FEM simulation of fatigue damage crack nucleation and growth in a pre-damaged material

Igor K. Korolev[1], **Alexander B. Freidin**[1], **Sergei V. Petinov**[1,2]

[1] Institute of Problems in Mechanical Engineering, RAS, 199178, V.O., Bolshoy pr., 61, St.Petersburg, Russia; i.korolev82@gmail.com, afreidin@yandex.ru
[2] St.Petersburg State Polytechnic University, 195251, Polytechnicheskaya, 29, St.Petersburg, Russia; spetinov@gmail.com

ABSTRACT

A FEM-based procedure of fatigue crack growth simulation in the field of progressive damage is developed. A new finite element grid is suggested to consider fatigue damage accumulation and crack growth based on a unified point approach. The grid design suggested allows saving information on accumulated damage and "natural" tracing of growing crack, which is timely and laborious using the standard FEM-procedures. The basic principles of meshing are formulated and following these principles a two-level finite element grid was designed. With application of the assumed grid fatigue behavior under cyclic loading of a thin rectangular plate with initial randomly distributed defects is analyzed. The plate is considered a composition of finite elements representing material elements with randomly distributed fatigue resistance parameters. Defects in material structure are modeled by elements with negligibly small stiffness. The fatigue properties of the material elements are described by the Basquin equation. The trajectories of growing cracks and the damage accumulation in the plate are presented at different phases of fatigue life.

The procedure developed is deemed an effective mechanism that allows both to model the crack formation from a defect and its further propagation in accordance with the damage accumulation during all period of loading. It allows to reduce drastically the influence of the mesh geometry on the crack growth trajectory.

INTRODUCTION

Fatigue crack growth in conjunction with the damage accumulation simulation, based on coupled action of mechanisms of slip in material grains and stress field attracted attention through the past decades, e.g., [1]. One of the effective ways of modeling the crack propagation is the use of finite-element method (FEM). The application of FEM for the analysis of crack propagation when the crack path may be affected by the inhomogeneous development of damage or by specifics of the stress field immediately assumes an operative reorganization of a finite element grid during the crack extensions. It allows avoiding the influence of the finite element grid topology on the crack trajectory. This is because the initial grid can not adequately describe the re-distribution the stress and strain field around the tip of the growing crack.

However, applicability of such approach is limited by a class of problems where there is no correlation between the crack path and the previous history of damage accumulation in a material.

When the crack extensions depend on the history of damage accumulation, the reorganization of grid is unacceptable since it leads to deleting the information on the developing degradation of the material.

This work is focused on construction of the finite element grid which, on the on the one hand, would allow naturally to save the information on the fatigue damage in material (finite elements) prior to failure and to model by this the whole process of fatigue crack development using the unique finite element grid, and on the other hand, would minimize the influence of the grid on the trajectories of fatigue cracks. Based on the formulated principles the original structure of FE grid is developed and verification of its consistency is presented.

Basics of the approach and development of the specific mesh type

The crack initiation and propagation is modeled based solely on assumption that the damage accumulation in material elements controls the process. As an example, the fatigue process is analyzed in a formally elastic plate with a central circular hole under cyclic zero-to-tension loading. First, the analysis is addressed to the homogeneous material modeled with the finite element grid differing by topology. Fatigue process is modeled by the sequence of damage accumulation in FE's using the Palmgren-Miner rule [2]:

$$D = \sum_i \frac{n_i}{N_i} \tag{1}$$

where D is the accumulated damage in an element, $n_i = n(S_i)$ is the number of load cycles with the stress range S_i , $N_i = N(S_i)$ is the number of cycles prior to failure of the "*i*" element with the stress range S_i.

Values of $N(S_i)$ are obtained from the S-N curve for the plate material approximated by the Basquin equation [3]:

$$N(S) = C / S^m \tag{2}$$

where C and m – the material empirical "constants", to be obtained from the experimental data, S – the stress range.

