses is expressed by:

$$K_{I,temp}(t)=\frac{-\alpha' E' q}{8\pi\lambda}\sqrt{\frac{2}{\pi}}\left[\int_{r_R}^{\infty}\frac{4at}{r^2\sqrt{r-r_R}}\left(e^{\frac{-r^2}{4at}}-e^{\frac{-r_R^2}{4at}}\right)dr\ldots\right.$$

$$\left.\ldots+\int_{r_R}^{\infty}\frac{1}{r^2\sqrt{r-r_R}}\left(-r_R^2 Ei\left(\frac{-r_R^2}{4at}\right)-r^2 Ei\left(\frac{-r^2}{4at}\right)\right)dr\right] \tag{21}$$

After integration (with the help of the Mathematica software) it becomes:

$$K_{I,temp}(t)=\frac{\alpha' E' q}{80\lambda}\sqrt{\frac{2}{\pi}}\left\{40\sqrt{r_R}+\frac{20at\left(e^{\frac{-r_R^2}{4at}}-1\right)}{r_R^{3/2}}+5\sqrt{r_R}Ei\left(\frac{-r_R^2}{4at}\right)\ldots\right. \tag{22}$$

$$\ldots+\frac{10(at)^{1/4}}{\Gamma\left(\frac{7}{4}\right)}\left(-3\ {}_2F_2\left[\left(\frac{-1}{4},\frac{1}{4}\right),\left(\frac{1}{2},\frac{3}{4}\right),\left(\frac{-r_R^2}{4at}\right)\right]+{}_2F_2\left[\left(\frac{1}{4},\frac{3}{4}\right),\left(\frac{1}{2},\frac{7}{4}\right),\left(\frac{-r_R^2}{4at}\right)\right]\right)\ldots$$

$$\left.\ldots-\frac{8r_R}{(at)^{1/4}\Gamma\left(\frac{1}{4}\right)}\left(5\ {}_2F_2\left[\left(\frac{1}{4},\frac{3}{4}\right),\left(\frac{5}{4},\frac{3}{2}\right),\left(\frac{-r_R^2}{4at}\right)\right]+{}_2F_2\left[\left(\frac{3}{4},\frac{5}{4}\right),\left(\frac{3}{2},\frac{9}{4}\right),\left(\frac{-r_R^2}{4at}\right)\right]\right)\right\}$$

with the hypergeometric function:

$${}_pF_q\left(\{a_1,\ldots,a_p\};\{b_1,\ldots,b_q\};z\right)=\sum_{k=0}^{+\infty}\frac{(a_1)_k\ldots(a_p)_k}{(b_1)_k\ldots(b_q)_k}\frac{z^k}{k!}, \tag{23}$$

where $(a)_k=\frac{\Gamma(a+k)}{\Gamma(a)}=a(a+1)(a+2)\ldots(a+k-1)$ is the Pochhammer symbol and $\Gamma(x)=\int_0^{+\infty}u^{x-1}e^{-x}du$ the Euler Gamma function. For instance in our case ${}_2F_2\left(\{a_1,a_2\};\{b_1,b_2\};z\right)=\sum_{k=0}^{+\infty}\frac{(a_1)_k(a_2)_k}{(b_1)_k(b_2)_k}\frac{z^k}{k!}$.

The evolution of $K_{I,temp}$ (the thermal correction on K_I) versus time is presented in Figure 5 for a unit line heat source ($q=1\ Wm^{-1}$) and the following typical material characteristics: $\rho=7800\ kg.m^{-3}$, $C=460 JK^{-1}kg^{-1}$, $\lambda=52\ Wm^{-1}K$-^{1}, $\alpha=12.10^{-6}$ and E=210 GPa. For instance, after times of 10s and 100 s the thermal correction on the stress intensity factor is respectively -1.2×10^{-3} MPa√m and -2.1×10^{-3} MPa√m for a reverse cyclic plastic zone radius of 4 µm and $q=1\ Wm^{-1}$. These values are negative because the temperature field generates compressive circumferential normal stresses near the crack tip.

Figure 5 shows the evolution versus time of the stress intensity factor $K_{I,temp}$ due to the temperature gradient for various values of the reverse cyclic plastic zone radius. This illustrates that $K_{I,temp}$ is not very sensitive to the size of the reverse cyclic plastic zone. This is due to the very large dimensions of the plate (infinite plate), compared to the size of the reverse cyclic plastic zone. The thermal boundary conditions do not consider the heat exchange between the specimen and the environment that is the reason why no thermal equilibrium is reached even after a long time (Figure 4). Further theoretical work in the analytic solution has to be done to take this phenomenon into account for being representative of small specimens (finite dimensions) for which thermal equilibrium is reached when a fatigue crack growth test is running during several hours in laboratory. This is the case of slow fatigue crack growth typically when the range of the stress intensity is close to the threshold.