By substitution Eq.2 into Eq.1 the damage accumulated in every of the elements:

$$D = \sum_i \frac{n_i}{N_i} = (1/C)\sum_i n_i S_i^m \tag{3}$$

Successive assumed failure of elements, i.e. when the damage in a sequential element becomes $D = 1$, is defined by decreased stiffness of the element by several decimal orders, and succession of failures indicates the crack extension. Acceleration of the progress of damage, characterized by the crack growth rate, was regarded the indication of the plate failure. This scenario was realized in the case when the crack origination was assumed at the notch root, but at a certain distance from the axis of symmetry.

First, a standard regular finite element grid is used. Fig.1 shows the initial stage of the crack extensions and the final state with well-developed crack under cyclic loading (the cyclic load is applied in vertical direction).

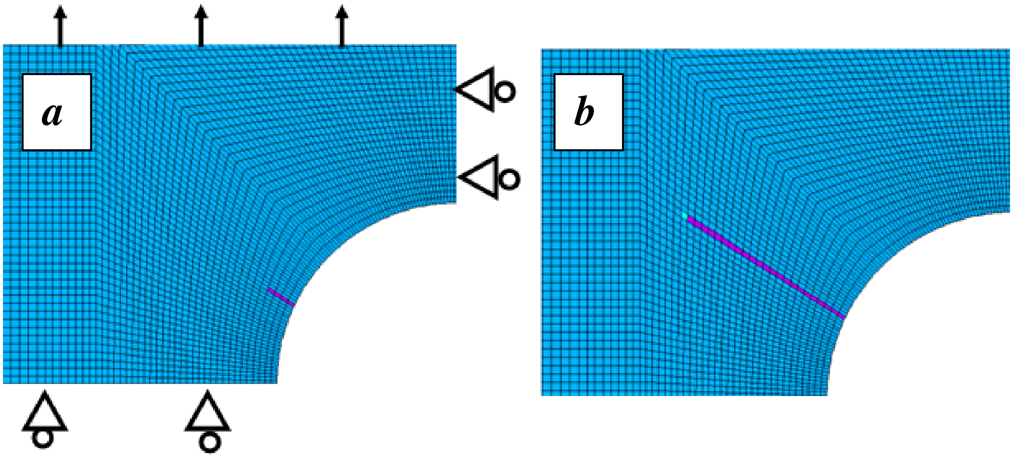

Fig.1. Crack growth simulation by the mechanism of the damage accumulation. Regular FE mesh

Fig.1, a, b show that the crack follows strictly the mesh nodal line, inclined to the direction of loading and direction of the maximum principal stress. Due analysis (by using finer mesh at assumed crack tip) indicates that the most intensive damage accumulation at every of the crack increments develops in successive element located directly at the crack path extension, i.e. along the grid nodal line. In a sense, it is because the crack is modeled by successive "killing" of elements.

Keeping with this principle, the following requirements for the meshing can be formulated in order to minimize the influence of the grid topology on the trajectory of growing crack:

1. The crack tip should be provided "freedom" to turn under the influence of the local stress field, accompanied by the accumulation of fatigue damage in successive elements
2. The isotropy of the grid. Any specified nodal lines in the mesh would control the crack extensions.

Based on the above principles a cell structure of the mesh was derived meeting the above requirements. A cell of the mesh structure is shown in Fig.2.

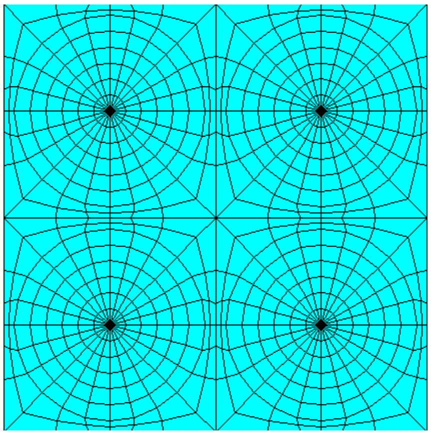

Fig.2. A cell of the proposed isotropic mesh type

The cell consists of "blocks-circles", in the center of "circles" there are triangular elements which allow for deviation of the crack tip. By this in the center of every "block-circles" the crack path would be corrected accordingly the local stress field; further, the crack would extend towards the next "block-circle". Thus, the crack trajectory will be represented by a zigzag line. If the size of the grid cell is small enough then the trajectory will be slightly differing from the "true" one.