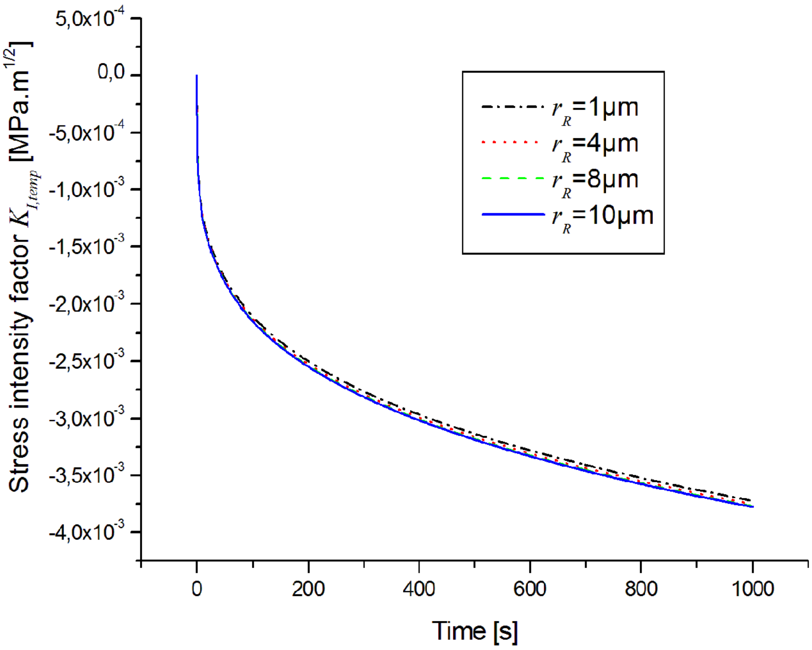


Figure 5: The stress intensity factor $K_{I,temp}$ due to thermal stresses versus time for different radius of the reverse cyclic plastic zone.

As written before, due to the compressive thermal stresses around the crack tip, it has been shown that the stress intensity factor during a fatigue loading has to be corrected by the factor $K_{I,temp}$. This factor determined by equation (22) would be a value superimposed on the usual stress intensity factor due to the fatigue cyclic loading noted $K_{I,cyc}$ in mode I.

$$K_I(t)=K_{I,temp}(t)+K_{I,cyc}(t) \tag{25}$$

$K_{I,temp}$ varies with time and can be considered as constant for long time. In the very beginning of loading, the value of $K_{I,temp}$ is small compared with $K_{I,cyc}$ and $\Delta K_I(t) \approx \Delta K_{I,cyc}$. There is also no significant effect of the temperature on the range of the stress intensity factor per load cycle. For long time *(t>>0),* $K_{I,temp}$ can be considered as constant during a loading period. Consequently the temperature has no effect on $\Delta K_I(t)$ but it has an effect on $K_{I,max}$ and $K_{I,min}$:

$$K_{I,max}=K_{I,cyc,max}+K_{I,temp} \quad \text{and} \quad K_{I,min}=K_{I,cyc,min}+K_{I,temp} \tag{26}$$

where $K_{I,min}$ and $K_{I,max}$ are the minimum and the maximum value of $K_I(t)$ over a loading period. However, $K_{I,temp}$ can affect crack closure by changing the load ratio $R_K=K_{I,min}/K_{I,max}$. The ratio R_K is affected by the temperature correction:

$$R_K=\frac{K_{I,min}}{K_{I,max}}=\frac{K_{I,cyc,min}+K_{I,temp}}{K_{I,cyc,max}+K_{I,temp}} \neq \frac{K_{I,cyc,min}}{K_{I,cyc,max}} \tag{27}$$

The evaluation of this correction needs a precise quantification of the heat source associated with the plastic dissipation and the thermal boundary conditions at the border of the plate. Experimental measurements of the temperature field, for instance by using pyrometry technique, need to be carried out in this way in a next study.

In this paper it has been shown that the effect of the heat source at the crack tip (within the reverse cyclic plastic zone) on the stress intensity factor is proportional to the line heat source q . The quantification of this is a key factor which is probably depending on the material behavior (plasticity, visco-plasticity if any). Another consequence of the thermal stresses is due to the fact that q is proportional to ΔK^4 [7,8]. When ΔK is changing significantly, for instance from the threshold 5 MPa√m up to 50 MPa√m (10 times more), the effect on the heat source is 10^4 times! The effect on the correction due to thermal stresses is thus significant. Furthermore, since the value of q is also proportional to the loading frequency, a frequency effect on the crack growth may be also linked with the heat source. This opens interesting investigations for further studies.

We have to keep in mind that the problem solved here assumed that the heat source is motionless. This means that the proposed solution is physically correct for slow crack growth. This is the case when ΔK is close to the threshold value. For instance, with dc/dN~10^{-9} m/cycle at a loading frequency between 1 Hz up to 100 Hz the velocity of the crack tip (i.e. heat source velocity) is between 10^{-6} mm.s^{-1} and 10^{-4} mm.s^{-1}. This means according to equation (4) that for a crack with a characteristic length between 1 mm and 10 mm that the Peclet number is small compare to the unity. In such a case, the motionless heat source hypothesis is correct and all the proposed results correct too.

According to the authors, considering the effect of thermal stresses may be a very important point for studying the crack growth close to the threshold and for the physical phenomena including crack closure and frequency effect.

CONCLUSION

The temperature variation field outside the reverse cyclic plastic zone in an infinite plate with a semi-infinite crack under a remotely applied tensile force (mode I) has been calculated analytically. This temperature field also applies to a large central through crack, as an estimation near each crack tip. It shows that due to the temperature gradient outside the plastic zone, a local compressive stress field is created. This may participate in the crack closure phenomenon. The mode I stress intensity factor has then been calculated by taking into account this field. Both the effective range of the stress intensity factor (considering closure), the maximum and minimum values of K_I and the stress intensity ratio $R_K = K_{I,min}/K_{I,max}$ may be affected by the thermal stresses. The proposed analytical solution shows that the correction on the stress intensity factor due to the heterogeneous temperature field around the crack tip is proportional to the heat source within the reverse cyclic plastic zone. Experimental investigation has to be carried out to quantify the heat source at the crack tip which is clearly a key factor in fracture mechanics. Further studies should also be carried out in thermomechanics to take into account the temperature field effect on fracture mechanics considerations.

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