Also this cell is consistent with the requirement to the mesh isotropy, since there is no predominant direction, nodal point line, capable of controlling the crack path.

Using this grid for the above notched plate gives the following results. Figs.3, 4 present the results of finite element-based modeling of fatigue crack extensions under uniaxial cyclic loading for different directions of applied stresses. Fig.3,a shows that the simulated crack propagation in general satisfactorily follows orientation of the planes normal to the maximum principal stress flow, apart from the area where the mesh is oriented. When the plate is loaded in horizontal direction (Fig.3, b) the initial "defect" (magenta line) occurs inactive, and the crack is initiated by the damage accumulation mechanism in the "proper" location.

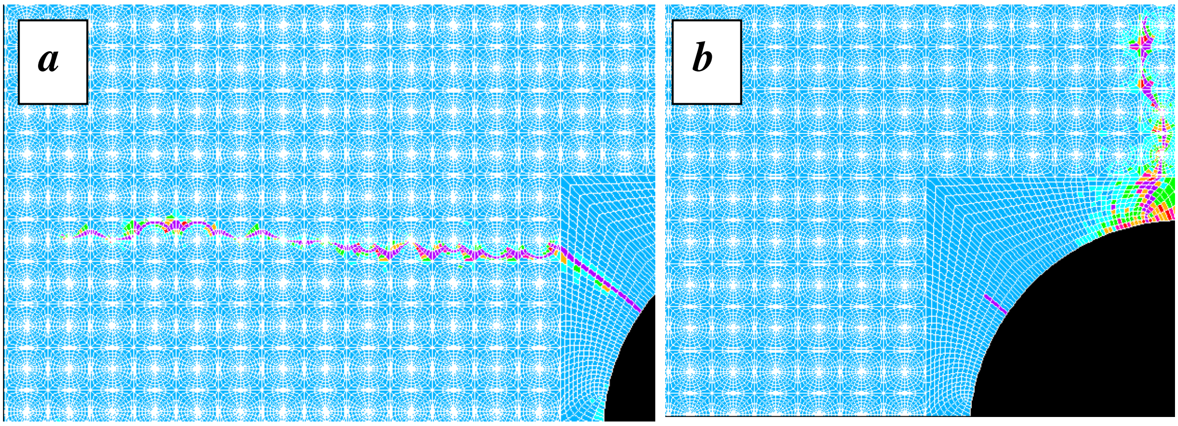

Fig.3. Fatigue crack simulated under uniaxial cyclic loading: (a) cyclic load is applied in vertical direction; (b) – horizontal direction of cyclic loading

Another example of “calibration” of the suggested procedure is the simulation of fatigue crack propagation under the in-phase biaxial loading. Fig.4 shows the results of the crack modeling: it is seen again that in general the crack trajectory complies with the Sih’s principle – it grows normally to direction of the maximum principal stress.

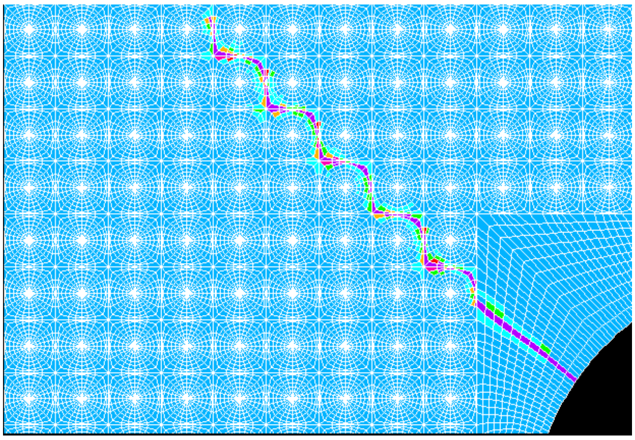

Fig.4. The development of fatigue crack under biaxial cyclic loading (the in-phase cyclic loads are applied both in horizontal and in vertical direction with equal amplitude)

However, the applied mesh type provides the wavy character of the crack path: the mesh makes the damage accumulation preferred along the boundaries of the mesh cells. To allow for the better crack trace simulation the proposed meshing would need in further optimization so that the damage accumulation and respective crack extensions would be less dependent on the properties of cells.

By applying the trial-and-error method an improved grid is developed which combines the advantages of the previous one and allow eliminating the revealed shortcomings. A cell of this advanced FE sub-structure is presented in Fig.5.

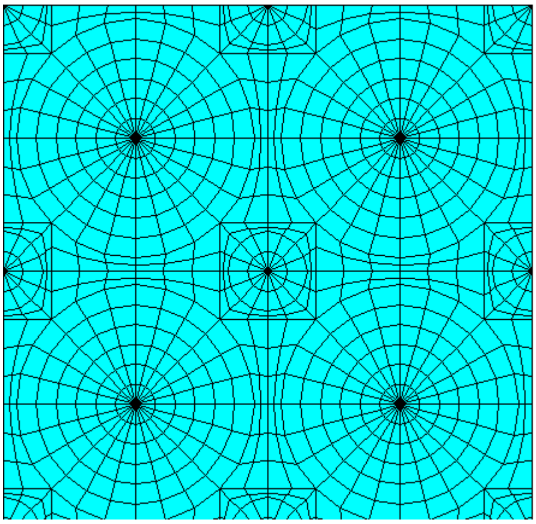

Fig.5. A cell of the improved finite element grid

This cell type has a two-level structure. The first level consists of “large blocks-circles”; in the center of these “circles” there are triangular elements which allow for trajectories of growing crack to deviate under influence of stress flow and accumulated damage in the part surrounding the crack tip. The second level consists of “small blocks-circles”; in the center of “circlets” there are also located triangular elements. The second level provides the possibility of growing crack to turn if it started to extend along the edge of a “large circle”.

By this, the proposed topology of the advanced finite-element grid for modeling the crack propagation would allow a trajectory to turn accordingly the local stress flow; the less wavy deviations are also expected. The two-level cell structure has to be better suited for the modeling fatigue crack morphology when the damage summation procedure is applied.

The modified mesh type was used to simulate behavior of initial micro-crack-like defects in material structure under cyclic loading. The defects and “material elements” (finite-elements) “fatigue properties” (coefficient C in Eq.2) were randomly distributed in a rectangular thin plate, as shown in Fig.6, a; cyclic load is applied in vertical direction.

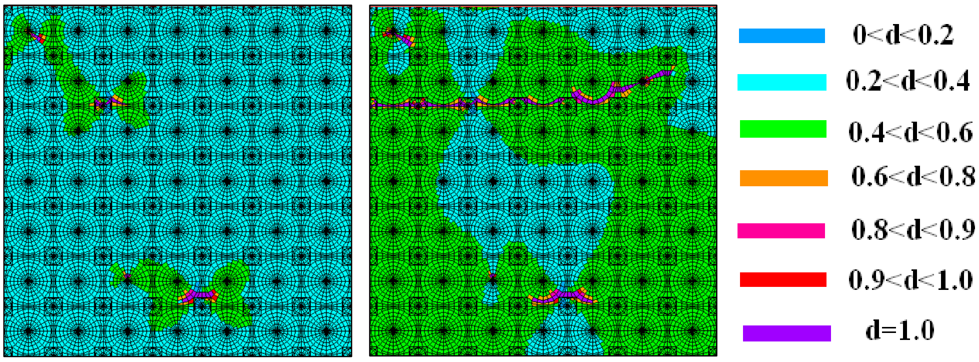

Fig.6. Development of defects in material structure into the propagating crack: (a) initial defect location; (b) “active’ defect turned into the propagating crack and “dormant” defects. Right hand – the damage intensity legend

The results given in Fig.7 show that of the defects assumed the only occurs “active” and grows into a propagating crack due to favorable combination of fatigue resistance of “material” elements surrounding the defects. The proposed mesh type makes it possible to trace the step-by-step damage accumulation, further formation of fracture nucleus near initial cracks/defects and their propagation.

The trajectory of crack initiated at the “active” defect, shown in Fig.6, in general, is controlled by the stress flow; its deviation from the straight line perpendicular to the loading direction may be explained by sensitivity to the progress of damages induced by the defect in the lower part of the plate. It is seen in this example that the influence of grid topology on the crack trajectory can be essentially reduced. Also, the modified meshing provides the better smoothness of the crack morphological features.

Verification of the developed grid consistency

The given in above examples may not be convincing in accuracy of the proposed approach. To add, a standard problem is considered to make comparison of stress intensity factors calculated:

- when the crack is simulated using the proposed approach
- when the regular mesh with singular elements is applied and
- obtained from the hand book sources, e.g., [4].

A rectangular plate with symmetrical edge cracks loaded in direction perpendicular to the crack planes is assumed. Fig.7 shows the geometry; due to the symmetry, a ¼ of the plate with respective boundary conditions is considered. A crack is located in the right bottom corner.

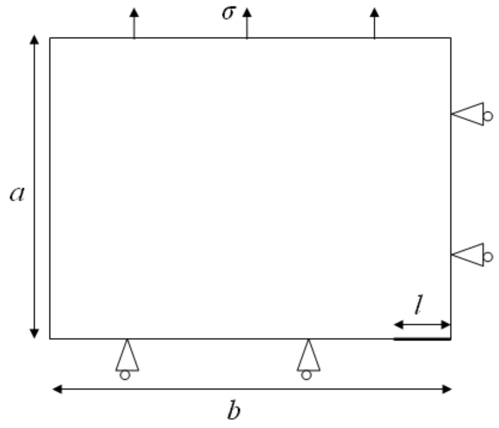

Fig.7. Edge crack propagation modeling

The plate dimensions are: 2a = 2b = 7*2l = 28 mm, initial crack length is: 2l = 4 mm, the size of grid cells is 2 mm; the plate is loaded by static stress σ = 1 MPa.

According to the handbook data [4] stress intensities ($K_I^{\grave{\imath}}$) for the plate with symmetrical cracks at the edges are given by:

$$K_I^{\grave{\imath}} = \sigma\sqrt{\pi l} \cdot F_I(\alpha), \quad \alpha = l/b, \quad F_I(\alpha) = (\sec(\alpha\pi/2))^{1/2}$$

When the proposed modified meshing is applied, the stress intensities, K_I^{grid}, can be estimated by using the extrapolation to the crack tip technique:

$$K_I^{grid} = \lim_{r\to 0} \sigma\sqrt{2\pi r}\,,$$

where r – the distance of the reference point from the crack tip, $\sigma = \sigma(r)$ is the (maximum principal) stress at the reference point range.

In case the plate with edge cracks is modeled by regular mesh with singular elements, stress intensities ($K_I^{\sin}$) are calculated by means of a procedure built in a software based on the method of nodal displacements. Respectively, the SIF values are as follows:

- handbook [4] data: $K_I^{\grave{I}} = 80.3\, kPa \cdot m^{1/2}$
- regular mesh with singular elements: $K_I^{\sin} = 81.6\, kPa \cdot m^{1/2}$
- proposed modified meshing technique: $K_I^{grid} = 78.5\, kPa \cdot m^{1/2}$

The example shows that relative difference of the results does not exceed 2.5% which can be regarded favorable for the proposed approach together with the assumed isotropic meshing.

Conclusion

A simple approach allowing for simulation of the fatigue crack initiation and growth in a non-uniform field of fatigue damages is developed based on application of the principles of FE meshing which provide realistic crack extensions independent of the mesh geometry.

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